CS 31: Intro to Systems C Programming

L03: Data representation

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Announcements

- HW1 is due Thursday before class
  - up to groups of four
  - invitations sent from gradescope

- Lab 1 is due Thursday, 11.59 PM

- Clickers will count for credit from this week

Reading Quiz

- Note the red border!
- 1 minute per question
- No talking, no laptops, phones during the quiz

<u>Check your frequency:</u>

- Iclicker2: frequency AA
- Iclicker+: green light next to selection

For new devices this should be okay,
For used you may need to reset frequency

Reset:
1. hold down power button until blue light flashes (2secs)
2. Press the frequency code: AA vote status light will indicate success

Agenda

Data representation

- number systems + conversion
- data types, storage
- sizes, representation
- signedness

Abstraction

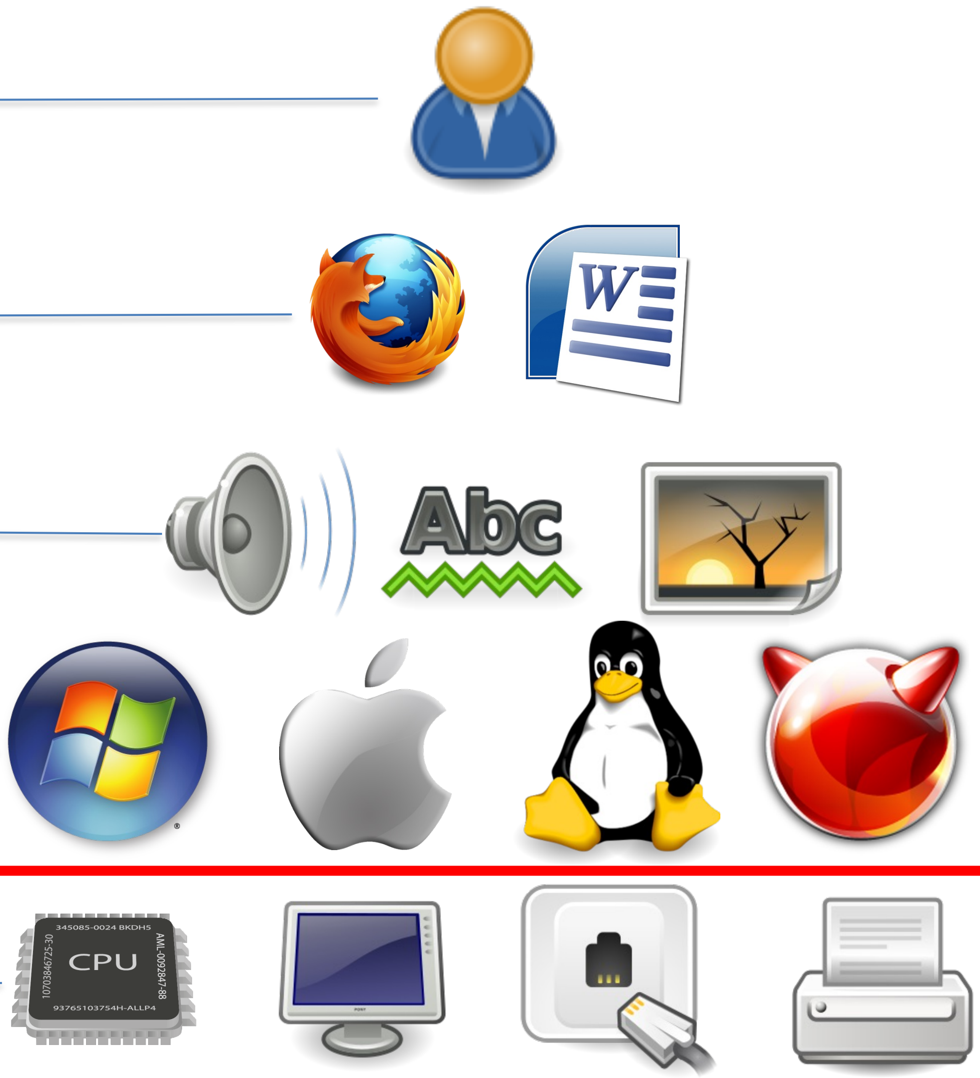
User / Programmer
Wants low complexity

Applications
Specific functionality

Software library
Reusable functionality

Operating system
Manage resources

Complex devices
Compute & I/O



Data Storage

- Lots of technologies out there:
  - Magnetic (hard drive, floppy disk)
  - Optical (CD / DVD / Blu-Ray)
  - Electronic (RAM, registers, …)
- Focus on electronic for now
  - We'll see (and build) digital circuits soon
- Relatively easy to differentiate two states
  - Voltage present
  - Voltage absent

Bits and Bytes

- Bit: a 0 or 1 value (binary)
  - HW represents as two different voltages
    - 1: the presence of voltage (high voltage)
    - 0: the absence of voltage (low voltage)

- Byte: 8 bits, the smallest addressable unit

Memory:	01010101	10101010	00001111	…
(address)	[0]	[1]	[2]	…

- Other names:
  - 4 bits: Nibble
  - “Word”: Depends on system, often 4 bytes

Files

Sequence of bytes… nothing more, nothing less

Binary Digits (BITs)

- One bit: two values (0 or 1)
- Two bits: four values (00, 01, 10, or 11)
- Three bits: eight values (000, 001, …, 110, 111)

How many unique values can we represent with 9 bits? Why?

- One bit: two values (0 or 1)
- Two bits: four values (00, 01, 10, or 11)
- Three bits: eight values (000, 001, …, 110, 111)

A. 18
B. 81
C. 256
D. 512
E. Some other number of values.

How many unique values can we represent with 9 bits? Why?

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- Two bits: four values (00, 01, 10, or 11)
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A. 18
B. 81
C. 256
D. 512
E. Some other number of values.

How many values?

1 bit: 0 1

How many values?

1 bit: $\underline{0}$ $\underline{1}$

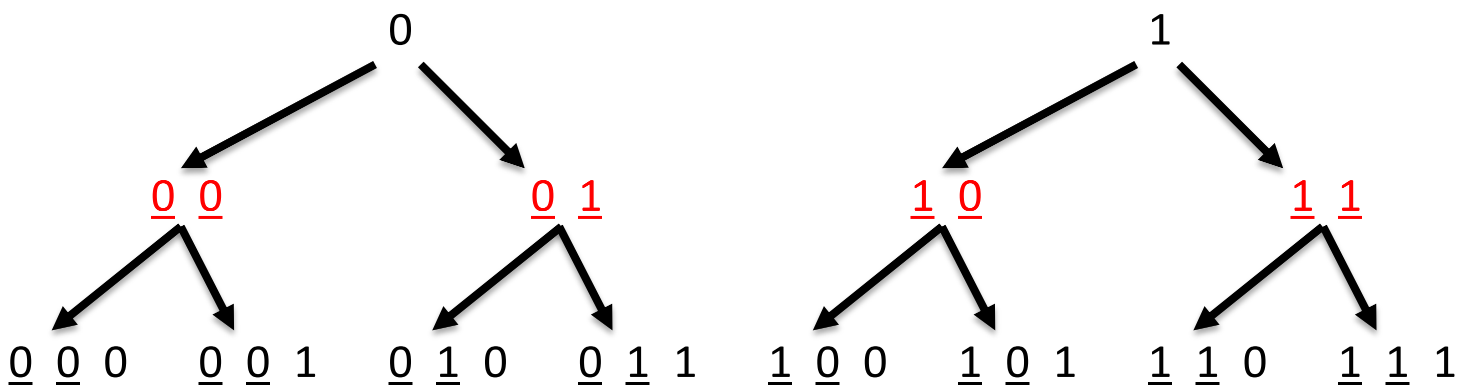
2 bits: $\underline{0}\ 0$ $\underline{0}\ 1$ $\underline{1}\ 0$ $\underline{1}\ 1$

How many values?

1 bit:

2 bits:

3 bits:



How many values?

1 bit: 0 1

2 bits: 0 0 0 1 1 0 1 1

3 bits: 0 0 0 0 0 1 0 1 0 0 1 1 1 0 0 1 0 1 1 1 0 1 1 1

4 bits:

0 0 0 0	0 0 0 1	0 0 1 0	0 0 1 1	16 values
0 1 0 0	0 1 0 1	0 1 1 0	0 1 1 1	
1 0 0 0	1 0 0 1	1 0 1 0	1 0 1 1	
1 1 0 0	1 1 0 1	1 1 1 0	1 1 1 1	

N bits: 2^N values

C types and their (typical!) sizes

- `1 byte:  char, unsigned char`
- `2 bytes: short, unsigned short`
- `4 bytes: int, unsigned int, float`
- `8 bytes: long long, unsigned long long, double`
- `4 or 8 bytes: long,`

```
unsigned long v1;
short s1;
long long ll;
```

WARNING: These sizes are **NOT** a guarantee. Don't always assume that every system will use these values!

```
// prints out number of bytes
printf("%lu %lu %lu\n", sizeof(v1), sizeof(s1), sizeof(ll));
```

How do we use this storage space (bits) to represent a value?

Let's start with what we know…

- Digits 0-9
- Positional numbering
- Digits are composed to make larger numbers
- Known as Base 10 representation

Decimal number system (Base 10)

- Sequence of digits in range [0, 9]

64025

Digit #0: 1’s place, “least significant digit”

Digit #1: 10’s place

Digit #4: “most significant digit”

Decimal: Base 10

A number, written as the sequence of N digits,

$$d_{n-1} \ldots d_2\, d_1\, d_0$$

where d is in {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, represents the value:

$$[d_{n-1} * 10^{n-1}] + [d_{n-2} * 10^{n-2}] + \ldots + [d_1 * 10^1] + [d_0 * 10^0]$$

64025 =

$6 * 10^4 \quad + \quad 4 * 10^3 \quad + \quad 0 * 10^2 \quad + \quad 2 * 10^1 \quad + \quad 5 * 10^0$

60000 + 4000 + 0 + 20 + 5

Binary: Base 2

- Used by computers to store digital values.

- Indicated by prefixing number with **0b**

- A number, written as the sequence of N digits, $d_{n-1}\ldots d_2 d_1 d_0$, where d is in {0,1}, represents the value:

$$[d_{n-1} * 2^{n-1}] + [d_{n-2} * 2^{n-2}] + \ldots + [d_2 * 2^2] + [d_1 * 2^1] + [d_0 * 2^0]$$

Converting Binary to Decimal

Most significant bit → $\underline{1}000111\underline{1}$ ← Least significant bit

7 6 5 4 3 2 1 0

Representation: $1 \times 2^7 + 0 \times 2^6 \ldots + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$128 + \quad + 8 + 4 + 2 + 1$

10001111 = 143

Hexadecimal: Base 16

Indicated by prefixing number with **0x**

A number, written as the sequence of N digits,

$$d_{n-1}\ldots d_2 d_1 d_0,$$

where d is in {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}, represents:

$$[d_{n-1} * 16^{n-1}] + [d_{n-2} * 16^{n-2}] + \ldots + [d_2 * 16^2] + [d_1 * 16^1] + [d_0 * 16^0]$$

Generalizing: Base b

The meaning of a digit depends on its position in a number.

A number, written as the sequence of N digits,

$$d_{n-1} \ldots d_2\, d_1\, d_0$$

in base b represents the value:

$$[d_{n-1} * b^{n-1}] + [d_{n-2} * b^{n-2}] + \ldots + [d_2 * b^2] + [d_1 * b^1] + [d_0 * b^0]$$

Base 10: $[d_{n-1} * 10^{n-1}] + [d_{n-2} * 10^{n-2}] + \ldots + [d_1 * 10^1] + [d_0 * 10^0]$

Other (common) number systems.

- Base 2: How data is stored in hardware.
- Base 8: Used to represent file permissions.
- Base 10: Preferred by people.
- Base 16: Convenient for representing memory addresses.
- Base 64: Commonly used on the Internet, (e.g. email attachments).

It’s **all** stored as binary in the computer.

Different representations (or visualizations) of the same information!

What is the value of 0b110101 in decimal?

A number, written as the sequence of N digits $d_{n-1}...d_2d_1d_0$ where d is in {0,1}, represents the value:

$$[d_{n-1} * 2^{n-1}] + [d_{n-2} * 2^{n-2}] + ... + [d_2 * 2^2] + [d_1 * 2^1] + [d_0 * 2^0]$$

A. 26
B. 53
C. 61
D. 106
E. 128

What is the value of 0b110101 in decimal?

A number, written as the sequence of N digits $d_{n-1}...d_2d_1d_0$ where d is in {0,1}, represents the value:

$$[d_{n-1} * 2^{n-1}] + [d_{n-2} * 2^{n-2}] + ... + [d_2 * 2^2] + [d_1 * 2^1] + [d_0 * 2^0]$$

A. 26
B. 53
C. 61
D. 106
E. 128

What is the value of 0x1B7 in decimal?

$[d_{n-1} * 16^{n-1}] + [d_{n-2} * 16^{n-2}] + \ldots + [d_2 * 16^2] + [d_1 * 16^1] + [d_0 * 16^0]$

(Note: $16^2 = 256$)

A. 397
B. 409
C. 419
D. 437
E. 439

DEC	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
HEX	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

What is the value of 0x1B7 in decimal?

$[d_{n-1} * 16^{n-1}] + [d_{n-2} * 16^{n-2}] + \ldots + [d_2 * 16^2] + [d_1 * 16^1] + [d_0 * 16^0]$

(Note: $16^2 = 256$)

A. 397
B. 409
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D. 437
E. 439

$1*16^2 + 11*16^1 + 7*16^0 =$

$256 + 176 + 7 = \underline{439}$

DEC	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
HEX	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Important Point…

- You can represent the same value in a variety of number systems or bases.

- It’s **all** stored as binary in the computer.
  - Presence/absence of voltage.

Hexadecimal: Base 16

- Fewer digits to represent same value
  - Same amount of information!

- Like binary, the base is power of 2

- Each digit is a “nibble”, or half a byte.

Each hex digit is a “nibble”

- One hex digit: 16 possible values (0-9, A-F)
- $16 = 2^4$, so each hex digit has exactly four bits worth of information.
- We can map each hex digit to a four-bit binary value.
  (helps for converting between bases)

Each hex digit is a “nibble”

Example value: 0x1B7

Four-bit value: 1

Four-bit value: B (decimal 11)

Four-bit value: 7

In binary: 0001 1011 0111

1 B 7

Converting Decimal -> Binary

- Two methods:
  - division by two remainder
  - powers of two and subtraction

Method 1: decimal value D, binary result b (b_i is ith digit):

```
i = 0
while (D > 0)
   if D is odd
          set b_i to 1
   if D is even
          set b_i to 0
   i++
   D = D/2
```

Example: Converting 105

idea: example: D = 105 $b_0 = 1$

Method 1: decimal value D, binary result b (b_i is ith digit):

```
i = 0
while (D > 0)
   if D is odd
         set b_i to 1
   if D is even
         set b_i to 0
   i++
   D = D/2
```

Example: Converting 105

```
idea:     D              example: D = 105        b0 = 1
          D = D/2                 D =  52        b1 = 0
```

Method 1: decimal value D, binary result b (b_i is ith digit):

```
i = 0
while (D > 0)
   if D is odd
          set b_i to 1
   if D is even
          set b_i to 0
   i++
   D = D/2
```

Example: Converting 105

```
idea:     D              example: D = 105       b0 = 1
          D = D/2                 D =  52       b1 = 0
          D = D/2                 D =  26       b2 = 0
          D = D/2                 D =  13       b3 = 1
          D = D/2                 D =   6       b4 = 0
          D = D/2                 D =   3       b5 = 1
          D = D/2                 D =   1       b6 = 1
          D = 0 (done)            D =   0       b7 = 0

                                       105   =   01101001
```

Method 2

- $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$
-

  To convert <u>105</u>:
  - Find largest power of two that's less than 105 (64)
  - Subtract 64 (105 – 64 = <u>41</u>), put a 1 in d_6
  - Subtract 32 (41 – 32 = <u>9</u>), put a 1 in d_5
  - Skip 16, it's larger than 9, put a 0 in d_4
  - Subtract 8 (9 – 8 = <u>1</u>), put a 1 in d_3
  - Skip 4 and 2, put a 0 in d_2 and d_1
  - Subtract 1 (1 – 1 = <u>0</u>), put a 1 in d_0 (Done)

1	1	0	1	0	0	1
d_6	d_5	d_4	d_3	d_2	d_1	d_0

What is the value of 357 in binary?

8 7654 3210

→ digit position

A. 1 0110 0011
B. 1 0110 0101
C. 1 0110 1001
D. 1 0111 0101
E. 1 1010 0101

$2^0 = 1, \quad 2^1 = 2, \quad 2^2 = 4, \quad 2^3 = 8, \quad 2^4 = 16,$

$2^5 = 32, \quad 2^6 = 64, \quad 2^7 = 128, \quad 2^8 = 256$

What is the value of 357 in binary?

8 7654 3210

→ digit position

A. 1 0110 0011
B. 1 0110 0101
C. 1 0110 1001
D. 1 0111 0101
E. 1 1010 0101

357 – 256 = 101
101 – 64 = 37
37 – 32 = 5
5 – 4 = 1

$\underline{1}$	$\underline{0}$	$\underline{1}$	$\underline{1}$	$\underline{0}$	$\underline{0}$	$\underline{1}$	$\underline{0}$	$\underline{1}$
d_8	d_7	d_6	d_5	d_4	d_3	d_2	d_1	d_0

$2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$,

$2^5 = 32$, $2^6 = 64$, $2^7 = 128$, $2^8 = 256$

So far: Unsigned Integers

<u>With N bits, can represent values: 0 to 2^n-1</u>

We can always add 0's to the front of a number without changing it:

10110= <u>0</u>10110 = <u>000</u>10110 = <u>00000</u>10110

So far: Unsigned Integers

With N bits, can represent values: 0 to 2^n-1

- `1 byte:  char, unsigned char`
- `2 bytes: short, unsigned short`
- `4 bytes: int, unsigned int, float`
- `8 bytes: long long, unsigned long long, double`
- `4 or 8 bytes: long, unsigned long`

Unsigned Integers

- Suppose we had <u>one byte</u>
  - Can represent 2^8 (256) values
  - If unsigned (strictly non-negative): 0 – 255

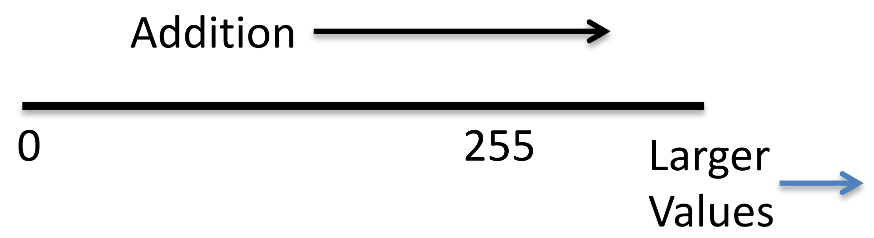
252 = 11111100

253 = 11111101

254 = 11111110

255 = 11111111

Traditional number line:



Unsigned Integers

Suppose we had one byte

- Can represent 2^8 (256) values
- If unsigned (strictly non-negative): 0 – 255

252 = 11111100

253 = 11111101

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255 = 11111111

What if we add one more?

Car odometer “rolls over”.

Any time we are dealing with a finite storage space we cannot represent an infinite number of values!

Unsigned Integers

Suppose we had one byte

- Can represent 2^8 (256) values
- If unsigned (strictly non-negative):

0 – 255

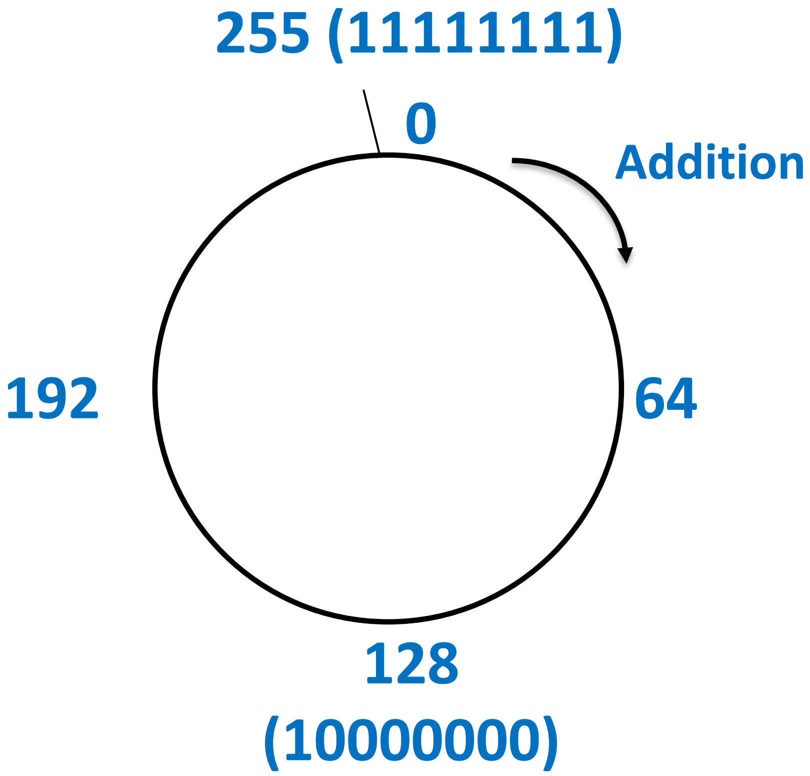
252 = 11111100

253 = 11111101

254 = 11111110

255 = 11111111

<u>What if we add one more?</u>



Modular arithmetic: Here, all values are modulo 256.

Unsigned Addition (4-bit)

- Addition works like grade school addition:

```
   1
   0110      6
+  0100   +  4
--------  -----
   1010     10
```

Four bits give us range: 0 - 15

Unsigned Addition (4-bit)

- Addition works like grade school addition:

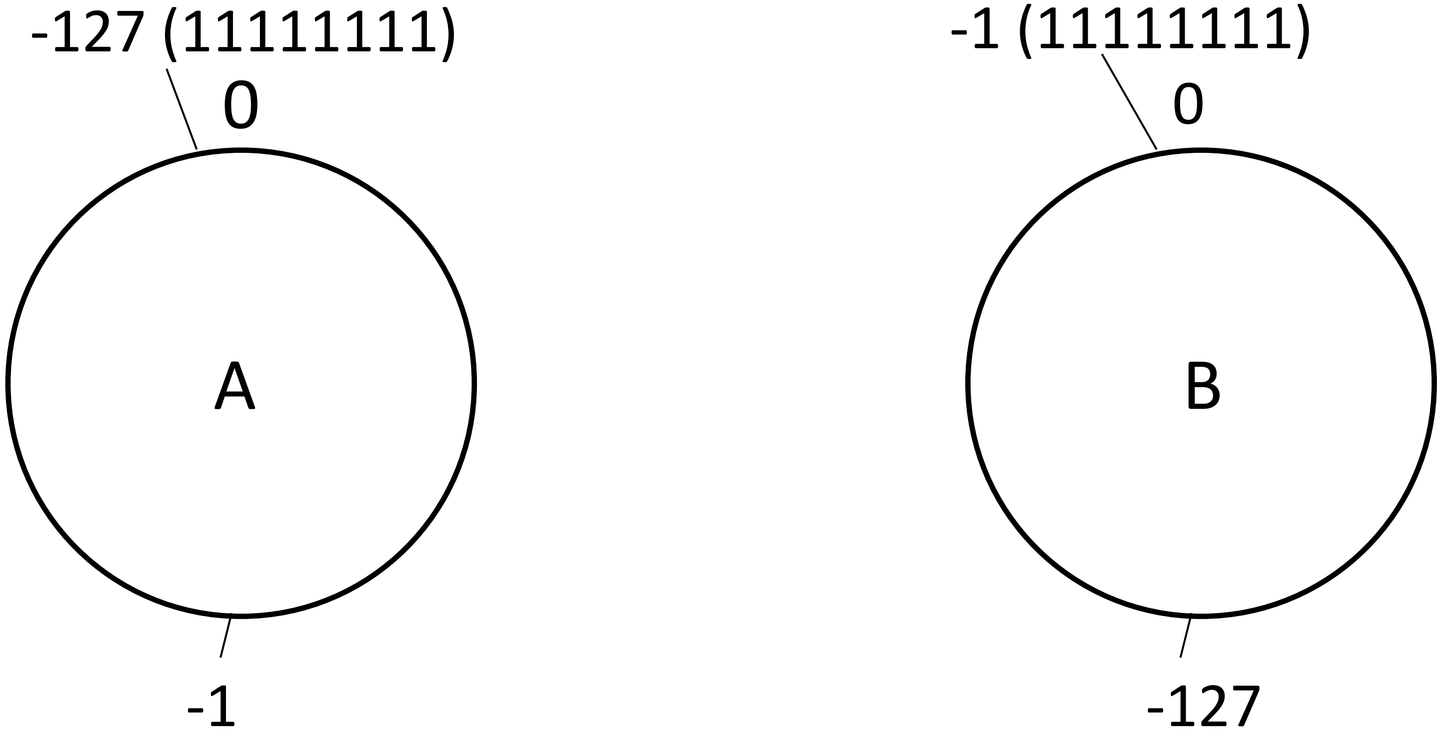
```
   1
   0110     6           1100     12
+  0100   + 4        +  1010    +10
   ----   ---        -------    ---
   1010    10        1 0110       6
^no carry out        ^carry out
```

Four bits give us range: 0 - 15

Overflow!

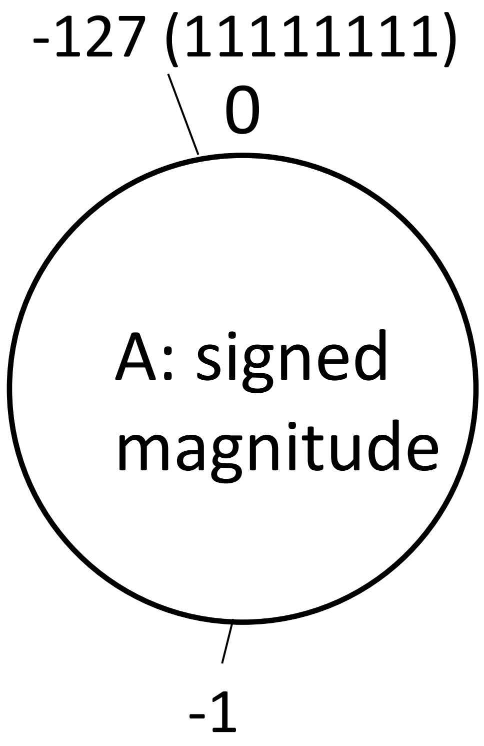
<u>Carry out is indicative of something having gone wrong when adding unsigned values</u>

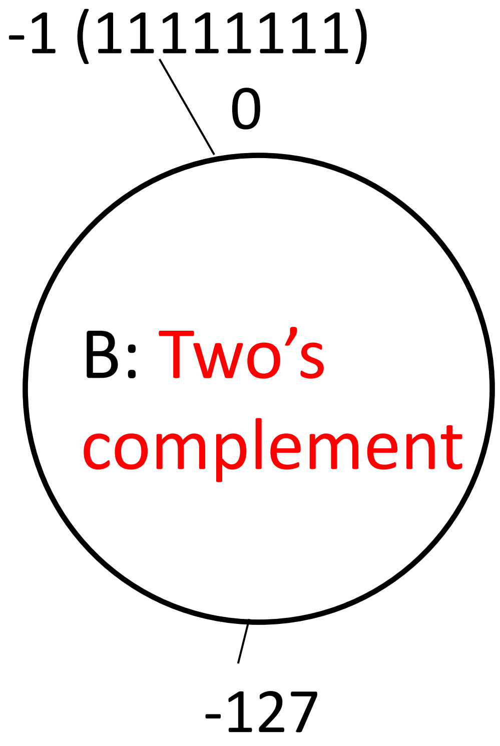
Suppose we want to support signed values (positive and negative) in 8 bits, where should we put -1 and -127 on the circle? Why?



C: Put them somewhere else.

Suppose we want to support signed values (positive and negative) in 8 bits, where should we put -1 and -127 on the circle? Why?





C: Put them somewhere else.

Signed Magnitude Representation (for 4 bit values)

- One bit (usually left-most) signals:
  - 0 for positive
  - 1 for negative

For one byte:

1 = 00000001, -1 = 10000001

Pros: Negation (negative value of a number) is very simple!

For one byte:

0 = 00000000

What about 10000000?

Major con: Two ways to represent zero!

Two's Complement Representation (for four bit values)

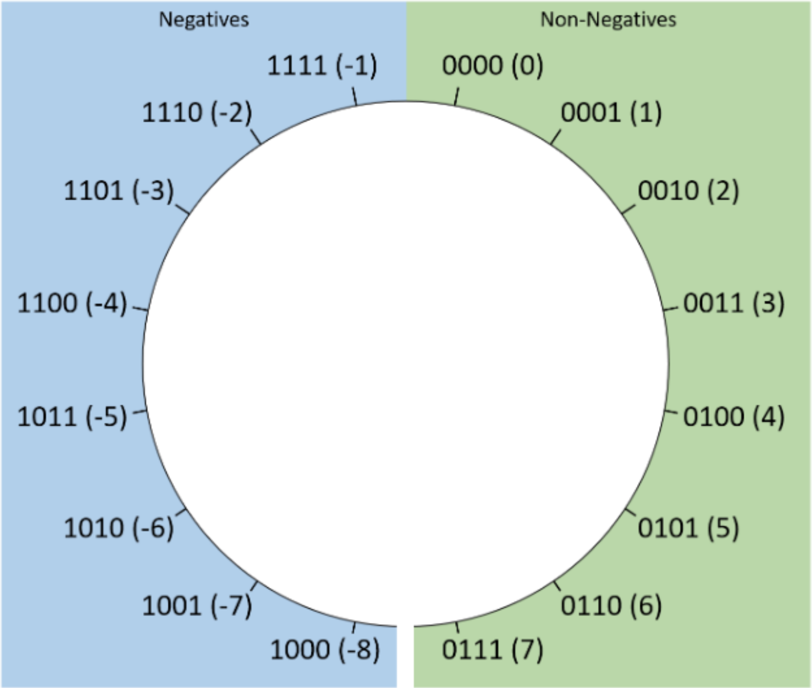
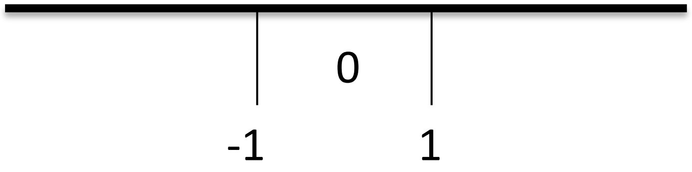


Figure 2. A logical layout of two's complement values for bit sequences of length four.

- Borrow nice property from number line:



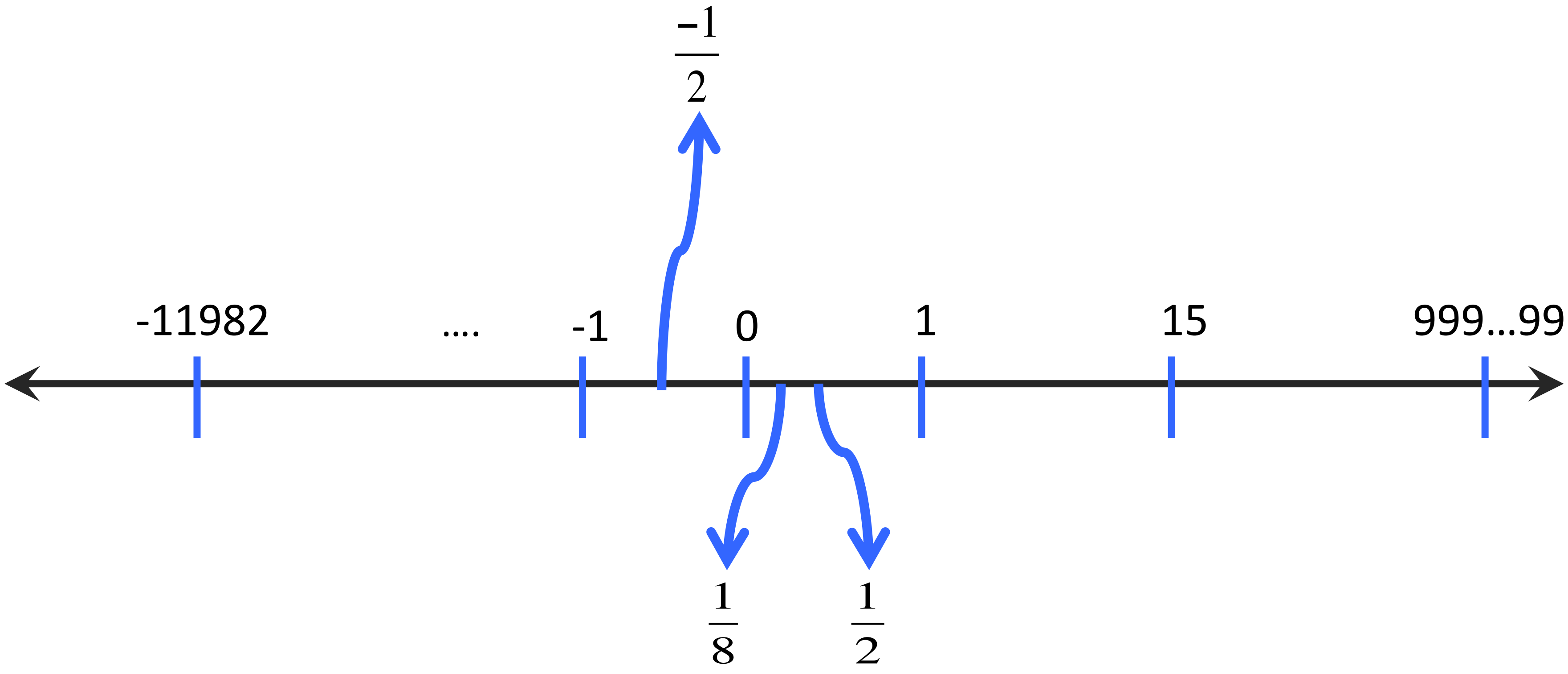
Only one instance of zero!
Implies: -1 and 1 on either side of it.

For an 8 bit range we can express 256 unique values:

- 128 non-negative values (0 to 127)
- 128 negative values (-1 to -128)

Additional Info: Fractional binary numbers

How do we represent fractions in binary?



Additional Info: Representing Signed Float Values

- One option (used for floats, <u>NOT integers</u>)
  - Let the first bit represent the sign
  - 0 means positive
  - 1 means negative

- For example:
  - <u>0</u>101 -> 5
  - <u>1</u>101 -> -5

- Problem with this scheme?

Additional Info: Floating Point Representation

1 bit for sign   sign | exponent | fraction |
8 bits for exponent
23 bits for precision

value = $(-1)^{sign}$ * 1.fraction * $2^{(exponent-127)}$

let's just plug in some values and try it out

```
0x40ac49ba: 0 10000001   01011000100100110111010
     sign = 0 exp = 129    fraction = 2902458
```

$= 1*1.2902458*2^2 = 5.16098$

<u>I don't expect you to memorize this</u>

Summary

- Images, Word Documents, Code, and Video can represented in bits.
- Byte or 8 bits is the smallest addressable unit
- N bits can represent 2^N unique values
- A number is written as a sequence of digits: in the decimal base system
  - [dn * 10 ^ n] + [dn-1 * 10 ^ n-1] + ... + [d2 * 10 ^ 2] + [d1 * 10 ^ 1] + [d0 * 10 ^ 0]
  - For any base system:
  - [dn * b ^ n] + [dn-1 * b ^ n-1] + ... + [d2 * b ^ 2] + [d1 * b ^ 1] + [d0 * b ^ 0]
- Hexadecimal values (represent 16 values): {0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F}
  - Each hexadecimal value can be represented by 4 bits. (2^4=16)
- A finite storage space we cannot represent an infinite number of values. For e.g., the max unsigned 8 bit value is 255.
  - Trying to represent a value >255 will result in an overflow.
- Two's Complement Representation: 128 non-negative values (0 to 127), and 128 negative values (-1 to -128).