CS46, Swarthmore College, Spring 2018
Lab 4 (due Wednesday 21 February)
Name: YOUR NAME(S) HERE

1. For each of the following languages, state if the language is regular or not regular. Support each claim with a proof.

   (a) $L_1 = \{w\overline{w} \mid \overline{w}$ is w with all as flipped to bs and all bs flipped to as$\}$ where $\Sigma = \{a, b\}$.

   (b) $L_2 = \{f(w) \mid f(w)$ is $w \in L$ with all bs flipped to as and all as flipped to bs$\}$ where L is some fixed regular language and $\Sigma = \{a, b\}$.

   (c) $L_3 = \{a^k u a^k \mid k \geq 1$ and $u \in \Sigma^*\}$ where $\Sigma = \{a, b\}$.

   (d) $L_4 = \{a^k b u a^k \mid k \geq 1$ and $u \in \Sigma^*\}$ where $\Sigma = \{a, b\}$.

   (e) $L_5 = \{w \mid w$ is not a palindrome $\}$ where $\Sigma = \{a, b\}$.

   (f) $L_6 = \{w \mid w = x_1\#x_2\#\cdots\#x_k$ for $k \geq 0$, each $x_i \in 1^*$, and $x_i \neq x_j$ for $i \neq j$, where $\Sigma = \{1, \#\}$.

2. Give a context-free grammar that generates the language

$$\{a^i b^j c^k d^l \mid i = j, j = k, \text{ or } k = l, \text{ where } i, j, k \geq 0\}$$

   You do not have to give a proof of correctness, but you should think about what would be required to write a proof. This will help you debug your grammar.

3. Regular expressions over an alphabet $\Sigma = \{a, b\}$ are just strings over another alphabet $\Sigma' = \{a, b, \emptyset, \varepsilon, \cup, \circ, *, (,)\}$. Define the language

$$L = \{w \mid w \text{ is a regular expression over } \Sigma\}.$$

   Show that L is context free.

4. Consider the class of context free languages

   (a) Using constructive proofs, build context free grammars that demonstrate the class of context free languages are closed under the regular operations of union, concatenation and Kleene star.

   (b) Theorem 1.25 proves that the class of regular languages is closed under intersection. Technically, it proves closure under union, but as the footnote in step 5 notes, a slight tweak makes this proof work for intersection too. Can a similar technique be applied to Pushdown Automata to show the set of context free languages are closed under intersection? Explain your answer briefly, but you do not need to give a full proof/counterargument.