Lab problems are an opportunity for discussion and trying many different solutions. They are not counted towards your grade, and you do not have to submit your solutions. The purpose of these lab problems is to get more comfortable with reasoning and writing about Turing machines, decidability, and recognizability, and particularly to get practice on mapping reductions.

1. **Mapping Reducibility** Show that \( L_{TM} \leq_m E_{TM} \).

2. **Turing Machine Equality** In class we showed that \( A_{TM} \leq_m EQ_{TM} \). Show that \( A_{TM} \leq_m \overline{EQ_{TM}} \).
   Conclude that \( EQ_{TM} \) is neither recognizable or co-recognizable.

3. **Classifying languages.** For each of the following languages, is the language decidable? Turing-recognizable? co-Turing-recognizable?
   Provided an argument for your answers. (Give the deciders/recognizers that you claim exist, and show why they work; if they do not exist, then prove why not.)
   
   (a) \( E_{TM} = \{ \langle M \rangle \mid L(M) = \emptyset \} \)
   (b) \( HUNDRED_{TM} = \{ \langle M, w \rangle \mid M \text{ never moves its head past the } 100^{th} \text{ tape square during its computation on } w \} \)

4. A **homomorphism** is a function \( f : \Sigma \to \Gamma^* \) from one alphabet to strings over another alphabet. We extend \( f \) to operate on strings by defining \( f(w) = f(w_1)f(w_2)\cdots f(w_n) \) where \( w = w_1w_2\cdots w_n \) and each \( w_i \in \Sigma \). We further extend \( f \) to operate on languages by defining \( f(\epsilon) = \epsilon \) and \( f(A) = \{ f(w) \mid w \in A \} \), for any language \( A \).
   
   (a) Show that the decidable languages are not closed under homomorphism. (That is, give an example language \( A \) and homomorphism \( f \) such that \( A \) is decidable, but \( f(A) \) is not decidable.)
   
   (b) A homomorphism is called **nonerasing** if it never maps a character to \( \epsilon \). (Equivalently, \( |f(\sigma)| \geq 1 \) for all \( \sigma \in \Sigma \).) Prove that the decidable languages are closed under nonerasing homomorphisms.