CS46 Homework 1

This homework is due at 11:59pm on Thursday, February 18.. Write your solution using LaTeX. Submit this homework using **github**. This is a **10 point** homework.

This is an individual homework. It's ok to discuss approaches at a high level. In fact, I encourage you to discuss general strategies. However, you should not reveal specific details of a solution, nor should you show your written solution to anyone else. Your write-up is your own. If you use any out-of-class references (anything except class notes, the textbook, or asking Joshua), then you **must** cite these in your post-homework survey. Please refer to the course webpage or ask me any questions you have about this policy.

The main **learning goal** of this homework is to cement your understanding of mathematical concepts that we will be using throughout the semester.

1. Give the cardinality of the following sets:
   (a) $\{a\}$
   (b) $\{\Smiley, \emptyset\}$
   (c) $\{a, b, c, \{b, \emptyset, \Laughey\}\}$
   (d) $\emptyset$
   (e) $\{\{\emptyset\}, \{a, \emptyset, \Neutral\}\}$

2. For each of these requirements, describe a relation (either formally or in English sentences) which is:
   (a) symmetric but not transitive.
   (b) transitive and reflexive but not symmetric.
   (c) reflexive, symmetric, and transitive.

   You can pick the domain of each relation, but make sure to specify it in your write-up.

3. Find the error in the following proof that all bears have the same color.

   Claim. *In any set of m bears, all bears are the same color.*

   Proof. (by induction on m)

   Base Case: ($m = 1$) In any set containing just one bear, all bears are clearly the same color.

   Induction Hypothesis: Assume that for all $n < m$ any set of n bears have the same color.

   Induction Step: (want to show the claim holds for $n = m$ as well) Let A be any set of m bears. Remove one bear (call it "Yogi")from this set, to obtain a set A_1 of $m - 1$ bears. By the induction hypothesis, all bears in A_1 have the same color. Now, replace Yogi with another bear (let's call him "Sparky") to get a new set of $m - 1$ bears A_2. By the induction hypothesis, all bears in A_2 have the same color too. Therefore, all bears in A must be the same color, and the proof is complete. $\square$

4. Let Σ be an alphabet (a set of letters). We define Σ^* as the set of all strings using letters from Σ. Let $\mathcal{C}$ be a collection of sets which are all subsets of Σ^*. We are given that $\Sigma^* \in \mathcal{C}$.

   Assume that $\mathcal{C}$ is closed under the operation set difference. (So if $A \in \mathcal{C}$ and $B \in \mathcal{C}$, then $A \backslash B \in \mathcal{C}$.)

   Using direct proof, show that:

   (a) If $A \in \mathcal{C}$, then $\bar{A} \in \mathcal{C}$. ($\mathcal{C}$ is closed under complement.)

   (b) If $A \in \mathcal{C}$ and $B \in \mathcal{C}$, then $A \cap B \in \mathcal{C}$. ($\mathcal{C}$ is closed under intersection.)

   (c) If $A \in \mathcal{C}$ and $B \in \mathcal{C}$, then $A \cup B \in \mathcal{C}$. ($\mathcal{C}$ is closed under union.)

5. **(extra credit)** Formally prove that $n^2 + n$ is divisible by 2 for all $n \in \mathbb{N}$.