CS46 Homework 6

This homework is due at 11:59PM on Thursday, April 8. This homework has three parts; this \LaTeX file is only part 1.

For this homework, you will work with a partner. It’s ok to discuss approaches at a high level with other students, but most of your discussions should just be with your partner. Your partnership’s write-up is your own: do not share it, and do not read other teams’ write-ups. If you use any out-of-class references (anything except class notes, the textbook, or asking Joshua), then you must cite these in your post-homework survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main learning goals of this homework is to work with and think about the power of Turing Machines and to connect theory to practice by implementing a CFG recognizer and by designing a CFG-based Twitter bot.

Part 1. Your solutions to this part should be written using \LaTeX and submitted using github as a file called \texttt{hw6.tex}. Write clear and unambiguous Turing machine descriptions. Your explanation of why and how a Turing machine works should be separate from your description of that Turing machine. Give the Turing machine description first, then explain it separately.

1. Another Turing machine extension. Let’s consider an extension which gives a Turing machine tape which is infinite in both directions. We keep everything else the same, and specify that the input string is given on a tape which is blank everywhere else, with the read/write head on the first character of the input. So the starting configuration on input \( w \) in state \( q_0 \) looks like:

\[
\cdots \square q_0 w \square \cdots
\]

Notice that now the tape head can always move left, no matter where it is.

Prove that Turing machines with doubly infinite tape are no more powerful than usual Turing machines (that is, with singly-infinite tape). Your proof should be formal (so if you are building a Turing machine, you must give a description for how to build all of the transitions.)

2. Closure properties. Show that the collection of Turing-recognizable languages is closed under the operations of:

(a) union
(b) concatenation
(c) Kleene star
(d) intersection

Parts 2 and 3 of this homework go in other files. Make sure you have committed and pushed all your files when you submit.