1. (Sipser 3.16) Show that the class of Turing-recognizable languages are closed under the following operations. Note that the book gives a solution for part a) that uses a deterministic TM. For part a), you should provide an alternate solution that uses a non-deterministic TM. You may additionally use non-determinism for parts b-d if needed too.

   (a) Union
   (b) Concatenation
   (c) Kleene Star
   (d) Intersection

2. (Sipser 3.18) Before starting this problem, please read the definition of enumerator in section 3.2, and the proof of Theorem 3.21. Also note that an enumerator for a finite language will eventually halt. An enumerator for an infinite language will never halt, but it will eventually print another string after printing any string \( w \). With those notes in mind, show that a language is decidable if and only if some enumerator enumerates the strings in the language in short lexicographical order. Be sure to show both directions of the proof. Think about what parts of the proof Theorem 3.21 become easier/harder when talking about decidable languages.