

# Visualizing Recursion

Friday, April 17, 2020 2:41 PM

$$rprintList(L) = \begin{cases} \text{return if } len(L) = 0 \\ \text{print first item + call } rprintList(L[1:]) \text{ otherwise} \end{cases}$$

**EX**

$$rprintList([2, 4, 8]) = \text{print } 2 + rprintList([4, 8])$$

$$\qquad \qquad \qquad \text{print } 4 + rprintList([8])$$

$$\qquad \qquad \qquad \text{print } 8 + rprintList([])$$

base case  
⇒ return

Output : 2  
              4  
              8

$$sum(n) = \begin{cases} \text{return } \emptyset \text{ if } n == 0 \\ \text{return } n + sum(n-1) \text{ otherwise} \end{cases}$$

**EX**

$$sum(4) = 4 + \text{sum}(3)$$

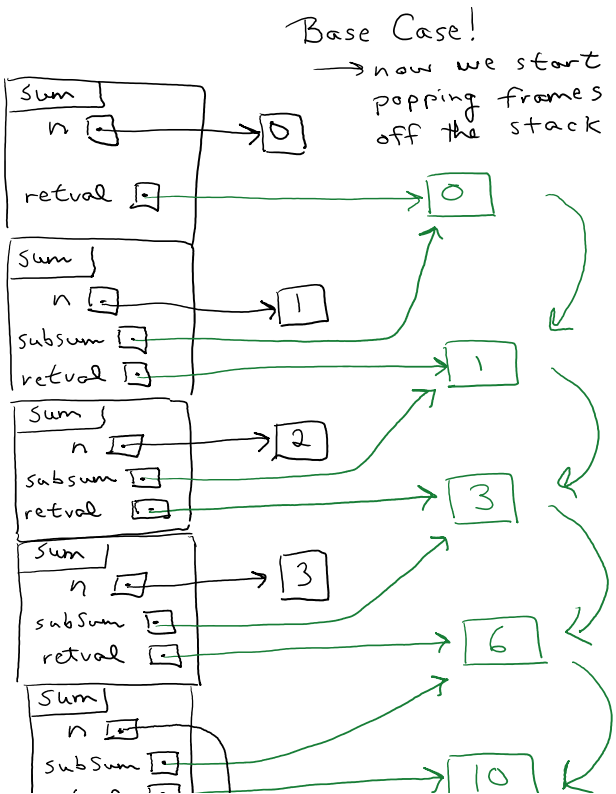
$$\qquad \qquad \qquad 3 + \text{sum}(2)$$

$$\qquad \qquad \qquad 2 + \text{sum}(1)$$

$$\qquad \qquad \qquad 1 + \text{sum}(0)$$

base case  
⇒ return

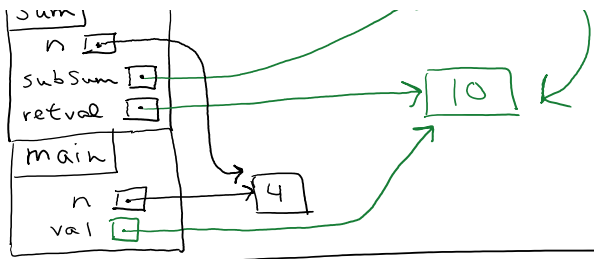
Stack Diagram for recursive sum(4):



note: Python re-uses values on the heap! but I keep them separate for clarity

note: subsum gets the return value from the last call to sum

note: retval is abbreviation for "return value"



"return value"

$$\text{mulList}(L) = \begin{cases} 1 & \text{if } \text{len}(L) == \emptyset \\ L[0] * \text{mulList}(L[1:]) & \text{otherwise} \end{cases}$$

EX  $\text{mulList}([2, 5, \frac{1}{2}]) = 2 * \text{mulList}([5, \frac{1}{2}]) = 5 * \text{mulList}([\frac{1}{2}]) = \frac{1}{2} * \text{mulList}([\ ])$

Annotations: 5.0, 2.5, 1/2, 1.0 (base case)

$$\text{allEven}(L) = \begin{cases} \text{True} & \text{if } L = [\ ] \\ L[0] \text{ is even AND } \text{allEven}(L[1:]) & \end{cases}$$

EX  $\text{allEven}([0, 2, 3]) = (\emptyset \text{ is even}) \text{ AND } (\text{allEven}([2, 3]) = (2 \text{ is even}) \text{ AND } (\text{allEven}([3]) = (3 \text{ is even}) \text{ AND } \text{allEven}([\ ])))$

Annotations: F, T, F, F, T, True (base case)

$$\text{countLetter}(\text{text}, \text{letter}) = \begin{cases} \emptyset & \text{if } \text{text} == "" \\ 1 + \text{countLetter}(\text{text}[1:], \text{letter}) & \text{if } \text{text}[0] == \text{letter} \\ \text{countLetter}(\text{text}[1:], \text{letter}) & \text{if } \text{text}[0] \neq \text{letter} \end{cases}$$

EX  $\text{countLetter}(\text{"hello"}, \text{"l"}) = \emptyset + \text{countLetter}(\text{"ello"}, \text{"l"}) = \emptyset + \text{countLetter}(\text{"llo"}, \text{"l"}) = 1 + \text{countLetter}(\text{"lo"}, \text{"l"}) = 1 + \text{countLetter}(\text{"o"}, \text{"l"}) = \emptyset + \text{countLetter}(\text{"", \text{"l"}) = \emptyset$

Annotations: 2, 2, 2, 1, 0, 0 (base case)

$$\text{badpassword}(\text{text}) = \begin{cases} "" & \text{if } \text{text} == "" \\ "3" + \text{badpassword}(\text{text}[1:]) & \text{if } \text{text}[0] == "e" \\ \dots + \text{badpassword}(\text{text}[1:]) & \text{if } \text{text}[0] \neq "e" \end{cases}$$

$$\text{badpassword}(\text{text}) = \begin{cases} \text{""} + \text{return} & \text{if } \text{text} == \text{"e"} \\ \text{"3"} + \text{badpassword}(\text{text}[1:]) & \text{if } \text{text}[0] == \text{"e"} \\ \text{text}[0] + \text{badpassword}(\text{text}[1:]) & \text{if } \text{text}[0] \neq \text{"e"} \end{cases}$$

EX

$$\text{badpassword}(\text{"fee"}) = \text{"f"} + \text{badpassword}(\text{"ee"})$$

$$\qquad \qquad \qquad \qquad \qquad \qquad \text{"3"} + \text{badpassword}(\text{"e"})$$

$$\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \text{"3"} + \text{badpassword}(\text{""})$$

$$\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \text{""} \text{ (base case)}$$