CS46 Lab 5

In typical labs this semester, you'll be working on a number of problems in groups of 3-4 students each. Lab problems are an opportunity for discussion and trying many different solutions. They are **not counted towards your grade**, and **you do not have to submit your solutions.** The purpose of these problems is to get more comfortable with CFGs and PDAs.

Do not expect to complete all (or even half) of the problems on this lab! Use this time to practice with CFGs and PDAs and get more comfortable constructing them. If you are already comfortable with context-free grammars, focus on building PDAs. If you want more CFG practice, focus on that. Or just tackle the problems that look the most interesting!

1. Describe PDAs for the following languages you saw in lab 4:

   (a) $L_1 = \{w \in \{a, b\}^* \mid w \text{ contains more } a\text{s than } b\text{s}\}$

   (b) $L_2 = \{w\#x \mid w^R \text{ is a substring of } x, \text{ where } w, x \in \{0, 1\}^*\} \subseteq \{0, 1, \#\}^*$

   (c) $L_3 = \{w \mid w \text{ contains twice as many } \text{☺} \text{ s as } \text{☹} \text{ s}\}$ be a language over $\Sigma = \{\text{☺}, \text{☹}\}$.

2. Let $L_{\text{eq-len}} = \{x\#y \mid |x| = |y|\}$, where $x, y \in \{a, b\}^*$.

   (a) Design a grammar for $L_{\text{eq-len}}$.

   (b) Describe a PDA for $L_{\text{eq-len}}$.

3. Let $L_{\text{neq-len}} = \{x\#y \mid |x| \neq |y|\}$, where $x, y \in \{a, b\}^*$.

   (a) Design a grammar for $L_{\text{neq-len}}$.

   (b) Describe a PDA for $L_{\text{neq-len}}$.

4. Let $L_{\text{neq}} = \{x\#y \mid x \neq y\}$, where $x, y \in \{a, b\}^*$.

   (a) Design a grammar for L_{neq}.

   (b) Describe a PDA for L_{neq}.

   Hint: There are multiple ways of solving this problem, both for CFGs and for PDAs. Think about what kind of hard evidence the CFG or PDA might use to certify that $x \neq y$.

   Here is one possible approach:

   - First, design a grammar or PDA for the language $\{a^p b^q b^p a^q \mid p, q \geq 0\}$. (Hint: it should be pretty simple.)
   - Next, figure out how to break up L_{neq} into subsets. Some subsets you should already have solved. For the remaining subset, find a way to write it which looks a lot like the language above)

5. If A and B are languages, define $A \diamond B = \{xy \mid x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$.

   (a) Show that if A and B are regular languages, then $A \diamond B$ is a context-free language. You will probably want to describe a PDA instead of a grammar, and use nondeterminism. You do not need to give a completely formal definition of your PDA, but instead describe at a high level how it works.

(b) If A and B are regular languages, is $A \diamond B$ regular? If so, give a proof. If not, give an example of two regular languages A and B for which $A \diamond B$ is not regular, and prove your claim.

(c) **(challenge)** If A is regular and B is context-free, what can you say about $A \diamond B$? Is it regular? Is it context-free?