

# CS46 Homework 5

This lab assignment is due at 11:59PM on Tuesday, 24 February. Write your solution using  $\text{\LaTeX}$ . Submit this assignment using **github**. There are total of **15 points** for this lab.

You may work with one partner on this lab. Your write-up is your own: do not share it, and do not read other teams' write-ups. If you use any out-of-class references (anything except class notes, the textbook, or asking the instructor), then you **must** cite these in your post-lab survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main **learning goal** of this lab is to prove languages are (or are not) regular and to explore context free languages through context-free grammars.

## 1 Written problems

These problems (worth 5 points) should be typeset in  $\text{\LaTeX}$  and submitted using **github**.

1. Consider the following languages. For each, is the language regular? Support your claim with a proof.
  - (a)  $L_1 = \{a^k u a^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$  where  $\Sigma = \{a, b\}$ .
  - (b)  $L_2 = \{a^k b u a^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$  where  $\Sigma = \{a, b\}$ .
  - (c)  $L_3 = \{a^n b^m a^m b^n \mid m, n \geq 0\}$  where  $\Sigma = \{a, b\}$ .
  - (d)  $L_5 = \{w \mid w \text{ is not a palindrome}\}$  where  $\Sigma = \{a, b\}$ .
  - (e)  $L_6 = \{w \mid w = x_1 \# x_2 \# \cdots \# x_k \text{ for } k \geq 0, \text{ each } x_i \in L(a^*), \text{ and } x_i \neq x_j \text{ for } i \neq j\}$ , where  $\Sigma = \{a, \#\}$ .

2. Give a context-free grammar that generates the language

$$\{a^i b^j c^k \mid i = j \text{ or } j = k, \text{ where } i, j, k \geq 0\}$$

You do not have to give a proof of correctness, but you should think about what would be required to write a proof. This will help you plan and debug your grammar.

3. Consider the class of context-free languages. Let  $G_1$  and  $G_2$  be two context-free grammars.
  - (a) Show that  $L(G_1) \cup L(G_2)$  is context-free, so the class of context-free languages is closed under union. (Use a constructive proof — build a new grammar for the union language.)
  - (b) Show that  $L(G_1) \circ L(G_2)$  is context-free, so the class of context-free languages is closed under concatenation. (Use a constructive proof — build a new grammar for the concatenation language.)
  - (c) Show that  $L(G_1)^*$  is context-free, so the class of context-free languages is closed under Kleene star. (Use a constructive proof — build a new grammar for the Kleene star language.)