CS46 Homework 11

This homework is due at 11:59pm on Tuesday, April 14. This is a 10-point homework.

You may work with one partner on this lab. Your write-up is your own: do not share it, and do not read other teams' write-ups. If you use any out-of-class references (anything except class notes, the textbook, or asking the instructor), then you **must** cite these in your post-lab survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main **learning goal** of this homework is to work with and think about Turing machines and decidability, and to start adding runtime considerations to the analysis of languages that we've been developing all semester.

1. Use the definition of big-O to prove that:

   (a) $2^n = O(5^n)$.

   (b) $n^2 \log n = O(n^3)$.

   (c) $\frac{1}{7} \cdot 3^n \neq O(n^2)$. (This should be a proof by contradiction.)

2. (Sipser 7.9) A **triangle** in a graph is three nodes that are all connected to each other by edges.

   (a) Describe a polynomial-time verifier V for TRIANGLE.

   (b) Show that TRIANGLE $\in$ P, where

   $$\text{TRIANGLE} = \{\langle G \rangle \mid G \text{ is a graph that contains a triangle}\}$$

   Refer back to chapter 0 for the details of the definition of a graph. You should be specific in your construction, as you will need to analyze the runtime of each line of your Turing machine.

   (c) A **quadrilateral** in a graph is four nodes (a, b, c, d) that are all connected by edges to form a quadrilateral, i.e., $(a, b), (b, c), (c, d)$, and (d, a) are edges in the graph. Is the language of graphs that contain a quadrilateral in P? Give a brief justification for your answer. It is not necessary to give a full proof like the previous two parts.