CS46 Lab 2

In typical labs this semester, you'll be working on a number of problems in groups of 3-4 students each. Lab problems are an opportunity for discussion and trying many different solutions. They are **not counted towards your grade**, and **you do not have to submit your solutions.**

The purpose of these problems is to get more comfortable with DFAs and with using the Automata Tutor site. You may want to try to solve these problems on paper *first*, then trying with the online tool. Once you are ready to test your solutions, the site will give you feedback on how to improve your solution. For each practice problem, you are allowed unlimited attempts.

1. For all of these problems, $\Sigma = \{a, b\}$.

   (a) Go to Automata Tutor and create a login (use your preferred first and last name, as we will use this tool for actual graded problems as well). Enroll in this course with:
   Course ID: CS46-S21
   Course Password: ODNG0142

   (b) Construct a DFA for the language $\{w \mid w \text{ contains the substring } ab\}$.

   (c) Construct a DFA for the language $\{w \mid w \text{ does } \textit{not} \text{ contain the substring } ab\}$.

   (d) Construct a DFA for the language $\{w \mid w \text{ contains the substring } baba\}$.

   (e) Construct a DFA for the language $\{w \mid w \text{ does } \textit{not} \text{ contain the substring } baba\}$.

   (f) Construct a DFA for the language $\{aa, abba\}$.
   You might consider breaking this problem into pieces:

      i. Construct a DFA for the language $\{aa\}$.
      ii. Construct a DFA for the language $\{abba\}$.
      iii. Use the proof idea from theorem 1.25 (regular languages are closed under union) to construct a new DFA for the union language from your two simpler DFAs.

   (g) Construct a DFA for the language $\{w \mid w \text{ contains exactly two } a\text{s and at least two } b\text{s}\}$.
   You might consider breaking this problem into pieces:

      i. Construct a DFA for the language $L_1 = \{w \mid w \text{ contains exactly two } a\text{s}\}$
      ii. Construct a DFA for the language $L_2 = \{w \mid w \text{ contains at least two } b\text{s}\}$.
      iii. We want to construct a DFA for $L_1 \cap L_2$, so we can use an idea like the footnote (page 46) on the proof of theorem 1.25 to construct the states and transitions for this new DFA.

   (h) Construct a DFA for the language $\{w \mid w \text{ begins with } a \text{ and ends with } b\}$.

   (i) Construct a DFA for the language $L = \{\varepsilon\}$ over the alphabet $\Sigma = \{a, b\}$.

   (j) Construct a DFA for the language $L = \{w \mid w \text{ does not contain exactly two } a\text{s}\}$ over the alphabet $\Sigma = \{a, b\}$.

   (k) Construct a DFA for the language $L = \{w \mid 3 \leq |w| \leq 5\}$ over the alphabet $\Sigma = \{a, b\}$.

   (l) Construct a DFA for the language $L = \{w \mid a \text{ appears } k \text{ times in } w \text{ where } k+1 \text{ is divisible by } 3\}$ over the alphabet $\Sigma = \{a, b\}$.

   (m) Construct a DFA for the language $L = \{w \mid \text{ every } b \text{ in } w \text{ is immediately followed by two } a\text{s}\}$ over the alphabet $\Sigma = \{a, b\}$.

(n) **Serious challenge.** With only 3 attempts: Construct a DFA for the language

$$L = \{w \mid w \text{ is a binary number equal to 1 mod 3}\}$$

over alphabet $\Sigma = \{0, 1\}$. (So $0 \notin L$, $1 \in L$, $100 \in L$, etc.)
(**Advice:** You absolutely need to figure this one out on paper first, or you will run out of attempts.)

2. We saw in class that $|\mathbb{N}| = |\mathbb{Z}|$ and That $|\mathbb{N}| \neq |\mathbb{R}|$, even though both $\mathbb{N}$ and $\mathbb{R}$ are infinite sets. In this problem, we will consider the set $\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z}\}$ of all rational numbers. We will argue that $\mathbb{Q}$ is countably infinite too.

   The idea is that we can "count" the elements of S according to their pairing with the elements of $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$. We will also use the term "enumerable" for this property (for the same reason: we can enumerate the elements of S).

   (a) Draw a Venn diagram of $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{R}$. Where does $\mathbb{Q}$ fit in this diagram?

   (b) Start by considering just the positive rational numbers $\mathbb{Q}^+ = \mathbb{Q} \cap \{x \mid x \geq 0\}$. Come up with a way to list the elements of $\mathbb{Q}^+$. Your list is allowed to have duplicates.
   Hint: The definition of $\mathbb{Q}$ says that every rational number $x \in \mathbb{Q}$ is representable as $\frac{a}{b}$ where $a, b \in \mathbb{Z}$. How can you use this to make sure your list includes every $x \in \mathbb{Q}$?

   (c) Remove duplicates from your list. (You don't need a rigorous description of how to do this, but you should consider how you would identify duplicated numbers and make sure that you don't eliminate some number completely from the list.)

   (d) Congratulations! Now that you have a list, you can set up your function $f : \mathbb{N} \to \mathbb{Q}^+$ and check that it is one-to-one and onto.

   (e) To finish the argument, explain how to extend our function f which shows that $\mathbb{Q}^+$ is countable to a function f' which shows that all of $\mathbb{Q}$ (including the negative rational numbers) is countable.
   Hint: We saw a "trick" for dealing with positive/negative numbers in the proof that $\mathbb{Z}$ is countable in class. Try a similar technique here.

3. Show that the union of countably infinitely many countably infinite sets is countably infinite.