Part 1: Automata Tutor

There is nothing to write in the \LaTeX{} document for the first part of this assignment. Submit your solutions online. If you are using a late day for this part, you must hand write your solutions or typeset your solutions in \LaTeX{} and/or graphviz dot notation. Hand written late solutions must be delivered to my office before 8am Friday to count as one late day.

Part 2: Written homework

1. For each of the following regular expressions over \( \Sigma = \{ a, b \} \), explain in English what language they describe. Briefly describe your thought process for arriving at your solution.

   (a) \( b^*a(b^*a^*)^*b(b^*a^*)^* \)
   (b) \( (b \cup abb^*)^*(a \cup \varepsilon) \)
   (c) \( (b(a \cup b))^*(b \cup \varepsilon) \)

2. Give regular expressions for the languages recognized by the machines \( M_1 \) and \( M_2 \) below. (Hint: use Lemma 1.60)

![Diagram](image.png)
3. Let $R$ and $S$ be regular expressions. Prove or disprove the following “identities”. To prove the identity, argue that a string on in the language defined on the left hand side is in the language defined on the right hand side, and vice versa. To disprove the identity, give a small counter example string with real examples of regular expressions for $R$ and $S$.

(a) $(R^*)^* = R^*$
(b) $(R \cup S)^* = R^* \cup S^*$
(c) $(R^* S^*)^* = (R \cup S)^*$

4. This problem has two parts.

(a) Using proof by contradiction and the pumping lemma for regular languages, show that the language $L_1 = \{ w \mid w = a^n b a^m, m > n \geq 0 \}$ is not regular.
(b) Show $L_2 = \{ w \mid w = a^n a^m, m > n \geq 0 \}$ is regular.