CS41 Fall 2012 Homework 5: Due in class Thursday October 11th.
You may work with one partner on this assignment. If you work together, you only need to submit one set of written solutions.

1. Solve
$$T(n) = \begin{cases} 2T(\frac{n}{2}) + c_1 & \text{if } n > 1 \\ c_2 & n = 1 \end{cases}$$

2. The recurrence in the previous problem looks similar to the recurrence for merge sort without merging. Unfortunately that version of merge sort doesn't actually sort. I claim the following algorithm does sort an array of size n: If $n = 1$, you are done. If $n = 2$, compare the two elements and swap if necessary. For $n \geq 3$, recursively sort the first 2/3 of the array, recursively sort the second 2/3 of the array (note there is some overlap in these two sub-arrays), and finally recursively sort the first 2/3 of the array again. Note there is no merge step in this algorithm.

   (a) Prove that this algorithm is correct in that it will always correctly sort an array of size $n \geq 1$. In each recursive call, interpret $\frac{2n}{3}$ as the ceiling $\lceil \frac{2n}{3} \rceil$.

   (b) Write a recurrence describing the run time of this sort algorithm

   (c) Solve the recurrence. Is this a viable replacement for merge sort?

   (d) Briefly consider using the values 3/5 or 3/4 instead of 2/3. How do they compare to the 2/3 solution? You do not have to formally write up you claims for this last part.

3. **Liars and Friars** For fall break, you escape to a tropical island to get away from Philly weather and Algorithms class. The island is populated by n inhabitants, where each inhabitant is either a liar or a friar. A friar always tells the truth, but liars cannot be trusted. You want to find the best place to eat for dinner and you certainly don't want to ask a liar (they will recommend bagel bar at Sharples, or some hipster coffee shop), so you would like to identify at least one friar. To help find a friar, you can pair up any two inhabitants A and B and ask each to identify the other. Each answers that the other is either a liar or a friar. If either A or B answers that the other is a liar, at least one of A and B is a liar. If both claim that the other is friar, A and B are either both friars or both liars.

   (a) Show if more than $n/2$ inhabitants are liars, you may not be able to identify a friar using this pairwise strategy. You may assume the liars can collude to convince you they are friars.

   (b) Now assume that more than $n/2$ inhabitants are friars. Show that $\lfloor n/2 \rfloor$ pairwise comparisons can reduce the problem of finding a single friar to a problem of nearly half the size. Describe how to find a single friar using this approach.

   (c) Show how to find all friars, assuming there are more than $n/2$ using no more than $O(n)$ pairwise comparisons. Give a recurrence which counts the number of comparisons and give a solution to this recurrence.