CS46 Lab 8

Lab problems are an opportunity for discussion and trying many different solutions. They are **not counted towards your grade**, and **you do not have to submit your solutions.**

The purpose of these problems is to get more comfortable with reasoning and writing about Turing machines. You should be *practicing writing out descriptions and proofs* for your solutions to these problems. If you are stumped or looking for guidance, some of these problems are in the "selected solutions" portion of the textbook.

1. Give an **implementation-level** description of a Turing machine that decides the following language over the alphabet $\{0, 1\}$:

$$\{w \mid w \text{ contains twice as many 0s as 1s}\}$$

2. Show that ALL_{DFA} is decidable, where ALL_{DFA} is defined as:

$$ALL_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$$

3. Show that $INFINITE_{DFA}$ is decidable, where $INFINITE_{DFA}$ is defined as:

$$INFINITE_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) \text{ is an infinite language}\}$$

4. In lecture we saw that if an enumerator E enumerates a language A then there is a Turing Machine that recognizes A.

   Define a *shortlex enumerator* to be a Turing Machine with a printer that prints out strings in shortlex (i.e., short lexicographical) order. Suppose a shortlex enumerator enumerates a language A. Show that A is decidable by giving a **high-level description** of a Turing Machine that decides A.