CS46, Swarthmore College, Spring 2026
Lab 1: due Tuesday, 27 January
Name: YOUR NAME HERE

This lab assignment is due at 11:59pm on Tuesday, 27 January. Write your solution using LaTeX. Submit this homework using **github**. This is a **10 point** assignment. This is an individual lab assignment. It's ok to discuss approaches at a high level. In fact, I encourage you to discuss general strategies. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. Your write-up is your own. If you use any out-of-class references (anything except class notes, the textbook, or asking the instructor), then you **must** cite these in your post-lab survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main **learning goal** of this assignment is to cement your understanding of mathematical concepts that we will be using throughout the semester. A secondary goal is to practice writing in LaTeX and submitting assignments via git. Please read Sipser Chapter 0 before starting this assignment.

1 Sets and Strings

1. Let A be a set of size three.
   (a) What is the size of the powerset of A?
   (b) What is the fewest number of elements in any element of 2^A?
   (c) What is the maximum number of elements in any element of 2^A?
2. Let $\Sigma = \{a, b, c\}$ be an alphabet over three letters.
   (a) How many unique strings w of length $|w| \leq 3$ can you make from Σ?
   (b) How does this compare to the number of elements in 2^Σ?
3. Let $A = \{a, b, c\}$ and $B = \{b, c, d\}$. Formally define the following sets by listing their elements:
   (a) $A \cap B$
   (b) $A \cup B$
   (c) $A \times B$

2 Relations and Functions

1. Review the definitions of reflexive, symmetric, and transitive on page 9 of the text before answering this question. Consider the following set of family members $A = \{$yourself, sister, mother, father's brother (uncle), paternal grandfather, maternal grandmother$\}$. Let $R \subseteq A \times A$ be the *ancestor* relation, meaning the ordered pair $(a, b) \in R$ if a is an ancestor of b.

   (a) List three ordered pairs from the set $A \times A$ that are in the relation R.

   (b) List three ordered pairs from the set $A \times A$ that are not in the relation R.

   (c) Is the ancestor relation symmetric? Explain.

   (d) Is the ancestor relation transitive? Explain.

2. A *natural isomorphism* between sets A and B is a simple bijective function, $f : A \mapsto B$, that can be considered a slight re-writing of elements from either set. For example, let $A = \{a, b, c\}$ and let $B = \{(a), (b), (c)\}$ be a set of singleton tuples. While there are many bijections between A and B, the one that maps $x \in A$ to $(x) \in B$ seems "natural". In addition to the term natural isomorphism, we define for two sets A and B the expression B^A to be the set of all functions from A to B. Let $B = \{0, 1\}$, and let $A = \{p, q, r\}$.

   (a) Give an example of one element of B^A.

   (b) Write the power set of A, 2^A.

   (c) Describe a natural isomorphism between B^A and 2^A.