# Kepler, Your Guide to Elliptical Orbits

Figure 1 shows an ellipse with semi-major axis $a$, semi-minor axis $b$ and eccentricity $\epsilon$. Eccentricity as defined as $\sqrt{(a^2 - b^2)/a^2}$. The figure also shows a planet moving around the sun in an elliptical orbit. The sun is located at one of the two foci of of the ellipse. Each focus is located a distance $\epsilon a$ away from the center of the ellipse. The position of the planet is given by $r$ and $\theta$ in relation to the sun (focus). To animate planets according to Kepler's laws, you will need to determine $r$ and $\theta$ as a function of time. The equation for $r(\theta)$ is relatively easy.

$$r(\theta) = \frac{a(1 - \epsilon^2)}{1 + \epsilon cos(\theta)}$$

If you can get $\theta(t)$, the rest is easy since $r(t) = r(\theta(t))$ and we already know $r(\theta)$. Unfortunately no easy and exact method exists for calculating $\theta(t)$ unless you consider elliptic integrals easy or exact.

Another approach is to use the differential form of the motion which says

$$\Delta\theta = k\ \Delta t / r^2,$$

where $k$ is a constant, $\Delta t$ is the time step and $r$ is the current radial position. The idea here is that we really don't care where the planet is at any given time, we just want to know "if it is at point $A$ now, where will it be in the next frame?". We can initialize a planet to be at a particular $\theta_0$ at time $t = 0$. From there, we can calculate $r(\theta)$ using the earlier equation. To animate a step we do the following iteration:

$$\begin{gathered} t_{i+1} \rightarrow t_i + \Delta t \\ \theta_{i+1} \rightarrow \theta_i + \Delta\theta_i = \theta_i + k\ \Delta t / r_i^2 \\ r_{i+1} \rightarrow r(\theta_{i+1}) \end{gathered}$$

This requires knowing, guessing, or calculating the constant $k$ which determines the period of the orbit. You can get a decent guess by holding $r$ constant at some average distance, say $(a + b)/2$ and calculating $k = 2\pi r^2 / T$ where $T$ is the period of orbit (e.g. 1 earth year).

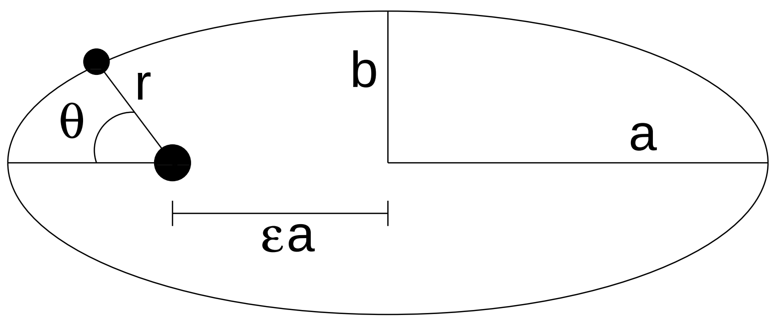


Figure 1: An ellipse

Using the techniques outlined above, you should be able to create elliptical orbits with the sun as a focus (Kepler law No.1) and make the planets move faster as they get closer to the sun (Kepler law No.2). Good luck and happy coding.