Name: YOUR NAME HERE

CS46 lab 3

This lab assignment is due at 11:59PM on Tuesday, 10 February. Write your solution using LaTeX. Submit this assignment using **github**. There are total of **10 points** for this lab.

This is an individual lab assignment. It's ok to discuss approaches at a high level. In fact, I encourage you to discuss general strategies. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. Your write-up is your own. If you use any out-of-class references (anything except class notes, the textbook, or asking the instructor), then you **must** cite these in your post-lab survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main **learning goal** of this lab is to develop the skills to design, understand, and analyze DFAs, and to think about the class of regular languages in general.

Part 1 — These problems should be completed[1] on Automata Tutor. You are allowed **three attempts** at each problem. I recommend that you *first* try to solve the problems on paper, *then* use the site to debug your solutions.

1. Construct a DFA for the language $\emptyset$ over alphabet $\Sigma = \{0, 1\}$.
2. Construct a DFA for the language $\{\varepsilon, 0\}$ over alphabet $\Sigma = \{0, 1\}$.
3. Construct a DFA for the language $\{w \mid w \text{ is either } a \text{ or } b\}$ over alphabet $\Sigma = \{a, b\}$.
4. Construct a DFA for the language $\{w \mid w \text{ is any string except } a \text{ or } b\}$ over alphabet $\Sigma = \{a, b\}$.
5. Construct a DFA for the language $\{w \mid w \text{ contains at least three 1s}\}$ over alphabet $\Sigma = \{0, 1\}$.
6. Construct a DFA for the language $\{w \mid \text{every } a \text{ in } w \text{ is immediately followed by a } b\}$ over alphabet $\Sigma = \{a, b\}$.
7. Construct a DFA for the language $\{w \mid b \text{ occurs } n \text{ times in } w, \text{ where } n \text{ is divisible by } 3\}$ over alphabet $\Sigma = \{a, b\}$.
8. Construct a DFA for the language $\{w \mid w \text{ contains at least two 0s and at most one 1}\}$ over alphabet $\Sigma = \{0, 1\}$.
9. Construct a DFA for the language $L = \{w \mid \text{ every odd position of } w = w_1 w_2 w_3 \dots w_n \text{ is a } 1\}$ over the alphabet $\Sigma = \{0, 1\}$.
10. Construct a DFA for the language $L = \{w \mid w \text{ is any non-empty string}\}$ over the alphabet $\Sigma = \{0, 1\}$.
11. Construct a DFA for the language $L = \{w \mid w \text{ begins and ends with the same symbol}\}$ over the alphabet $\Sigma = \{0, 1\}$. This language includes the empty string.

[1] If you want to use late days on this assignment, you will need to submit solutions to these problems via github. The automatatutor site has only one deadline.

Part 2 — These problems should be typeset in LaTeX and submitted using **github**.

12. Consider the language $C = \text{op}(A, B)$ where "op" is some operation that regular languages are closed under. Suppose we know the following about A and C. What, if anything, can we conclude about B?

    (You should support your answer with a brief explanation. Even though we have not yet seen any specific languages that are *not* regular, you can approach this problem using just the definition of "regular language" and "closed".)

    (a) A is regular and C is regular.

    (b) A is regular and C is not regular.

    (c) A is not regular and C is regular.

    (d) A is not regular and C is not regular.

13. We have shown in class that the class of regular languages is closed under union, intersection, concatenation, and star.

    (a) Show via direct proof that the set of regular languages is closed under the *complement* operation. Begin by assuming a language A is regular. Describe how to construct a machine $M = (Q, \Sigma, \delta, q_0, F)$ that recognizes $\overline{A}$. Define all elements of M and indicate if your constructed machine is a DFA or NFA.

    (b) Show that the set of regular languages is closed under set difference. That is, if A and B are regular languages, then $A \setminus B = \{w \mid w \in A \text{ and } w \notin B\}$, is also a regular language. You do not need to provide a formal description of the machine. Instead, use the closure properties you already know.

    (c) For a string $w = w_1 w_2 \dots w_n$, the *reverse* of w is the string $w^R = w_n w_{n-1} \dots w_1$. For a language A, the *reverse* of A is the language $A^R = \{w^R \mid w \in A\}$. Show that the set of regular languages is closed under the reverse operation. S