CS41 Fall 2012 Homework 4: Due in class Thursday October 4th.
You may work with one partner on this assignment. If you work together, you only need to submit one set of written solutions.

1. Consider the problem of making change for n cents using the fewest number of coins. Assume that n and the coin values are positive integers (cents).

   (a) Describe a greedy algorithm to solve the problem given the US coin denominations of quarters (25), dimes (10), nickels (5), and pennies (1). Prove that your algorithm is optimal.

   (b) Suppose the country of Algrithmistan uses denominations that are powers of c for some integer c. This country has $k+1$ coins with value $c^0, c^1, c^2, \ldots c^k$. Show that your greedy solution would also work in Algrithmistan.

   (c) Design a currency system of your choosing with at least three coin denominations such that a greedy solution does not yield an minimal number of coins for some amount of n cents. Assume one of your coins has a value of one, so a solution exists for all values of n

2. Suppose a weighted graph $G = (V, E)$, with n vertices, is *almost* a tree if it is (1) connected and (2) has at most n+5 edges. Give an algorithm that runs in $O(n)$ time that constructs a minimum spanning tree for G. You may assume all edge weights are distinct if needed.

3. Given a set $S = \{d_1, d_2, \ldots, d_n\}$ of n positive integers, consider the problem of creating an undirected graph $G = (V, E)$ with n vertices, whose degrees are given precisely by S, e.g., vertex v_i has degree d_i. The graph G should not have multiple edges between pairs of vertices, and should not have loop edges of the form $e = (v, v)$.

   (a) Draw a graph with four vertices where $S = \{3, 1, 2, 2\}$.

   (b) Argue that it is not possible to construct a graph G for the $S = \{3, 3, 1, 1\}$

   (c) Describe a polynomial time algorithm, $O(n^4)$ or better, that determines if it is possible to construct a graph G given S. If it is possible to construct the graph, you algorithm should return the edges of G. Hint: proceed inductively. Process one d_i, simplify S, and repeat.