# CS46 Homework 4

This homework is due at 11:59pm on Thursday March 11. Write your solution using LaTeX. Submit this homework using **github** as a file called **hw4.tex**. This is a **10 point** homework.

For this homework, you will work with a partner. It's ok to discuss approaches at a high level with other students, but most of your discussions should just be with your partner. Your partnership's write-up is your own: do not share it, and do not read other teams' write-ups. If you use any out-of-class references (anything except class notes, the textbook, or asking Joshua), then you **must** cite these in your post-homework survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main **learning goal** of this homework is to work with our tools for proving that languages are/are not regular, and designing CFGs. As always, we shall continue to improve our proof-writing, clarity, and organization skills.

**Note:** You must submit your solutions in a file named **hw4.tex**, and your submission must compile without errors using `pdflatex`. Any .pdf submissions will be ignored. Any .tex files not named hw3.tex, .tex files that don't compile, or not submitting a homework submussion poll will earn up to a **-0.5** point deduction.

1. Give a context-free grammar that generates the language

$$\{a^i b^j c^k \mid i = j \text{ or } j = k, \text{ where } i, j, k \geq 0\}$$

   You do not have to give a proof of correctness, but you should think about what would be required to write a proof. This will help you debug your grammar.

   **Hint:** What if the language was just $\{a^i b^j c^k | i = j, \text{ where } i, j, k \geq 0\}$? How would you build this CFG? What about if just $j = k$ instead of $i = j$?

2. Consider the class of context-free languages.

   (a) Show that the class of context-free languages is closed under union. Do this constructively by building a CFG that derives the union of two other CFGs.
   Argue that your construction does indeed accept the union of two other CFGs.

   (b) Constructively show that the class of context-free languages is closed under concatenation. Argue that your solution works.

   (c) Constructively show that the class of context-free languages is closed under Kleene star. Argue that your solution works.

3. Consider the following languages. For each, is the language regular? Support your claim with a proof.

   (a) $L_1 = \{a^k u a^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$ where $\Sigma = \{a, b\}$.

   (b) $L_2 = \{a^k b u a^k \mid k \geq 1 \text{ and } u \in \Sigma^*\}$ where $\Sigma = \{a, b\}$.

   (c) $L_3 = \{a^n b^m a^m b^n \mid m, n \geq 0\}$ where $\Sigma = \{a, b\}$.

   (d) $L_4 = \{a^{m-n} \mid \frac{m}{n} = 5\}$ where $\Sigma = \{a, b\}$.

   (e) $L_5 = \{w \mid w \text{ is not a palindrome}\}$ where $\Sigma = \{a, b\}$.

(f) $L_6 = \{w \mid w = x_1\#x_2\#\cdots\#x_k$ for $k \geq 0$, each $x_i \in L(a^*)$, and $x_i \neq x_j$ for $i \neq j\}$, where $\Sigma = \{a, \#\}$.

4. **(extra credit)** Consider the grammar $G$ with rules: $\left\{\begin{array}{ll} S & \to AS \mid \varepsilon \\ A & \to 0A \mid A1 \mid \varepsilon \end{array}\right.$

   (a) Show that $G$ is ambiguous.

   (b) Give an equivalent grammar to $G$ which is not ambiguous. (No proof is required, but you should explain why it's not ambiguous and why it generates exactly the same strings as $G$.)