CS41 Homework 10

This homework is due at 11:59PM on Wednesday, December 7. Write your solution using \LaTeX. Submit this homework in a file named hw10.tex using github.

This is a partnered homework. You should primarily be discussing problems with your homework partner. It’s ok to discuss approaches at a high level with others. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. The only exception to this rule is work you’ve done with a lab teammate while in lab. In this case, note (in your homework submission poll) who you’ve worked with and what parts were solved during lab.

1. In the Four-Coloring problem, the input is a graph $G = (V, E)$, and you should output yes iff the vertices in $G$ can be colored using at most four colors such that each edge $(u, v) \in E$ is bichromatic. Prove that Four-Coloring is NP-complete.

2. Path-Selection (K&T 8.9) Consider the following problem. You are managing a communication network, modeled by a directed graph $G = (V, E)$. There are $c$ users who are interested in making use of this network. User $i$ (for each $i = 1,\ldots,c$) issues a request to reserve a specific path $P_i$ in $G$ on which to transmit data.

You are interested in accepting as many of these path requests as possible, subject to the following restriction: if you accept both $P_i$ and $P_j$, then $P_i$ and $P_j$ cannot share any nodes.

Thus, the Path-Selection problem asks: Given a directed graph $G = (V, E)$, set of requests $P_1,\ldots,P_c$—each of which is a path in $G$, and a number $k$, output yes iff it is possible to select at least $k$ paths so that no two of the selected paths share any nodes.

Prove that Path-Selection is NP-complete.

3. Intersection-Inference (K&T 8.16) Consider the problem of reasoning about the identity of a set from the size of its intersections with other sets. You are given a finite set $U$ of size $n$, and a collection $A_1,\ldots,A_m \subseteq U$ of subsets of $U$. You are also given integers $c_1,\ldots,c_m$. The question is: does there exists $X \subseteq U$ such that for each $i = 1,2,\ldots,m$, the cardinality of $X \cap A_i$ equals $c_i$. We will call this an instance of the Intersection-Inference problem, with input $U,\{A_i\},\{c_i\}$.

Prove that Intersection-Inference is NP-complete, Hint: reduce from the following problem, which you may assume is NP-complete:

Problem One-In-Three-Sat:

Inputs: $n$ variables $x_1,\ldots,x_n$ and $m$ clauses $c_1,\ldots,c_m$ where each clauses is the or of three literals e.g., $c_i = (x_1 \lor \overline{x_2} \lor x_3)$.

Output: yes iff there is a truth assignment to the variables such that for each clauses there is exactly one satisfied variable.

Hint: Let $U$ be the set of literals. You'll have to work to ensure that a variable and its negation cannot both end up in $X$. 

1