CS46 Lab 10

Lab problems are an opportunity for discussion and trying many different solutions. They are **not counted towards your grade**, and **you do not have to submit your solutions.** The purpose of these lab problems is to get more comfortable with reasoning and writing about Turing machines, decidability, and recognizability, and particularly to get practice on mapping reductions.

1. **Mapping Reducibility** Show that $L_{\text{TM}} \leq_{\text{m}} E_{\text{TM}}$.

2. **Turing Machine Equality** In class we showed that $A_{\text{TM}} \leq_{\text{m}} EQ_{\text{TM}}$. Show that $A_{\text{TM}} \leq_{\text{m}} \overline{EQ}_{\text{TM}}$. Conclude that EQ_{TM} is neither recognizable or co-recognizable.

3. **Classifying languages.** For each of the following languages, is the language decidable? Turing-recognizable? co-Turing-recognizable?

   Provided an argument for your answers. (Give the deciders/recognizers that you claim exist, and show why they work; if they do not exist, then prove why not.)

   (a) $E_{\text{TM}} = \{\langle M \rangle \mid L(M) = \emptyset\}$

   (b) $HUNDRED_{\text{TM}} = \{\langle M, w\rangle \mid M$ never moves its head past the 100^{th} tape square during its computation on $w\}$

4. A **homomorphism** is a function $f : \Sigma \to \Gamma^*$ from one alphabet to strings over another alphabet. We extend f to operate on strings by defining $f(w) = f(w_1)f(w_2)\cdots f(w_n)$ where $w = w_1w_2\cdots w_n$ and each $w_i \in \Sigma$. We further extend f to operate on languages by defining $f(\epsilon) = \epsilon$ and $f(A) = \{f(w) \mid w \in A\}$, for any language A.

   (a) Show that the decidable languages are not closed under homomorphism. (That is, give an example language A and homomorphism f such that A is decidable, but $f(A)$ is not decidable.)

   (b) A homomorphism is called **nonerasing** if it never maps a character to ε. (Equivalently, $|f(\sigma)| \geq 1$ for all $\sigma \in \Sigma$.) Prove that the decidable languages are closed under nonerasing homomorphisms.