# CS46 Homework 10

This homework is due at 11:59PM on Thursday, May 6. This is a **7 point** homework. For this homework, you will work with a partner. It's ok to discuss approaches at a high level with other students, but most of your discussions should just be with your partner. Your partnership's write-up is your own: do not share it, and do not read other teams' write-ups. If you use any out-of-class references (anything except class notes, the textbook, or asking Joshua), then you **must** cite these in your post-homework survey. Please refer to the course webpage or ask me any questions you have about this policy.

**Note:** You must submit your solutions in a file named **hw10.tex**, and your submission must compile without errors using `pdflatex`. Any .pdf submissions will be ignored. Any .tex files not named hw10.tex, .tex files that don't compile, or not submitting a post-homework survey will earn up to a **-0.5** point deduction.

1. (Sipser 7.9) A **triangle** in a graph is three nodes that are all connected to each other by edges. Show that TRIANGLE $\in$ P, where

$$\text{TRIANGLE} = \{\langle G \rangle \mid G \text{ contains a triangle}\}$$

2. **Closure properties.**

   (a) Prove that P is closed under concatenation.

   (b) Prove that NP is closed under union.

   **Note:** Your proofs do not need to be completely formal. High-level descriptions will suffice as long as they are clear and rigorous.

3. Give polynomial-time verifiers for the following problems:

   (a) THREE-COLOR. Say a graph $G$ is *three colorable* if the vertices of $G$ can be colored using one of three colors (say, RED, BLUE, or GREEN) such that each edge has different-colored endpoints.

   Let THREE-COLOR $= \{\langle G \rangle |$ G is a three colorable graph$\}$.

   (b) FACTOR. Let FACTOR $= \{\langle N, k \rangle |$ N, k are written in binary and N is divisible by d for some $1 < d \leq k\}$.

4. **Closure properties. (extra credit)**

   (a) Prove that P is closed under complement.

   (b) Prove that NP is closed under concatenation.

5. **(extra credit)** Does coNP = NP? Support your answer with a proof.