CS46 Lab 1

In typical labs this semester, you'll be working on a number of problems in groups of 3-4 students each. Lab problems are an opportunity for discussion and trying many different solutions. They are **not counted towards your grade**, and **you do not have to submit your solutions.**

You are welcome to consider these problems in any order. The later problems require more discussion. The purpose of these problems is to get more comfortable with set notation, thinking about sets, making proof-like arguments, and pondering the mathematical mysteries of infinity and $\emptyset$.

1. True or false:
   (a) $\emptyset \in \emptyset$
   (b) $\emptyset \subseteq \emptyset$
   (c) $\emptyset \in \{\emptyset\}$
   (d) $\emptyset \subseteq \{\emptyset\}$
   (e) $\{\{\emptyset\}\} \subseteq \{\emptyset, \{\emptyset\}\}$
   (f) $\{\{\emptyset\}\} \subseteq \{\{\emptyset, \{\emptyset\}\}\}$

2. Set operations.
   (a) If A is a set of size k, how many elements are in the powerset of A? Explain your answer.
   (b) How many bit strings of length n are there? Explain your answer.
   (c) Let $A = \{a, b, c\}$ and $B = \{x, y, z, a\}$. What are the sets $A \cap B$, $A \cup B$, and $A \times B$? Formally describe them by listing their elements.
   (d) List the elements of the powerset of $A = \{a, b, \emptyset\}$.
   (e) Let $\Sigma = \{a, b\}$. List all strings over Σ of length 3 in short lexicographic order.

3. Consider two sets A and B. Using direct proof, show that

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

4. Find the problems in the following proofs.
   (a) *... in which we question the true nature of numbers...*

   Claim 1. *It turns out that* $1 = 0$.

   Proof. (directly)

Let x and y be any two non-zero numbers such that $x = y$. Then:

$x = y$	our starting assumption
$x^2 = x \cdot y$	multiplying by x on both sides
$x^2 - y^2 = xy - y^2$	subtracting y^2 from both sides
$(x + y) \cdot (x - y) = (x - y) \cdot y$	factoring
$(x + y) = y$	dividing by $x - y$ on both sides
$y + y = y$	substituting since $x = y$
$2y = y$	
$2 = 1$	since y was nonzero
$1 = 0$	subtracting 1 from both sides

□

(b) *This proof owes all credit to Professor Danner's cackling brilliance.*

Claim 2. *Linear search is $O(1)$ runtime.*

Proof. (by induction on n, the length of the list we are searching)

base case: If $n = 1$, then linear searching a list of length $n = 1$ takes 1 operation, which is $O(1)$.

inductive hypothesis: Assume that if we have a list of length k some constant, then linear search on that list takes $O(1)$ time.

inductive step: Consider a list of length $n = k + 1$. When we do linear search on it, we first do linear search on the list of length k that is the first "part" (the first k elements) of this list. This takes $O(1)$, according to the inductive hypothesis. If we haven't found the item we're looking for yet, then we do one more check to see if it is the last item in the list. This check takes $O(1)$.

Thus total runtime is $O(1) + O(1) = O(1)$. □