CS46 Homework 2

This homework is due at 11:59pm on Thursday, February 25. Write your solution using \LaTeX. Submit this homework using github. This is a 10 point homework.

This is an individual homework. It’s ok to discuss approaches at a high level. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. Your write-up is your own. If you use any out-of-class references (anything except class notes, the textbook, or asking Joshua), then you must cite these in your post-homework survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main learning goal of this homework is to develop the skills to design, understand, and analyze DFAs and regular languages.

Part 1 — These problems should be completed on Automata Tutor. You are allowed three attempts at each problem. I recommend that you first try to solve the problems on paper, then use the site to debug your solutions.

1. Construct a DFA for the language $\emptyset$ over alphabet $\Sigma = \{0, 1\}$.
2. Construct a DFA for the language $\{\varepsilon, 0\}$ over alphabet $\Sigma = \{0, 1\}$.
3. Construct a DFA for the language $\{w \mid w \text{ is either } a \text{ or } b\}$ over alphabet $\Sigma = \{a, b\}$.
4. Construct a DFA for the language $\{w \mid w \text{ contains at least three } a\}$ over alphabet $\Sigma = \{a, b\}$.
5. Construct a DFA for the language $\{w \mid \text{every } b \text{ in } w \text{ is immediately followed by a } a\}$ over alphabet $\Sigma = \{a, b\}$.
6. Construct a DFA for the language $\{w \mid a \text{ an even number of times}\}$ over alphabet $\Sigma = \{a, b\}$.
7. Construct a DFA for the language $\{w \mid \text{length of } w \leq 4\}$ over alphabet $\Sigma = \{a, b\}$.
8. Construct a DFA for the language $\{w \mid w \text{ contains at least two } 1\text{s and at most one } 0\}$ over alphabet $\Sigma = \{0, 1\}$.
9. Construct a DFA for the language $L = \{w \mid \text{every odd position of } w = w_1w_2w_3\ldots w_n \text{ is a } 1\}$ over the alphabet $\Sigma = \{0, 1\}$.
10. Construct a DFA for the language $L = \{w \mid w \text{ is any non-empty string}\}$ over the alphabet $\Sigma = \{0, 1\}$.
11. Construct a DFA for the language $L = \{w \mid w \text{ begins and ends with the same symbol}\}$ over the alphabet $\Sigma = \{0, 1\}$. This language includes the empty string.
12. Extra credit. Construct a DFA for the language $L = \{w \mid w \text{ is a binary number equal to } 2 \text{ mod } 5\}$ over alphabet $\Sigma = \{0, 1\}$. (So $0 \not\in L$, $10 \in L$, $100 \not\in L$, $111 \in L$, etc.)

\footnote{If you want to use late days on this assignment, you will need to submit solutions to these problems via github. The automatatutor site has only one deadline.}
Part 2 — These problems should be typeset in \LaTeX and submitted using github.

13. Write a concise English description of the language recognized by DFA $M_1$.

![Figure 1: DFA $M_1$](image1)

14. Write a concise English description of the language recognized by DFA $M_2$.

![Figure 2: DFA $M_2$](image2)

15. Write a concise English description of the language recognized by DFA $M_3$.

![Figure 3: DFA $M_3$](image3)
16. Let $\Sigma = \{a, b, c, \ldots, z\}$. For any language $A \subseteq \Sigma^*$, let the super of $A$ be defined as:

$$\text{super}(A) = \{\text{super}w \mid w \in A\}$$

For example, if $A = \{\text{speed}, \text{duper}, \text{conducting}\}$, then

$$\text{super}(A) = \{\text{superspeed}, \text{superduper}, \text{superconducting}\}$$

Prove that the class of regular languages is closed under the "super" operator. (That is, prove that if $A$ is regular, then super($A$) is regular. You should describe how to construct a machine that recognizes super($A$), define all elements of your machine $M = (Q, \Sigma, \delta, q_0, F)$, and argue why this machine recognizes super($A$).)

17. (extra credit) For languages $A$ and $B$, let the perfect shuffle of $A$ and $B$ be the language:

$$\{w \mid w = \sigma_1 \gamma_1 \sigma_2 \gamma_2 \cdots \sigma_k \gamma_k \text{ where } \sigma_1 \cdots \sigma_k \in A \text{ and } \gamma_1 \cdots \gamma_k \in B \text{ and each } \sigma_i, \gamma_i \in \Sigma\}$$

Prove that the class of regular languages is closed under perfect shuffle.