

# CS91T Week 1 In-Lab Exercises

January 19, 2023

We will use labs in CS91T in several different ways:

- To introduce a bi-weekly lab assignment and get you started working on the assignment,
- To give one of the three tests this semester,
- To work on in-lab exercises.

This week we'll do in-lab exercise. When working on in-lab exercises, you'll be working on a number of problems in groups of 3-4 students each. You will not be handing in solutions. The primary purpose of in-lab exercises are to have a low-pressure space to discuss randomized algorithm design, and to gain experience collaborating with others on algorithm design and analysis.

The focus of lab this week is on asymptotic analysis and on review of quant skills you picked up from CS35 and other courses.

1. Clone the **examples** repo from the **CS91T-S23** github org.

```
$ git clone git@github.swarthmore.edu:CS91T-S23/examples.git
```

2. **Asymptotic Proofs.** Let  $f(n) = 6n^2 - 19n + 7$  and  $g(n) = \frac{n^3}{2} + 1$ . Prove that  $f(n) = O(g(n))$ .
3. **Asymptotic analysis.** Assume you have functions  $f$  and  $g$  such that  $f(n)$  is  $O(g(n))$ . For each of the following statements, decide whether you think it is true or false and give a proof or counterexample.

(a)  $f(n)^2$  is  $O((g(n))^2)$ .

(b)  $2^{f(n)}$  is  $O(2^{g(n)})$ .

(c)  $\log_2(f(n))$  is  $O(\log_2(g(n)))$ .

4. **Logarithmic Properties**

$\log_2(n)$  is the unique real number  $x$  such that  $2^x = n$ . Show that the following properties hold for all positive real numbers  $a, b$ .

- $\log_2(ab) = \log_2(a) + \log_2(b)$ .
- $-\log_2(a) = \log_2(1/a)$ .
- $\log_2(a^b) = b \log_2(a)$ .
- $a^{\log_2(b)} = b^{\log_2(a)}$ . (*really*)

## 5. Induction

Using induction, show that the following summations hold for all  $n \geq 0$ .

- $\sum_{k=0}^n k = \frac{n(n+1)}{2} .$

- $\sum_{k=0}^n 2^k = 2^{n+1} - 1 .$

- For all positive  $c \neq 1$ ,  $\sum_{k=0}^n c^k = \frac{c^{n+1} - 1}{c - 1}.$

6. Using `LearningLaTeX.tex` as a guide, provide a writeup of one or more of the proofs from lab.