1. Recall that every Turing-recognizable language $R$ has an enumerator $E$ that enumerators it.
   
   (a) (Sipser 3.19) Show that every infinite Turing-recognizable language $R$ has an infinite decidable subset $D$.
   
   (b) Is it possible for $R$ to contain only a finite number of pairwise disjoint infinite decidable subsets? Briefly explain your answer.

2. Given a grammar $G$, we say that a variable $V \in G$ is useless if there is no string $w$ for which a possible derivation of $w$ contains the variable $V$. Formulate this problem of finding grammars containing useless variables as a language and show that this language is decidable.

3. (Sipser 5.2) Show that $\text{EQ}_{\text{CFG}}$ is co-Turing-recognizable. I highly recommend non-determinism.

4. Consider the language $L = \{ \langle M, w \rangle \mid M$ is a single tape TM that never modifies the portion of the tape that contains the original input $w \}$.
   
   (a) Show that $L$ is co-Turing-recognizable, by briefly describing the elements of $\overline{L}$ and then describing a recognizer for $\overline{L}$.
   
   (b) Is $L$ decidable? Prove your answer. Note if you can show that $L$ is Turing-recognizable, you can apply Theorem 4.22 and part (a) to show $L$ is decidable.