In lab exercises

1. Selection sort on a array of \( n \) items can be described recursively as follows: find the smallest element of the array, move it to the front, selection sort the rest of the array. Write a recurrence equation that describes the run time of selection sort and then solve the recurrence.

2. Solve

\[
T(n) = \begin{cases}
2T(\frac{n}{2}) + c_1 n^2 & \text{if } n > 1 \\
0 & \text{if } n = 1
\end{cases}
\]

3. Solve

\[
T(n) = \begin{cases}
2T(n-1) + n & \text{if } n > 1 \\
0 & \text{if } n = 1
\end{cases}
\]

4. Solve

\[
T(n) = \begin{cases}
T(n-1) + T(n-2) + 1 & \text{if } n > 2 \\
0 & \text{otherwise}
\end{cases}
\]

5. Solve

\[
T(n) = \begin{cases}
2T(\frac{n}{2}) + c_1 & \text{if } n > 4 \\
0 & \text{otherwise}
\end{cases}
\]

6. Solve

\[
T(n) = \begin{cases}
2T(\sqrt{n}) + \lg n & \text{if } n > 2 \\
0 & \text{otherwise}
\end{cases}
\]

This one is a bit evil. Start by letting \( m = \lg n \). Rewrite the recurrence in terms of \( m \). Now let \( S(m) = T(2^m) \) and rewrite the recurrence in terms of \( S(m) \). This recurrence should look familiar, solve it. Finally express your answer in terms of \( T \) and \( n \).

7. Liars and Friars

For fall break, you escape to a tropical island to get away from Philly weather and Algorithms class. The island is populated by \( n \) inhabitants, where each inhabitant is either a liar or a friar. A friar always tells the truth, but liars cannot be trusted. You want to find the best place to eat for dinner and you certainly don’t want to ask a liar (they will recommend bagel bar at Sharples, or some hipster coffee shop), so you would like to identify at least one friar. To help find a friar, you can pair up any two inhabitants \( A \) and \( B \) and ask each to identify the other. Each answers that the other is either a liar or a friar. If either \( A \) or \( B \) answers that the other is a liar, at least one of \( A \) and \( B \) is a liar. If both claim that the other is a friar, \( A \) and \( B \) are either both friars or both liars.

(a) Show if more than \( n/2 \) inhabitants are liars, you may not be able to identify a friar using this pairwise strategy. You may assume the liars can collude to convince you they are liars.

(b) Now assume that more than \( n/2 \) inhabitants are friars. Show that \( \lfloor n/2 \rfloor \) pairwise comparisons can reduce the problem of finding a single friar to a problem of nearly half the size. Describe how to find a single friar using this approach.

(c) Show how to find all friars, assuming there are more than \( n/2 \) using no more than \( O(n) \) pairwise comparisons. Give a recurrence which counts the number of comparisons and give a solution to this recurrence.