In typical labs this semester, you’ll be working on a number of problems in groups of 3-4 students each. You will not be handing in solutions. The primary purpose of these labs are to have a low-pressure space to discuss algorithm design, and to gain experience collaborating with others on algorithm design and analysis. However, it will be common to have some overlap between lab exercises and homework sets. There will typically be many more problems than you have time to complete during the lab. I encourage you to work on any problems your group wants, in any order.

1. First, get git set up. See the ”set up git” help page linked off the course website.

2. Clone the examples repo from the CS41-F22 github org.

   $ git clone git@github.swarthmore.edu:CS41-F22/examples.git

3. Using LearningLaTeX.tex as a guide, provide a writeup of the analysis of one of the Hiking Problem algorithms from class.

4. Induction

   Using induction, show that the following summations hold for all $n \geq 0$.

   \[
   \sum_{k=0}^{n} k = \frac{n(n + 1)}{2}.
   \]

   \[
   \sum_{k=0}^{n} 2^k = 2^{n+1} - 1.
   \]

   For all positive $c \neq 1$, \[
   \sum_{k=0}^{n} c^k = \frac{c^{n+1} - 1}{c - 1}.
   \]

5. Logarithmic Properties

   $\log_2(n)$ is the unique real number $x$ such that $2^x = n$. Show that the following properties hold for all positive real numbers $a, b$.

   \[
   \log_2(ab) = \log_2(a) + \log_2(b).
   \]

   \[
   - \log_2(a) = \log_2(1/a).
   \]

   \[
   \log_2(a^b) = b \log_2(a).
   \]

   \[
   a^{\log_2(b)} = b^{\log_2(a)}. \text{ (really)}
   \]
6. **Counterfeit Coins.** You are given $n$ coins and a balance scale. To use this scale, you put a number of coins in a pile on the left part of the scale, and a number of coins in a pile on the right. The scale indicates which pile is heavier.

Most of the coins are identical in every aspect; however, one of the coins is counterfeit and much heavier than the rest. Design an algorithm to identify the counterfeit coin that uses the scale at most $\log_3 n$ times.