How to Think Like a Computer Scientist: Learning with Python Documentation

Release 2nd Edition

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CHAPTER

ONE

LEARNING WITH PYTHON

2nd Edition (Using Python 2.x)

by Jeffrey Elkner, Allen B. Downey, and Chris Meyers

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Foreword

By David Beazley

As an educator, researcher, and book author, I am delighted to see the completion of this book. Python is a fun and extremely easy-to-use programming language that has steadily gained in popularity over the last few years. Developed over ten years ago by Guido van Rossum, Python's simple syntax and overall feel is largely derived from ABC, a teaching language that was developed in the 1980's. However, Python was also created to solve real problems and it borrows a wide variety of features from programming languages such as C++, Java, Modula-3, and Scheme. Because of this, one of Python's most remarkable features is its broad appeal to professional software developers, scientists, researchers, artists, and educators.

Despite Python's appeal to many different communities, you may still wonder why Python? or why teach programming with Python? Answering these questions is no simple task—especially when popular opinion is on the side of more masochistic alternatives such as C++ and Java. However, I think the most direct answer is that programming in Python is simply a lot of fun and more productive.

When I teach computer science courses, I want to cover important concepts in addition to making the material interesting and engaging to students. Unfortunately, there is a tendency for introductory programming courses to focus far too much attention on mathematical abstraction and for students to become frustrated with annoying problems related to low-level details of syntax, compilation, and the enforcement of seemingly arcane rules. Although such abstraction and formalism is important to professional software engineers and students who plan to continue their study of computer science, taking such an approach in an introductory course mostly succeeds in making computer science boring. When I teach a course, I don't want to have a room of uninspired students. I would much rather see them trying to solve interesting problems by exploring different ideas, taking unconventional approaches, breaking the rules, and learning from their mistakes. In doing so, I don't want to waste half of the semester trying to sort out obscure syntax problems, unintelligible compiler error messages, or the several hundred ways that a program might generate a general protection fault.

One of the reasons why I like Python is that it provides a really nice balance between the practical and the conceptual. Since Python is interpreted, beginners can pick up the language and start doing neat things almost immediately without getting lost in the problems of compilation and linking. Furthermore, Python comes with a large library of modules that can be used to do all sorts of tasks ranging from web-programming to graphics. Having such a practical focus is a great way to engage students and it allows them to complete significant projects. However, Python can also serve as an excellent foundation for introducing important computer science concepts. Since Python fully supports procedures and classes, students can be gradually introduced to topics such as procedural abstraction, data structures, and object-oriented programming — all of which are applicable to later courses on Java or C++. Python even borrows a number of features from functional programming languages and can be used to introduce concepts that would be covered in more detail in courses on Scheme and Lisp.

In reading Jeffrey's preface, I am struck by his comments that Python allowed him to see a higher level of success and a lower level of frustration and that he was able to move faster with better results. Although these comments refer to his introductory course, I sometimes use Python for these exact same reasons in advanced graduate level computer science courses at the University of Chicago. In these courses, I am constantly faced with the daunting task of covering a lot of difficult course material in a blistering nine week quarter. Although it is certainly possible for me to inflict a lot of pain and suffering by using a language like C++, I have often found this approach to be counterproductive—especially when the course is about a topic unrelated to just programming. I find that using Python allows me to better focus on the actual topic at hand while allowing students to complete substantial class projects.

Although Python is still a young and evolving language, I believe that it has a bright future in education. This book is an important step in that direction. David Beazley University of Chicago Author of the *Python Essential Reference*

Preface

By Jeffrey Elkner

This book owes its existence to the collaboration made possible by the Internet and the free software movement. Its three authors—a college professor, a high school teacher, and a professional programmer—never met face to face to work on it, but we have been able to collaborate closely, aided by many other folks who have taken the time and energy to send us their feedback.

We think this book is a testament to the benefits and future possibilities of this kind of collaboration, the framework for which has been put in place by Richard Stallman and the Free Software Foundation.

How and why I came to use Python

In 1999, the College Board's Advanced Placement (AP) Computer Science exam was given in C++ for the first time. As in many high schools throughout the country, the decision to change languages had a direct impact on the computer science curriculum at Yorktown High School in Arlington, Virginia, where I teach. Up to this point, Pascal was the language of instruction in both our first-year and AP courses. In keeping with past practice of giving students two years of exposure to the same language, we made the decision to switch to C++ in the first year course for the 1997-98 school year so that we would be in step with the College Board's change for the AP course the following year.

Two years later, I was convinced that C++ was a poor choice to use for introducing students to computer science. While it is certainly a very powerful programming language, it is also an extremely difficult language to learn and teach. I found myself constantly fighting with C++'s difficult syntax and multiple ways of doing things, and I was losing many students unnecessarily as a result. Convinced there had to be a better language choice for our first-year class, I went looking for an alternative to C++.

I needed a language that would run on the machines in our GNU/Linux lab as well as on the Windows and Macintosh platforms most students have at home. I wanted it to be free software, so that students could use it at home regardless of their income. I wanted a language that was used by professional programmers, and one that had an active developer community around it. It had to support both procedural and object-oriented programming. And most importantly, it had to be easy to learn and teach. When I investigated the choices with these goals in mind, Python stood out as the best candidate for the job.

I asked one of Yorktown's talented students, Matt Ahrens, to give Python a try. In two months he not only learned the language but wrote an application called pyTicket that enabled our staff to report technology problems via the Web. I knew that Matt could not have finished an application of that scale in so short a time in C++, and this accomplishment, combined with Matt's positive assessment of Python, suggested that Python was the solution I was looking for.

Finding a textbook

Having decided to use Python in both of my introductory computer science classes the following year, the most pressing problem was the lack of an available textbook.

Free documents came to the rescue. Earlier in the year, Richard Stallman had introduced me to Allen Downey. Both of us had written to Richard expressing an interest in developing free educational materials. Allen had already written a first-year computer science textbook, *How to Think Like a Computer Scientist*. When I read this book, I knew immediately that I wanted to use it in my class. It was the clearest and most helpful computer science text I had seen. It emphasized the processes of thought involved in programming rather than the features of a particular language. Reading it immediately made me a better teacher.

How to Think Like a Computer Scientist was not just an excellent book, but it had been released under the GNU public license, which meant it could be used freely and modified to meet the needs of its user. Once I decided to use Python, it occurred to me that I could translate Allen's original Java version of the book into the new language. While I would not have been able to write a textbook on my own, having Allen's book to work from made it possible for me to do so, at the same time demonstrating that the cooperative development model used so well in software could also work for educational materials.

Working on this book for the last two years has been rewarding for both my students and me, and my students played a big part in the process. Since I could make instant changes whenever someone found a spelling error or difficult passage, I encouraged them to look for mistakes in the book by giving them a bonus point each time they made a suggestion that resulted in a change in the text. This had the double benefit of encouraging them to read the text more carefully and of getting the text thoroughly reviewed by its most important critics, students using it to learn computer science.

For the second half of the book on object-oriented programming, I knew that someone with more real programming experience than I had would be needed to do it right. The book sat in an unfinished state for the better part of a year until the open source community once again provided the needed means for its completion.

I received an email from Chris Meyers expressing interest in the book. Chris is a professional programmer who started teaching a programming course last year using Python at Lane Community College in Eugene, Oregon. The prospect of teaching the course had led Chris to the book, and he started helping out with it immediately. By the end of the school year he had created a companion project on our Website at http://openbookproject.net called *Python for Fun* and was working with some of my most advanced students as a master teacher, guiding them beyond where I could take them.

Introducing programming with Python

The process of translating and using *How to Think Like a Computer Scientist* for the past two years has confirmed Python's suitability for teaching beginning students. Python greatly simplifies

programming examples and makes important programming ideas easier to teach.

The first example from the text illustrates this point. It is the traditional hello, world program, which in the Java version of the book looks like this:

```
class Hello {

  public static void main (String[] args) {
      System.out.println ("Hello, world.");
  }
}
```

in the Python version it becomes:

```
print "Hello, World!"
```

Even though this is a trivial example, the advantages of Python stand out. Yorktown's Computer Science I course has no prerequisites, so many of the students seeing this example are looking at their first program. Some of them are undoubtedly a little nervous, having heard that computer programming is difficult to learn. The Java version has always forced me to choose between two unsatisfying options: either to explain the *class Hello*, *public static void main*, *String[] args*, *{*, and *}*, statements and risk confusing or intimidating some of the students right at the start, or to tell them, Just don't worry about all of that stuff now; we will talk about it later, and risk the same thing. The educational objectives at this point in the course are to introduce students to the idea of a programming statement and to get them to write their first program, thereby introducing them to the programming environment. The Python program has exactly what is needed to do these things, and nothing more.

Comparing the explanatory text of the program in each version of the book further illustrates what this means to the beginning student. There are seven paragraphs of explanation of Hello, world! in the Java version; in the Python version, there are only a few sentences. More importantly, the missing six paragraphs do not deal with the big ideas in computer programming but with the minutia of Java syntax. I found this same thing happening throughout the book. Whole paragraphs simply disappear from the Python version of the text because Python's much clearer syntax renders them unnecessary.

Using a very high-level language like Python allows a teacher to postpone talking about low-level details of the machine until students have the background that they need to better make sense of the details. It thus creates the ability to put first things first pedagogically. One of the best examples of this is the way in which Python handles variables. In Java a variable is a name for a place that holds a value if it is a built-in type, and a reference to an object if it is not. Explaining this distinction requires a discussion of how the computer stores data. Thus, the idea of a variable is bound up with the hardware of the machine. The powerful and fundamental concept of a variable is already difficult enough for beginning students (in both computer science and algebra). Bytes and addresses do not help the matter. In Python a variable is a name that refers to a thing. This is a far more intuitive concept for beginning students and is much closer to the meaning of variable that they learned in their math courses. I had much less difficulty teaching variables this year than I did in the past, and I spent less time helping students with problems using them.

Another example of how Python aids in the teaching and learning of programming is in its syntax for functions. My students have always had a great deal of difficulty understanding functions. The main problem centers around the difference between a function definition and a function call, and the related distinction between a parameter and an argument. Python comes to the rescue with syntax that is nothing short of beautiful. Function definitions begin with the keyword `def`, so I simply tell my students, When you define a function, begin with `def`, followed by the name of the function that you are defining; when you call a function, simply call (type) out its name. Parameters go with definitions; arguments go with calls. There are no return types, parameter types, or reference and value parameters to get in the way, so I am now able to teach functions in less than half the time that it previously took me, with better comprehension.

Using Python improved the effectiveness of our computer science program for all students. I saw a higher general level of success and a lower level of frustration than I experienced teaching with either C++ or Java. I moved faster with better results. More students left the course with the ability to create meaningful programs and with the positive attitude toward the experience of programming that this engenders.

Building a community

I have received email from all over the globe from people using this book to learn or to teach programming. A user community has begun to emerge, and many people have been contributing to the project by sending in materials for the companion Website at http://openbookproject.net/pybiblio.

With the continued growth of Python, I expect the growth in the user community to continue and accelerate. The emergence of this user community and the possibility it suggests for similar collaboration among educators have been the most exciting parts of working on this project for me. By working together, we can increase the quality of materials available for our use and save valuable time. I invite you to join our community and look forward to hearing from you. Please write to me at jeff@elkner.net.

Jeffrey Elkner
Governor's Career and Technical Academy in Arlington
Arlington, Virginia

Contributor List

To paraphrase the philosophy of the Free Software Foundation, this book is free like free speech, but not necessarily free like free pizza. It came about because of a collaboration that would not have been possible without the GNU Free Documentation License. So we would like to thank the Free Software Foundation for developing this license and, of course, making it available to us.

We would also like to thank the more than 100 sharp-eyed and thoughtful readers who have sent us suggestions and corrections over the past few years. In the spirit of free software, we decided to express our gratitude in the form of a contributor list. Unfortunately, this list is not complete, but we are doing our best to keep it up to date. It was also getting too large to include everyone who sends in a typo or two. You have our gratitude, and you have the personal satisfaction of making a book you found useful better for you and everyone else who uses it. New additions to the list for the 2nd edition will be those who have made on-going contributions.

If you have a chance to look through the list, you should realize that each person here has spared you and all subsequent readers from the confusion of a technical error or a less-than-transparent explanation, just by sending us a note.

Impossible as it may seem after so many corrections, there may still be errors in this book. If you should stumble across one, we hope you will take a minute to contact us. The email address is jeff@elkner.net . Substantial changes made due to your suggestions will add you to the next version of the contributor list (unless you ask to be omitted). Thank you!

Second Edition

- An email from Mike MacHenry set me straight on tail recursion. He not only pointed out an error in the presentation, but suggested how to correct it.
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First Edition

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- Julie Peters caught a typo in the Preface.

The way of the program

The goal of this book is to teach you to think like a computer scientist. This way of thinking combines some of the best features of mathematics, engineering, and natural science. Like mathematicians, computer scientists use formal languages to denote ideas (specifically computations). Like engineers, they design things, assembling components into systems and evaluating tradeoffs among alternatives. Like scientists, they observe the behavior of complex systems, form hypotheses, and test predictions.

The single most important skill for a computer scientist is **problem solving**. Problem solving means the ability to formulate problems, think creatively about solutions, and express a solution clearly and accurately. As it turns out, the process of learning to program is an excellent opportunity to practice problem-solving skills. That's why this chapter is called, The way of the program.

On one level, you will be learning to program, a useful skill by itself. On another level, you will use programming as a means to an end. As we go along, that end will become clearer.

The Python programming language

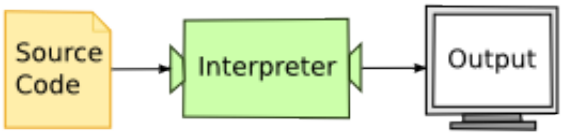
The programming language you will be learning is Python. Python is an example of a **high-level language**; other high-level languages you might have heard of are C++, PHP, and Java.

As you might infer from the name high-level language, there are also **low-level languages**, sometimes referred to as machine languages or assembly languages. Loosely speaking, computers can only execute programs written in low-level languages. Thus, programs written in a high-level language have to be processed before they can run. This extra processing takes some time, which is a small disadvantage of high-level languages.

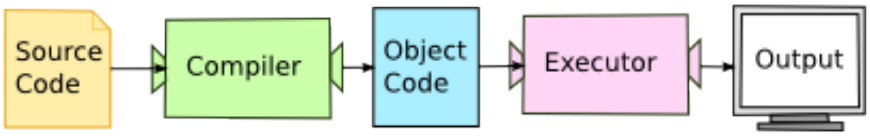
But the advantages are enormous. First, it is much easier to program in a high-level language. Programs written in a high-level language take less time to write, they are shorter and easier to read, and they are more likely to be correct. Second, high-level languages are **portable**, meaning that they can run on different kinds of computers with few or no modifications. Low-level programs can run on only one kind of computer and have to be rewritten to run on another.

Due to these advantages, almost all programs are written in high-level languages. Low-level languages are used only for a few specialized applications.

Two kinds of programs process high-level languages into low-level languages: **interpreters** and **compilers**. An interpreter reads a high-level program and executes it, meaning that it does what the program says. It processes the program a little at a time, alternately reading lines and performing computations.



A compiler reads the program and translates it completely before the program starts running. In this case, the high-level program is called the **source code**, and the translated program is called the **object code** or the **executable**. Once a program is compiled, you can execute it repeatedly without further translation.



Many modern languages use both processes. They are first compiled into a lower level language, called **byte code**, and then interpreted by a program called a **virtual machine**. Python uses both processes, but because of the way programmers interact with it, it is usually considered an interpreted language.

There are two ways to use the Python interpreter: *shell mode* and *script mode*. In shell mode, you type Python statements into the **Python shell** and the interpreter immediately prints the result:

```
$ python
Python 2.5.1 (r251:54863, May  2 2007, 16:56:35)
[GCC 4.1.2 (Ubuntu 4.1.2-0ubuntu4)] on linux2
Type "help", "copyright", "credits" or "license" for more information.
>>> print(1 + 1)
2
```

The first line of this example is the command that starts the Python interpreter at a Unix command prompt. The next three lines are messages from the interpreter. The fourth line starts with `>>>`, which is the **Python prompt**. The interpreter uses the prompt to indicate that it is ready for instructions. We typed `print(1 + 1)`, and the interpreter replied `2`.

Alternatively, you can write a program in a file and use the interpreter to execute the contents of the file. Such a file is called a **script**. For example, we used a text editor to create a file named `firstprogram.py` with the following contents:

```
print(1 + 1)
```

By convention, files that contain Python programs have names that end with `.py`.

To execute the program, we have to tell the interpreter the name of the script:

```
$ python firstprogram.py
2
```

These examples show Python being run from a Unix command line. In other development environments, the details of executing programs may differ. Also, most programs are more interesting than this one.

The examples in this book use both the Python interpreter and scripts. You will be able to tell which is intended since shell mode examples will always start with the Python prompt.

Working in shell mode is convenient for testing short bits of code because you get immediate feedback. Think of it as scratch paper used to help you work out problems. Anything longer than a few lines should be put into a script.

What is a program?

A **program** is a sequence of instructions that specifies how to perform a computation. The computation might be something mathematical, such as solving a system of equations or finding the roots of a polynomial, but it can also be a symbolic computation, such as searching and replacing text in a document or (strangely enough) compiling a program.

The details look different in different languages, but a few basic instructions appear in just about every language:

input Get data from the keyboard, a file, or some other device.

output Display data on the screen or send data to a file or other device.

math Perform basic mathematical operations like addition and multiplication.

conditional execution Check for certain conditions and execute the appropriate sequence of statements.

repetition Perform some action repeatedly, usually with some variation.

Believe it or not, that's pretty much all there is to it. Every program you've ever used, no matter how complicated, is made up of instructions that look more or less like these. Thus, we can describe programming as the process of breaking a large, complex task into smaller and smaller subtasks until the subtasks are simple enough to be performed with one of these basic instructions.

That may be a little vague, but we will come back to this topic later when we talk about **algorithms**.

What is debugging?

Programming is a complex process, and because it is done by human beings, it often leads to errors. For whimsical reasons, programming errors are called **bugs** and the process of tracking them down and correcting them is called **debugging**.

Three kinds of errors can occur in a program: syntax errors, runtime errors, and semantic errors. It is useful to distinguish between them in order to track them down more quickly.

Syntax errors

Python can only execute a program if the program is syntactically correct; otherwise, the process fails and returns an error message. **syntax** refers to the structure of a program and the rules about that structure. For example, in English, a sentence must begin with a capital letter and end with a period. this sentence contains a **syntax error**. So does this one

For most readers, a few syntax errors are not a significant problem, which is why we can read the poetry of e. e. cummings without spewing error messages. Python is not so forgiving. If there is a single syntax error anywhere in your program, Python will print an error message and quit, and you will not be able to run your program. During the first few weeks of your programming career, you will probably spend a lot of time tracking down syntax errors. As you gain experience, though, you will make fewer errors and find them faster.

Runtime errors

The second type of error is a runtime error, so called because the error does not appear until you run the program. These errors are also called **exceptions** because they usually indicate that something exceptional (and bad) has happened.

Runtime errors are rare in the simple programs you will see in the first few chapters, so it might be a while before you encounter one.

Semantic errors

The third type of error is the **semantic error**. If there is a semantic error in your program, it will run successfully, in the sense that the computer will not generate any error messages, but it will not do the right thing. It will do something else. Specifically, it will do what you told it to do.

The problem is that the program you wrote is not the program you wanted to write. The meaning of the program (its semantics) is wrong. Identifying semantic errors can be tricky because it requires you to work backward by looking at the output of the program and trying to figure out what it is doing.

Experimental debugging

One of the most important skills you will acquire is debugging. Although it can be frustrating, debugging is one of the most intellectually rich, challenging, and interesting parts of programming.

In some ways, debugging is like detective work. You are confronted with clues, and you have to infer the processes and events that led to the results you see.

Debugging is also like an experimental science. Once you have an idea what is going wrong, you modify your program and try again. If your hypothesis was correct, then you can predict the result of the modification, and you take a step closer to a working program. If your hypothesis was wrong, you have to come up with a new one. As Sherlock Holmes pointed out, When you have eliminated the impossible, whatever remains, however improbable, must be the truth. (A. Conan Doyle, *The Sign of Four*)

For some people, programming and debugging are the same thing. That is, programming is the process of gradually debugging a program until it does what you want. The idea is that you should start with a program that does *something* and make small modifications, debugging them as you go, so that you always have a working program.

For example, Linux is an operating system kernel that contains millions of lines of code, but it started out as a simple program Linus Torvalds used to explore the Intel 80386 chip. According to Larry Greenfield, one of Linus's earlier projects was a program that would switch between printing AAAA and BBBB. This later evolved to Linux (*The Linux Users' Guide* Beta Version 1).

Later chapters will make more suggestions about debugging and other programming practices.

Formal and natural languages

Natural languages are the languages that people speak, such as English, Spanish, and French. They were not designed by people (although people try to impose some order on them); they evolved naturally.

Formal languages are languages that are designed by people for specific applications. For example, the notation that mathematicians use is a formal language that is particularly good at denoting relationships among numbers and symbols. Chemists use a formal language to represent the chemical structure of molecules. And most importantly:

> *Programming languages are formal languages that have been designed to express computations.*

Formal languages tend to have strict rules about syntax. For example, `3+3=6` is a syntactically correct mathematical statement, but `3=+6$` is not. H_2O is a syntactically correct chemical name, but $_2Zz$ is not.

Syntax rules come in two flavors, pertaining to **tokens** and structure. Tokens are the basic elements of the language, such as words, numbers, and chemical elements. One of the problems with `3=+6$` is that `$` is not a legal token in mathematics (at least as far as we know). Similarly, $_2Zz$ is not legal because there is no element with the abbreviation `Zz`.

The second type of syntax rule pertains to the structure of a statement— that is, the way the tokens are arranged. The statement `3=+6$` is structurally illegal because you can't place a plus sign

immediately after an equal sign. Similarly, molecular formulas have to have subscripts after the element name, not before.

When you read a sentence in English or a statement in a formal language, you have to figure out what the structure of the sentence is (although in a natural language you do this subconsciously). This process is called **parsing**.

For example, when you hear the sentence, The other shoe fell, you understand that the other shoe is the subject and fell is the verb. Once you have parsed a sentence, you can figure out what it means, or the semantics of the sentence. Assuming that you know what a shoe is and what it means to fall, you will understand the general implication of this sentence.

Although formal and natural languages have many features in common — tokens, structure, syntax, and semantics — there are many differences:

ambiguity Natural languages are full of ambiguity, which people deal with by using contextual clues and other information. Formal languages are designed to be nearly or completely unambiguous, which means that any statement has exactly one meaning, regardless of context.

redundancy In order to make up for ambiguity and reduce misunderstandings, natural languages employ lots of redundancy. As a result, they are often verbose. Formal languages are less redundant and more concise.

literalness Natural languages are full of idiom and metaphor. If someone says, The other shoe fell, there is probably no shoe and nothing falling. Formal languages mean exactly what they say.

People who grow up speaking a natural language—everyone—often have a hard time adjusting to formal languages. In some ways, the difference between formal and natural language is like the difference between poetry and prose, but more so:

Poetry Words are used for their sounds as well as for their meaning, and the whole poem together creates an effect or emotional response. Ambiguity is not only common but often deliberate.

Prose The literal meaning of words is more important, and the structure contributes more meaning. Prose is more amenable to analysis than poetry but still often ambiguous.

Programs The meaning of a computer program is unambiguous and literal, and can be understood entirely by analysis of the tokens and structure.

Here are some suggestions for reading programs (and other formal languages). First, remember that formal languages are much more dense than natural languages, so it takes longer to read them. Also, the structure is very important, so it is usually not a good idea to read from top to bottom, left to right. Instead, learn to parse the program in your head, identifying the tokens and interpreting the structure. Finally, the details matter. Little things like spelling errors and bad punctuation, which you can get away with in natural languages, can make a big difference in a formal language.

The first program

Traditionally, the first program written in a new language is called Hello, World! because all it does is display the words, Hello, World! In Python, it looks like this:

```
print("Hello, World!")
```

This is an example of a **print statement**, which doesn't actually print anything on paper. It displays a value on the screen. In this case, the result is the words

```
Hello, World!
```

The quotation marks in the program mark the beginning and end of the value; they don't appear in the result.

Some people judge the quality of a programming language by the simplicity of the Hello, World! program. By this standard, Python does about as well as possible.

Glossary

algorithm A general process for solving a category of problems.

bug An error in a program.

byte code An intermediate language between source code and object code. Many modern languages first compile source code into byte code and then interpret the byte code with a program called a *virtual machine*.

compile To translate a program written in a high-level language into a low-level language all at once, in preparation for later execution.

debugging The process of finding and removing any of the three kinds of programming errors.

exception Another name for a runtime error.

executable Another name for object code that is ready to be executed.

formal language Any one of the languages that people have designed for specific purposes, such as representing mathematical ideas or computer programs; all programming languages are formal languages.

high-level language A programming language like Python that is designed to be easy for humans to read and write.

interpret To execute a program in a high-level language by translating it one line at a time.

low-level language A programming language that is designed to be easy for a computer to execute; also called machine language or assembly language.

natural language Any one of the languages that people speak that evolved naturally.

object code The output of the compiler after it translates the program.

parse To examine a program and analyze the syntactic structure.

portability A property of a program that can run on more than one kind of computer.

print statement An instruction that causes the Python interpreter to display a value on the screen.

problem solving The process of formulating a problem, finding a solution, and expressing the solution.

program a sequence of instructions that specifies to a computer actions and computations to be performed.

Python shell An interactive user interface to the Python interpreter. The user of a Python shell types commands at the prompt (>>>), and presses the return key to send these commands immediately to the interpreter for processing.

runtime error An error that does not occur until the program has started to execute but that prevents the program from continuing.

script A program stored in a file (usually one that will be interpreted).

semantic error An error in a program that makes it do something other than what the programmer intended.

semantics The meaning of a program.

source code A program in a high-level language before being compiled.

syntax The structure of a program.

syntax error An error in a program that makes it impossible to parse — and therefore impossible to interpret.

token One of the basic elements of the syntactic structure of a program, analogous to a word in a natural language.

Exercises

1. Write an English sentence with understandable semantics but incorrect syntax. Write another English sentence which has correct syntax but has semantic errors.
2. Start a Python shell. Type `1 + 2` and then hit return. Python *evaluates* this *expression*, prints the result, and then prints another prompt. `*` is the *multiplication operator*, and `**` is the *exponentiation operator*. Experiment by entering different expressions and recording what is printed by the Python interpreter. What happens if you use the `/` operator? Are the results what you expect? Explain.
3. Type `1 2` and then hit return. Python tries to evaluate the expression, but it can't because the expression is not syntactically legal. Instead, it prints the error message:

```
  File "<stdin>", line 1
    1 2
      ^
SyntaxError: invalid syntax
```

In many cases, Python indicates where the syntax error occurred, but it is not always right, and it doesn't give you much information about what is wrong.

So, for the most part, the burden is on you to learn the syntax rules.

In this case, Python is complaining because there is no operator between the numbers.

See if you can find a few more examples of things that will produce error messages when you enter them at the Python prompt. Write down what you enter at the prompt and the last line of the error message that Python reports back to you.

4. Type `print('hello')`. Python executes this statement, which has the effect of printing the letters h-e-l-l-o. Notice that the quotation marks that you used to enclose the string are not part of the output. Now type `"hello"` and describe your result. Make note of when you see the quotation marks and when you don't.

5. Type `print(cheese)` without the quotation marks. The output will look something like this:

```
Traceback (most recent call last):
  File "<stdin>", line 1, in ?
NameError: name 'cheese' is not defined
```

This is a run-time error; specifically, it is a NameError, and even more specifically, it is an error because the name *cheese* is not defined. If you don't know what that means yet, you will soon.

6. Type `'This is a test...'` at the Python prompt and hit enter. Record what happens.

Now create a python script named `test1.py` with the following contents (be sure to save it before you try to run it):

```
'This is a test...'
```

What happens when you run this script? Now change the contents to:

```
print('This is a test...')
```

and run it again.

What happened this time?

Whenever an *expression* is typed at the Python prompt, it is *evaluated* and the result is printed on the line below. `'This is a test...'` is an expression, which evaluates to `'This is a test...'` (just like the expression `42` evaluates to `42`). In a script, however, evaluations of expressions are not sent to the program output, so it is necessary to explicitly print them.

Variables, expressions and statements

Values and data types

A **value** is one of the fundamental things — like a letter or a number — that a program manipulates. The values we have seen so far are `2` (the result when we added `1 + 1`), and `"Hello, World!"`.

These values belong to different **data types**: `2` is an *integer*, and `"Hello, World!"` is a *string*, so-called because it contains a string of letters. You (and the interpreter) can identify strings because they are enclosed in quotation marks.

The print statement also works for integers.

```
>>> print(4)
4
```

If you are not sure what type a value has, the interpreter can tell you.

```
>>> type("Hello, World!")
<type 'str'>
>>> type(17)
<type 'int'>
```

Not surprisingly, strings belong to the type **str** and integers belong to the type **int**. Less obviously, numbers with a decimal point belong to a type called **float**, because these numbers are represented in a format called *floating-point*.

```
>>> type(3.2)
<type 'float'>
```

What about values like `"17"` and `"3.2"`? They look like numbers, but they are in quotation marks like strings.

```
>>> type("17")
<type 'str'>
>>> type("3.2")
<type 'str'>
```

They're strings.

Strings in Python can be enclosed in either single quotes (') or double quotes ("):

```
>>> type('This is a string.')
<type 'str'>
>>> type("And so is this.")
<type 'str'>
```

Double quoted strings can contain single quotes inside them, as in `"Bruce's beard"`, and single quoted strings can have double quotes inside them, as in `'The knights who say "Ni!"'`.

When you type a large integer, you might be tempted to use commas between groups of three digits, as in `1,000,000`. This is not a legal integer in Python, but it is legal:

```
>>> print(1,000,000)
(1, 0, 0)
```

Well, that's not what we expected at all! Python interprets `1,000,000` as a three separate items to be printed. So remember not to put commas in your integers.

Variables

One of the most powerful features of a programming language is the ability to manipulate **variables**. A variable is a name that refers to a value.

The **assignment statement** creates new variables and gives them values:

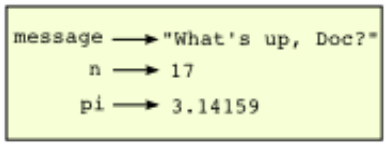
```
>>> message = "What's up, Doc?"
>>> n = 17
>>> pi = 3.14159
```

This example makes three assignments. The first assigns the string `"What's up, Doc?"` to a new variable named `message`. The second gives the integer `17` to `n`, and the third gives the floating-point number `3.14159` to `pi`.

The **assignment operator**, =, should not be confused with an equals sign (even though it uses the same character). Assignment operators link a *name*, on the left hand side of the operator, with a *value*, on the right hand side. This is why you will get an error if you enter:

```
>>> 17 = n
```

A common way to represent variables on paper is to write the name with an arrow pointing to the variable's value. This kind of figure is called a **state diagram** because it shows what state each of the variables is in (think of it as the variable's state of mind). This diagram shows the result of the assignment statements:



The print statement also works with variables.

```
>>> print(message)
What's up, Doc?
>>> print(n)
```

```
17
>>> print(pi)
3.14159
```

In each case the result is the value of the variable. Variables also have types; again, we can ask the interpreter what they are.

```
>>> type(message)
<type 'str'>
>>> type(n)
<type 'int'>
>>> type(pi)
<type 'float'>
```

The type of a variable is the type of the value it refers to.

Variable names and keywords

Programmers generally choose names for their variables that are meaningful — they document what the variable is used for.

Variable names can be arbitrarily long. They can contain both letters and numbers, but they have to begin with a letter. Although it is legal to use uppercase letters, by convention we don't. If you do, remember that case matters. `Bruce` and `bruce` are different variables.

The underscore character (_) can appear in a name. It is often used in names with multiple words, such as `my_name` or `price_of_tea_in_china`.

If you give a variable an illegal name, you get a syntax error:

```
>>> 76trombones = "big parade"
SyntaxError: invalid syntax
>>> more$ = 1000000
SyntaxError: invalid syntax
>>> class = "Computer Science 101"
SyntaxError: invalid syntax
```

`76trombones` is illegal because it does not begin with a letter. `more$` is illegal because it contains an illegal character, the dollar sign. But what's wrong with `class`?

It turns out that `class` is one of the Python **keywords**. Keywords define the language's rules and structure, and they cannot be used as variable names.

Python has thirty-one keywords:

and	as	assert	break	class	continue
def	del	elif	else	except	exec
finally	for	from	global	if	import
in	is	lambda	not	or	pass
print	raise	return	try	while	with
yield					

You might want to keep this list handy. If the interpreter complains about one of your variable names and you don't know why, see if it is on this list.

Statements

A **statement** is an instruction that the Python interpreter can execute. We have seen two kinds of statements: print and assignment.

When you type a statement on the command line, Python executes it and displays the result, if there is one. The result of a print statement is a value. Assignment statements don't produce a result.

A script usually contains a sequence of statements. If there is more than one statement, the results appear one at a time as the statements execute.

For example, the script

```
print(1)
x = 2
print(x)
```

produces the output:

```
1
2
```

Again, the assignment statement produces no output.

Evaluating expressions

An **expression** is a combination of values, variables, and operators. If you type an expression on the command line, the interpreter **evaluates** it and displays the result:

```
>>> 1 + 1
2
```

The *evaluation of an expression* produces a value, which is why expressions can appear on the right hand side of assignment statements. A value all by itself is a simple expression, and so is a variable.

```
>>> 17
17
>>> x
2
```

Confusingly, evaluating an expression is not quite the same thing as printing a value.

```
>>> message = "What's up, Doc?"
>>> message
"What's up, Doc?"
>>> print(message)
What's up, Doc?
```

When the Python shell displays the value of an expression, it uses the same format you would use to enter a value. In the case of strings, that means that it includes the quotation marks. But the print statement prints the value of the expression, which in this case is the contents of the string.

In a script, an expression all by itself is a legal statement, but it doesn't do anything. The script

```
17
3.2
"Hello, World!"
1 + 1
```

produces no output at all. How would you change the script to display the values of these four expressions?

Operators and operands

Operators are special symbols that represent computations like addition and multiplication. The values the operator uses are called **operands**.

The following are all legal Python expressions whose meaning is more or less clear:

```
20+32    hour-1    hour*60+minute    minute/60    5**2    (5+9)*(15-7)
```

The symbols +, −, and /, and the use of parenthesis for grouping, mean in Python what they mean in mathematics. The asterisk (*) is the symbol for multiplication, and ** is the symbol for exponentiation.

When a variable name appears in the place of an operand, it is replaced with its value before the operation is performed.

Addition, subtraction, multiplication, and exponentiation all do what you expect, but you might be surprised by division. The following operation has an unexpected result:

```
>>> minute = 59
>>> minute/60
0
```

The value of `minute` is 59, and 59 divided by 60 is 0.98333, not 0. The reason for the discrepancy is that Python is performing **integer division**.

When both of the operands are integers, the result must also be an integer, and by convention, integer division always rounds *down*, even in cases like this where the next integer is very close.

A possible solution to this problem is to calculate a percentage rather than a fraction:

```
>>> minute*100/60
98
```

Again the result is rounded down, but at least now the answer is approximately correct. Another alternative is to use floating-point division. We'll see in the Chapter 4 how to convert integer values and variables to floating-point values.

Order of operations

When more than one operator appears in an expression, the order of evaluation depends on the **rules of precedence**. Python follows the same precedence rules for its mathematical operators that mathematics does. The acronym PEMDAS is a useful way to remember the order of operations:

1. **P**arentheses have the highest precedence and can be used to force an expression to evaluate in the order you want. Since expressions in parentheses are evaluated first, `2 * (3-1)` is 4, and `(1+1)**(5-2)` is 8. You can also use parentheses to make an expression easier to read, as in `(minute * 100) / 60`, even though it doesn't change the result.
2. **E**xponentiation has the next highest precedence, so `2**1+1` is 3 and not 4, and `3*1**3` is 3 and not 27.
3. **M**ultiplication and **D**ivision have the same precedence, which is higher than **A**ddition and **S**ubtraction, which also have the same precedence. So `2*3-1` yields 5 rather than 4, and `2/3-1` is -1, not 1 (remember that in integer division, 2/3=0).
4. Operators with the same precedence are evaluated from left to right. So in the expression `minute*100/60`, the multiplication happens first, yielding 5900/60, which in turn yields 98. If the operations had been evaluated from right to left, the result would have been `59*1`, which is 59, which is wrong.

Operations on strings

In general, you cannot perform mathematical operations on strings, even if the strings look like numbers. The following are illegal (assuming that `message` has type string):

```
message-1   "Hello"/123   message*"Hello"   "15"+2
```

Interestingly, the + operator does work with strings, although it does not do exactly what you might expect. For strings, the + operator represents **concatenation**, which means joining the two operands by linking them end-to-end. For example:

```
fruit = "banana"
baked_good = " nut bread"
print(fruit + baked_good)
```

The output of this program is `banana nut bread`. The space before the word `nut` is part of the string, and is necessary to produce the space between the concatenated strings.

The * operator also works on strings; it performs repetition. For example, `'Fun'*3` is `'FunFunFun'`. One of the operands has to be a string; the other has to be an integer.

On one hand, this interpretation of + and * makes sense by analogy with addition and multiplication. Just as `4*3` is equivalent to `4+4+4`, we expect `"Fun"*3` to be the same as `"Fun"+"Fun"+"Fun"`, and it is. On the other hand, there is a significant way in which string concatenation and repetition are different from integer addition and multiplication. Can you think of a property that addition and multiplication have that string concatenation and repetition do not?

Input

The built-in function `raw_input()` can be used for getting keyboard input. This function takes one argument (the string used as a prompt to the user) and returns whatever the user enters **as a string**:

```
name = raw_input("Please enter your name: ")
print(name)
age = raw_input("Please enter your age: ")
print(age)
```

A sample run of this script would look something like this:

```
$ python tryinput.py
Please enter your name: Jeff
Jeff
Please enter your age: 50
50
```

NOTE: in the above example, the variable *age* is assigned the string "50". It's easy to see this using the interactive python shell and the *type()* function. If you need an integer or a float, you could convert the given user input:

```
>>> age = raw_input("How old are you? ")
How old are you? 50
>>> type(age)
<type 'str'>
>>> age = int(age)
```

```
>>> type(age)
<type 'int'>
>>>
```

Composition

So far, we have looked at the elements of a program — variables, expressions, and statements — in isolation, without talking about how to combine them.

One of the most useful features of programming languages is their ability to take small building blocks and **compose** them. For example, we know how to add numbers and we know how to print; it turns out we can do both at the same time:

```
>>>  print(17 + 3)
20
```

In reality, the addition has to happen before the printing, so the actions aren't actually happening at the same time. The point is that any expression involving numbers, strings, and variables can be used inside a print statement. You've already seen an example of this:

```
print("Number of minutes since midnight: ")
print(hour*60+minute)
```

You can also put arbitrary expressions on the right-hand side of an assignment statement:

```
percentage = (minute * 100) / 60
```

This ability may not seem impressive now, but you will see other examples where composition makes it possible to express complex computations neatly and concisely.

> Warning: There are limits on where you can use certain expressions. For example, the left-hand side of an assignment statement has to be a *variable* name, not an expression. So, the following is illegal: `minute+1 = hour`.

Comments

As programs get bigger and more complicated, they get more difficult to read. Formal languages are dense, and it is often difficult to look at a piece of code and figure out what it is doing, or why.

For this reason, it is a good idea to add notes to your programs to explain in natural language what the program is doing. These notes are called **comments**, and they are marked with the # symbol:

```
# compute the percentage of the hour that has elapsed
percentage = (minute * 100) / 60
```

In this case, the comment appears on a line by itself. You can also put comments at the end of a line:

```
percentage = (minute * 100) / 60     # caution: integer division
```

Everything from the # to the end of the line is ignored — it has no effect on the program. The message is intended for the programmer or for future programmers who might use this code. In this case, it reminds the reader about the ever-surprising behavior of integer division.

Glossary

assignment operator = is Python's assignment operator, which should not be confused with the mathematical comparison operator using the same symbol.

assignment statement A statement that assigns a value to a name (variable). To the left of the assignment operator, =, is a name. To the right of the assignment operator is an expression which is evaluated by the Python interpreter and then assigned to the name. The difference between the left and right hand sides of the assignment statement is often confusing to new programmers. In the following assignment:

```
n = n + 1
```

`n` plays a very different role on each side of the =. On the right it is a *value* and makes up part of the *expression* which will be evaluated by the Python interpreter before assigning it to the name on the left.

comment Information in a program that is meant for other programmers (or anyone reading the source code) and has no effect on the execution of the program.

composition The ability to combine simple expressions and statements into compound statements and expressions in order to represent complex computations concisely.

concatenate To join two strings end-to-end.

data type A set of values. The type of a value determines how it can be used in expressions. So far, the types you have seen are integers (type `int`), floating-point numbers (type `float`), and strings (type `str`).

evaluate To simplify an expression by performing the operations in order to yield a single value.

expression A combination of variables, operators, and values that represents a single result value.

float A Python data type which stores *floating-point* numbers. Floating-point numbers are stored internally in two parts: a *base* and an *exponent*. When printed in the standard format, they look like decimal numbers. Beware of rounding errors when you use `floats`, and remember that they are only approximate values.

int A Python data type that holds positive and negative whole numbers.

integer division An operation that divides one integer by another and yields an integer. Integer division yields only the whole number of times that the numerator is divisible by the denominator and discards any remainder.

keyword A reserved word that is used by the compiler to parse program; you cannot use keywords like `if`, `def`, and `while` as variable names.

operand One of the values on which an operator operates.

operator A special symbol that represents a simple computation like addition, multiplication, or string concatenation.

rules of precedence The set of rules governing the order in which expressions involving multiple operators and operands are evaluated.

state diagram A graphical representation of a set of variables and the values to which they refer.

statement An instruction that the Python interpreter can execute. Examples of statements include the assignment statement and the print statement.

str A Python data type that holds a string of characters.

value A number or string (or other things to be named later) that can be stored in a variable or computed in an expression.

variable A name that refers to a value.

variable name A name given to a variable. Variable names in Python consist of a sequence of letters (a..z, A..Z, and _) and digits (0..9) that begins with a letter. In best programming practice, variable names should be chosen so that they describe their use in the program, making the program *self documenting*.

Exercises

1. Record what happens when you print an assignment statement:

```
>>> print(n = 9)
```

   How about this?

```
>>> print(7 + 5)
```

   Or this?

```
>>> print("this"+"that")
```

   Can you think of a general rule for what can follow the `print` statement? What does the `print` statement return?

2. Take the sentence: *All work and no play makes Jack a dull boy.* Store each word in a separate variable, then print out the sentence on one line using print.

3. Add parenthesis to the expression `6 * 1 - 2` to change its value from 4 to -6.

4. Place a comment before a line of code that previously worked, and record what happens when you rerun the program.

5. Start the Python interpreter and enter `bruce + 4` at the prompt. This will give you an error:

```
NameError: name 'bruce' is not defined
```

   Assign a value to `bruce` so that `bruce + 4` evaluates to `10`.

6. Write a program (Python script) named `madlib.py`, which asks the user to enter a series of nouns, verbs, adjectives, adverbs, plural nouns, past tense verbs, etc., and then generates a paragraph which is syntactically correct but semantically ridiculous (see http://madlibs.org for examples).

Functions

Definitions and use

In the context of programming, a **function** is a named sequence of statements that performs a desired operation. This operation is specified in a **function definition**. In Python, the syntax for a function definition is:

```
def NAME( LIST OF PARAMETERS ):
    STATEMENTS
```

You can make up any names you want for the functions you create, except that you can't use a name that is a Python keyword. The list of parameters specifies what information, if any, you have to provide in order to use the new function.

There can be any number of statements inside the function, but they have to be indented from the `def`. In the examples in this book, we will use the standard indentation of four spaces. Function definitions are the first of several **compound statements** we will see, all of which have the same pattern:

1. A **header**, which begins with a keyword and ends with a colon.
2. A **body** consisting of one or more Python statements, each indented the same amount – *4 spaces is the Python standard* – from the header.

In a function definition, the keyword in the header is `def`, which is followed by the name of the function and a list of *parameters* enclosed in parentheses. The parameter list may be empty, or it may contain any number of parameters. In either case, the parentheses are required.

The first couple of functions we are going to write have no parameters, so the syntax looks like this:

```
def new_line():
    print("")           # a print statement with an empty string prints a new line
```

This function is named `new_line`. Its body contains only a single statement, which outputs a newline character. (That's what happens when you use a `print` command with an empty string.)

Defining a new function does not make the function run. To do that we need a **function call**. Function calls contain the name of the function being executed followed by a list of values, called *arguments*, which are assigned to the parameters in the function definition. Our first examples have an empty parameter list, so the function calls do not take any arguments. Notice, however, that the *parentheses are required in the function call*:

```
print("First Line.")
new_line()
print("Second Line.")
```

The output of this program is:

```
First line.

Second line.
```

The extra space between the two lines is a result of the `new_line()` function call. What if we wanted more space between the lines? We could call the same function repeatedly:

```
print("First Line.")
new_line()
new_line()
new_line()
print("Second Line.")
```

Or we could write a new function named `three_lines` that prints three new lines:

```
def three_lines():
    new_line()
    new_line()
    new_line()

print("First Line.")
three_lines()
print("Second Line.")
```

This function contains three statements, all of which are indented by four spaces. Since the next statement is not indented, Python knows that it is not part of the function.

You should notice a few things about this program:

- You can call the same procedure repeatedly. In fact, it is quite common and useful to do so.
- You can have one function call another function; in this case `three_lines` calls `new_line`.

So far, it may not be clear why it is worth the trouble to create all of these new functions. Actually, there are a lot of reasons, but this example demonstrates two:

1. Creating a new function gives you an opportunity to name a group of statements. Functions can simplify a program by hiding a complex computation behind a single command and by using English words in place of arcane code.
2. Creating a new function can make a program smaller by eliminating repetitive code. For example, a short way to print nine consecutive new lines is to call `three_lines` three times.

Pulling together the code fragments from the previous section into a script named `tryme1.py`, the whole program looks like this:

```
def new_line():
    print("")

def three_lines():
    new_line()
    new_line()
    new_line()

print("First Line.")
three_lines()
print("Second Line.")
```

This program contains two function definitions: `new_line` and `three_lines`. Function definitions get executed just like other statements, but the effect is to create the new function. The statements inside the function do not get executed until the function is called, and the function definition generates no output.

As you might expect, you have to create a function before you can execute it. In other words, the function definition has to be executed before the first time it is called.

Flow of execution

In order to ensure that a function is defined before its first use, you have to know the order in which statements are executed, which is called the **flow of execution**.

Execution always begins at the first statement of the program. Statements are executed one at a time, in order from top to bottom.

Function definitions do not alter the flow of execution of the program, but remember that statements inside the function are not executed until the function is called. Although it is not common, you can define one function inside another. In this case, the inner definition isn't executed until the outer function is called.

Function calls are like a detour in the flow of execution. Instead of going to the next statement, the flow jumps to the first line of the called function, executes all the statements there, and then comes back to pick up where it left off.

That sounds simple enough, until you remember that one function can call another. While in the middle of one function, the program might have to execute the statements in another function. But while executing that new function, the program might have to execute yet another function!

Fortunately, Python is adept at keeping track of where it is, so each time a function completes, the program picks up where it left off in the function that called it. When it gets to the end of the program, it terminates.

What's the moral of this sordid tale? When you read a program, don't read from top to bottom. Instead, follow the flow of execution.

Parameters, arguments, and the `import` statement

Most functions require arguments, values that control how the function does its job. For example, if you want to find the absolute value of a number, you have to indicate what the number is. Python has a built-in function for computing the absolute value:

```
>>> abs(5)
5
>>> abs(-5)
5
```

In this example, the arguments to the `abs` function are 5 and -5.

Some functions take more than one argument. For example the built-in function `pow` takes two arguments, the base and the exponent. Inside the function, the values that are passed get assigned to variables called **parameters**.

```
>>> pow(2, 3)
8
>>> pow(7, 4)
2401
```

Another built-in function that takes more than one argument is `max`.

```
>>> max(7, 11)
11
>>> max(4, 1, 17, 2, 12)
17
>>> max(3 * 11, 5**3, 512 - 9, 1024**0)
503
```

`max` can be sent any number of arguments, separated by commas, and will return the maximum value sent. The arguments can be either simple values or expressions. In the last example, 503 is returned, since it is larger than 33, 125, and 1.

Here is an example of a user-defined function that has a parameter:

```
def print_twice(param):
    print(param)
    print(param)
```

This function takes a single **argument** and assigns it to the parameter named `param`. The value of the parameter (at this point we have no idea what it will be) is printed twice, each followed by a newline. The name `param` was chosen to reinforce the idea that it is a *parameter*, but in general, you will want to choose a name for your parameters that describes their use in the function.

The interactive Python shell provides us with a convenient way to test our functions. We can use the **import statement** to bring the functions we have defined in a script into the interpreter session. To see how this works, assume the `print_twice` function is defined in a script named `chap03.py`. We can now test it interactively by *importing* it into our Python shell session:

```
>>> from chap03 import *
>>> print_twice('Spam')
Spam
Spam
>>> print_twice(5)
5
5
>>> print_twice(3.14159)
3.14159
3.14159
```

In a function call, the value of the argument is assigned to the corresponding parameter in the function definition. In effect, it is if `param = 'Spam'` is executed when `print_twice('Spam')` is called, `param = 5` in `print_twice(5)`, and `param = 3.14159` in `print_twice(3.14159)`.

Any type of argument that can be printed can be sent to `print_twice` In the first function call, the argument is a string. In the second, it's an integer. In the third, it's a `float`.

As with built-in functions, we can use an expression as an argument for `print_twice`:

```
>>> print_twice('Spam' * 4)
SpamSpamSpamSpam
SpamSpamSpamSpam
```

`'Spam'*4` is first evaluated to `'SpamSpamSpamSpam'`, which is then passed as an argument to `print_twice`.

Composition

Just as with mathematical functions, Python functions can be **composed**, meaning that you use the result of one function as the input to another.

```
>>> print_twice(abs(-7))
7
7
>>> print_twice(max(3, 1, abs(-11), 7))
11
11
```

In the first example, `abs(-7)` evaluates to 7, which then becomes the argument to `print_twice`. In the second example we have two levels of composition, since `abs(-11)` is first evaluated to 11 before `max(3, 1, 11, 7)` is evaluated to 11 and `print_twice(11)` then displays the result.

We can also use a variable as an argument:

```
>>> sval = 'Eric, the half a bee.'
>>> print_twice(sval)
Eric, the half a bee.
Eric, the half a bee.
```

Notice something very important here. The name of the variable we pass as an argument (`sval`) has nothing to do with the name of the parameter (`param`). Again, it is as if `param = sval` is executed when `print_twice(sval)` is called. It doesn't matter what the value was named in the caller, in `print_twice` its name is `param`.

Variables and parameters are local

When you create a **local variable** inside a function, it only exists inside the function, and you cannot use it outside. For example:

```
def cat_twice(part1, part2):
    cat = part1 + part2
    print_twice(cat)
```

This function takes two arguments, concatenates them, and then prints the result twice. We can call the function with two strings:

```
>>> chant1 = "Pie Jesu domine, "
>>> chant2 = "Dona eis requiem."
>>> cat_twice(chant1, chant2)
Pie Jesu domine, Dona eis requiem.
Pie Jesu domine, Dona eis requiem.
```

When `cat_twice` terminates, the variable `cat` is destroyed. If we try to print it, we get an error:

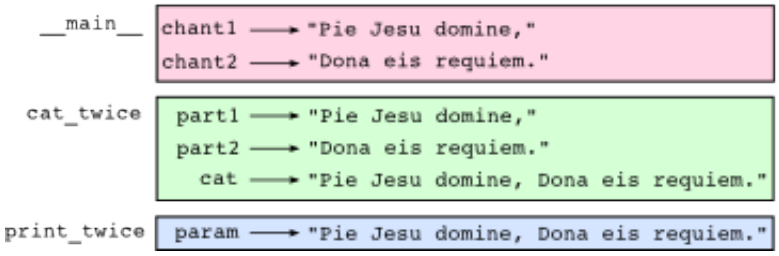
```
>>> print(cat)
NameError: name 'cat' is not defined
```

Parameters are also local. For example, outside the function `print_twice`, there is no such thing as `param`. If you try to use it, Python will complain.

Stack diagrams

To keep track of which variables can be used where, it is sometimes useful to draw a **stack diagram**. Like state diagrams, stack diagrams show the value of each variable, but they also show the function to which each variable belongs.

Each function is represented by a **frame**. A frame is a box with the name of a function beside it and the parameters and variables of the function inside it. The stack diagram for the previous example looks like this:



The order of the stack shows the flow of execution. `print_twice` was called by `cat_twice`, and `cat_twice` was called by `__main__`, which is a special name for the topmost function. When you create a variable outside of any function, it belongs to `__main__`.

Each parameter refers to the same value as its corresponding argument. So, `part1` has the same value as `chant1`, `part2` has the same value as `chant2`, and `param` has the same value as `cat`.

If an error occurs during a function call, Python prints the name of the function, and the name of the function that called it, and the name of the function that called *that*, all the way back to the top most function.

To see how this works, create a Python script named `tryme2.py` that looks like this:

```
def print_twice(param):
    print(param)
    print(param)
    print(cat)

def cat_twice(part1, part2):
    cat = part1 + part2
    print_twice(cat)

chant1 = "Pie Jesu domine, "
chant2 = "Dona eis requim."
cat_twice(chant1, chant2)
```

We've added the statement, `print(cat)` inside the `print_twice` function, but `cat` is not defined there. Running this script will produce an error message like this:

```
Traceback (innermost last):
  File "tryme2.py", line 12, in <module>
    cat_twice(chant1, chant2)
  File "tryme2.py", line 8, in cat_twice
    print_twice(cat)
  File "tryme2.py", line 4, in print_twice
    print(cat)
NameError: global name 'cat' is not defined
```

This list of functions is called a **traceback**. It tells you what program file the error occurred in, and what line, and what functions were executing at the time. It also shows the line of code that caused the error.

Notice the similarity between the traceback and the stack diagram. It's not a coincidence. In fact, another common name for a traceback is a *stack trace*.

Glossary

argument A value provided to a function when the function is called. This value is assigned to the corresponding parameter in the function.

body The second part of a compound statement. The body consists of a sequence of statements all indented the same amount from the beginning of the header. The standard amount of indentation used within the Python community is 4 spaces.

compound statement A statement that consists of two parts:

1. header - which begins with a keyword determining the statement type, and ends with a colon.
2. body - containing one or more statements indented the same amount from the header.

The syntax of a compound statement looks like this:

```
    keyword expression:
        statement
        statement ...
```

flow of execution The order in which statements are executed during a program run.

frame A box in a stack diagram that represents a function call. It contains the local variables and parameters of the function.

function A named sequence of statements that performs some useful operation. Functions may or may not take parameters and may or may not produce a result.

function call A statement that executes a function. It consists of the name of the function followed by a list of arguments enclosed in parentheses.

function composition Using the output from one function call as the input to another.

function definition A statement that creates a new function, specifying its name, parameters, and the statements it executes.

header The first part of a compound statement. Headers begin with a keyword and end with a colon (:)

import A statement which permits functions and variables defined in a Python script to be brought into the environment of another script or a running Python shell. For example, assume the following is in a script named `tryme.py`:

```
def print_thrice(thing):
    print(thing)
    print(thing)
    print(thing)

n = 42
s = "And now for something completely different..."
```

Now begin a python shell from within the same directory where `tryme.py` is located:

```
$ ls
tryme.py
$ python
>>>
```

Three names are defined in `tryme.py`: `print_thrice`, `n`, and `s`. If we try to access any of these in the shell without first importing, we get an error:

```
>>> n
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'n' is not defined
>>> print_thrice("ouch!")
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'print_thrice' is not defined
```

If we import everything from `tryme.py`, however, we can use everything defined in it:

```
>>> from tryme import *
>>> n
42
>>> s
'And now for something completely different...'
>>> print_thrice("Yipee!")
```

```
Yipee!
Yipee!
Yipee!
>>>
```

Note that you do not include the `.py` from the script name in the import statement.

local variable A variable defined inside a function. A local variable can only be used inside its function.

parameter A name used inside a function to refer to the value passed as an argument.

stack diagram A graphical representation of a stack of functions, their variables, and the values to which they refer.

traceback A list of the functions that are executing, printed when a runtime error occurs. A traceback is also commonly refered to as a *stack trace*, since it lists the functions in the order in which they are stored in the runtime stack.

Exercises

1. Using a text editor, create a Python script named `tryme3.py` . Write a function in this file called `nine_lines` that uses `three_lines` to print nine blank lines. Now add a function named `clear_screen` that prints out twenty-five blank lines. The last line of your program should be a *call* to `clear_screen`.
2. Move the last line of `tryme3.py` to the top of the program, so the *function call* to `clear_screen` appears before the *function definition*. Run the program and record what error message you get. Can you state a rule about *function definitions* and *function calls* which describes where they can appear relative to each other in a program?
3. Starting with a working version of `tryme3.py` , move the definition of `new_line` after the definition of `three_lines`. Record what happens when you run this program. Now move the definition of `new_line` below a call to `three_lines()`. Explain how this is an example of the rule you stated in the previous exercise.
4. Fill in the *body* of the *function definition* for `cat_n_times` so that it will print the string, s, n times:

```
def cat_n_times(s, n):
    <fill in your code here>
```

Save this function in a script named `import_test.py`. Now at a unix prompt, make sure you are in the same directory where the `import_test.py` is located (`ls` should show `import_test.py`). Start a Python shell and try the following:

```
>>> from import_test import *
>>> cat_n_times('Spam', 7)
SpamSpamSpamSpamSpamSpamSpam
```

If all is well, your session should work the same as this one. Experiment with other calls to `cat_n_times` until you feel comfortable with how it works.

Conditionals

The modulus operator

The **modulus operator** works on integers (and integer expressions) and yields the remainder when the first operand is divided by the second. In Python, the modulus operator is a percent sign (`%`). The syntax is the same as for other operators:

```
>>> quotient = 7 / 3
>>> print(quotient)
2
>>> remainder = 7 % 3
>>> print(remainder)
1
```

So 7 divided by 3 is 2 with 1 left over.

The modulus operator turns out to be surprisingly useful. For example, you can check whether one number is divisible by another—if `x % y` is zero, then `x` is divisible by `y`.

Also, you can extract the right-most digit or digits from a number. For example, `x % 10` yields the right-most digit of `x` (in base 10). Similarly `x % 100` yields the last two digits.

Boolean values and expressions

The Python type for storing true and false values is called `bool`, named after the British mathematician, George Boole. George Boole created *Boolean algebra*, which is the basis of all modern computer arithmetic.

There are only two **boolean values**: `True` and `False`. Capitalization is important, since `true` and `false` are not boolean values.

```
>>> type(True)
<type 'bool'>
>>> type(true)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'true' is not defined
```

A **boolean expression** is an expression that evaluates to a boolean value. The operator == compares two values and produces a boolean value:

```
>>> 5 == 5
True
>>> 5 == 6
False
```

In the first statement, the two operands are equal, so the expression evaluates to `True`; in the second statement, 5 is not equal to 6, so we get `False`.

The == operator is one of the **comparison operators**; the others are:

```
x != y               # x is not equal to y
x > y                # x is greater than y
x < y                # x is less than y
x >= y               # x is greater than or equal to y
x <= y               # x is less than or equal to y
```

Although these operations are probably familiar to you, the Python symbols are different from the mathematical symbols. A common error is to use a single equal sign (=) instead of a double equal sign (==). Remember that = is an assignment operator and == is a comparison operator. Also, there is no such thing as =< or =>.

Logical operators

There are three **logical operators**: `and`, `or`, and `not`. The semantics (meaning) of these operators is similar to their meaning in English. For example, `x > 0 and x < 10` is true only if `x` is greater than 0 *and* less than 10.

`n % 2 == 0 or n % 3 == 0` is true if *either* of the conditions is true, that is, if the number is divisible by 2 *or* 3.

Finally, the `not` operator negates a boolean expression, so `not (x > y)` is true if `(x > y)` is false, that is, if `x` is less than or equal to `y`.

Conditional execution

In order to write useful programs, we almost always need the ability to check conditions and change the behavior of the program accordingly. **Conditional statements** give us this ability. The simplest form is the ** `if` statement**:

```
if x > 0:
    print("%s is positive" % (x))
```

(See Chapter 7.13 for more information on string formatting.)

The boolean expression after the `if` statement is called the **condition**. If it is true, then the indented statement gets executed. If not, nothing happens.

The syntax for an `if` statement looks like this:

```
if BOOLEAN EXPRESSION:
    STATEMENTS
```

As with the function definition from last chapter and other compound statements, the `if` statement consists of a header and a body. The header begins with the keyword `if` followed by a *boolean expression* and ends with a colon (:).

The indented statements that follow are called a **block**. The first unindented statement marks the end of the block. A statement block inside a compound statement is called the **body** of the statement.

Each of the statements inside the body are executed in order if the boolean expression evaluates to `True`. The entire block is skipped if the boolean expression evaluates to `False`.

There is no limit on the number of statements that can appear in the body of an `if` statement, but there has to be at least one. Occasionally, it is useful to have a body with no statements (usually as a place keeper for code you haven't written yet). In that case, you can use the `pass` statement, which does nothing.

```
if True:           # This is always true
    pass           # so this is always executed, but it does nothing
```

Alternative execution

A second form of the `if` statement is alternative execution, in which there are two possibilities and the condition determines which one gets executed. The syntax looks like this:

```
if x % 2 == 0:
    print("%s is even" % (x))
else:
    print("%s is odd" % (x))
```

If the remainder when `x` is divided by 2 is 0, then we know that `x` is even, and the program displays a message to that effect. If the condition is false, the second set of statements is executed. Since the condition must be true or false, exactly one of the alternatives will be executed. The alternatives are called **branches**, because they are branches in the flow of execution.

As an aside, if you need to check the parity (evenness or oddness) of numbers often, you might *wrap this code in a function*:

```
def print_parity(x):
    if x % 2 == 0:
        print("%s is even" % (x))
```

```
    else:
        print("%s is odd" % (x))
```

For any value of x, print_parity displays an appropriate message. When you call it, you can provide any integer expression as an argument.

```
>>> print_parity(17)
17 is odd
>>> y = 41
>>> print_parity(y+1)
42 is even
```

Chained conditionals

Sometimes there are more than two possibilities and we need more than two branches. One way to express a computation like that is a **chained conditional**:

```
if x < y:
    print("%s is less than %s" % (x, y))
elif x > y:
    print("%s is greater than %s" % (x, y))
else:
    print("%s and %s are equal" % (x,y))
```

elif is an abbreviation of else if . Again, exactly one branch will be executed. There is no limit of the number of elif statements but only a single (and optional) else statement is allowed and it must be the last branch in the statement:

```
if choice == 'a':
    function_a()
elif choice == 'b':
    function_b()
elif choice == 'c':
    function_c()
else:
    print("Invalid choice.")
```

Each condition is checked in order. If the first is false, the next is checked, and so on. If one of them is true, the corresponding branch executes, and the statement ends. Even if more than one condition is true, only the first true branch executes.

Nested conditionals

One conditional can also be **nested** within another. We could have written the trichotomy example as follows:

```
if x == y:
    print("%s and %s are equal" % (x,y))
else:
    if x < y:
        print("%s is less than %s" % (x, y))
    else:
        print("%s is greater than %s" % (x, y))
```

The outer conditional contains two branches. The first branch contains a simple output statement. The second branch contains another `if` statement, which has two branches of its own. Those two branches are both output statements, although they could have been conditional statements as well.

Although the indentation of the statements makes the structure apparent, nested conditionals become difficult to read very quickly. In general, it is a good idea to avoid them when you can.

Logical operators often provide a way to simplify nested conditional statements. For example, we can rewrite the following code using a single conditional:

```
if 0 < x:
    if x < 10:
        print("x is a positive single digit.")
```

The `print` statement is executed only if we make it past both the conditionals, so we can use the `and` operator:

```
if 0 < x and x < 10:
    print("x is a positive single digit.")
```

These kinds of conditions are common, so Python provides an alternative syntax that is similar to mathematical notation:

```
if 0 < x < 10:
    print("x is a positive single digit.")
```

This condition is semantically the same as the compound boolean expression and the nested conditional.

The `return` statement

The `return` statement allows you to terminate the execution of a function before you reach the end. One reason to use it is if you detect an error condition:

```
def print_square_root(x):
    if x <= 0:
        print("Positive numbers only, please.")
        return
```

```
    result = x**0.5
    print("The square root of %s is %s" % (x, result))
```

The function `print_square_root` has a parameter named `x`. The first thing it does is check whether `x` is less than or equal to 0, in which case it displays an error message and then uses `return` to exit the function. The flow of execution immediately returns to the caller, and the remaining lines of the function are not executed.

Keyboard input

In *Input* we were introduced to Python's built-in function that gets input from the keyboard: `raw_input()`. Let's look at this again in greater depth.

When `raw_input()` is called, the program stops and waits for the user to type something. When the user presses Return or the Enter key, the program resumes and `raw_input()` returns what the user typed **as a string**:

```
>>> my_input = raw_input()
3.14159
>>> print(my_input)
3.14159
>>> type(my_input)
<type 'str'>
```

It is always a good idea to tell the user what to input. We can supply a prompt as a single argument to `raw_input()`:

```
>>> name = raw_input("What...is your name? ")
What...is your name? Arthur, King of the Britons!
>>> print(name)
Arthur, King of the Britons!
```

Notice that the prompt is a string, so it must be enclosed in quotation marks.

If we expect the user response to be an integer or a float, we can still call `raw_input()`, and then use conversion functions to convert what `raw_input()` returns to other types.

Type conversion

Each Python type comes with a built-in command that attempts to convert values of another type into that type. The `int(ARGUMENT)` command, for example, takes any value and converts it to an integer, if possible, or complains otherwise:

```
>>> int("32")
32
```

```
>>> int("Hello")
ValueError: invalid literal for int() with base 10: 'Hello'
```

`int` can also convert floating-point values to integers, but remember that it truncates the fractional part:

```
>>> int(-2.3)
-2
>>> int(3.99999)
3
>>> int("42")
42
>>> int(1.0)
1
```

The `float(ARGUMENT)` command converts integers and strings to floating-point numbers:

```
>>> float(32)
32.0
>>> float("3.14159")
3.14159
>>> float(1)
1.0
```

It may seem odd that Python distinguishes the integer value `1` from the floating-point value `1.0`. They may represent the same number, but they belong to different types. The reason is that they are represented differently inside the computer.

The `str(ARGUMENT)` command converts any argument given to it to type `string`:

```
>>> str(32)
'32'
>>> str(3.14149)
'3.14149'
>>> str(True)
'True'
>>> str(true)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'true' is not defined
```

`str(ARGUMENT)` will work with any value and convert it into a string. As mentioned earlier, `True` is boolean value; `true` is not.

For boolean values, the situation is especially interesting:

```
>>> bool(1)
True
>>> bool(0)
False
```

```
>>> bool("Ni!")
True
>>> bool("")
False
>>> bool(3.14159)
True
>>> bool(0.0)
False
```

Python assigns boolean values to values of other types. For numerical types like integers and floating-points, zero values are false and non-zero values are true. For strings, empty strings are false and non-empty strings are true.

NOTE: calling one of these conversion functions on data stored in a variable does **not** change what's stored in the variable! It takes what is stored in the variable and converts it, but does not automatically reassign the result back to the variable:

```
>>> x = "3.14159"
>>> float(x)
3.14159
>>> type(x)
<type 'str'>
```

If you want x to contain the float 3.14159, you can convert it and reassign it like this:

```
>>> x = float(x)
>>> print(x)
3.14159
>>> type(x)
<type 'float'>
```

GASP

GASP (**G**raphics **A**PI for **S**tudents of **P**ython) will enable us to write programs involving graphics. Before you can use GASP, it needs to be installed on your machine. If you are running Ubuntu GNU/Linux, see *GASP* in Appendix A. Current instructions for installing GASP on other platforms can be found at http://dev.laptop.org/pub/gasp/downloads.

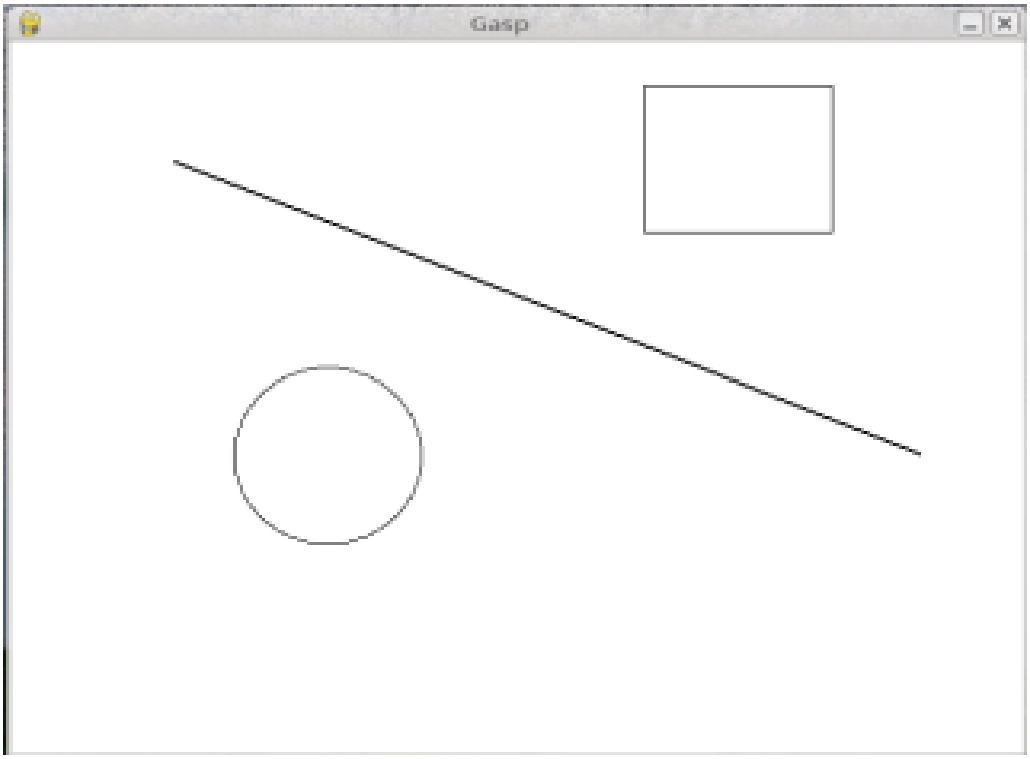
After installing gasp, try the following python script:

```
from gasp import *

begin_graphics()

Circle((200, 200), 60)
Line((100, 400), (580, 200))
Box((400, 350), 120, 100)
```

```
update_when('key_pressed')
end_graphics()
```

The second to the last command pauses and waits until a key is pressed. Without it, the screen would flash by so quickly you wouldn't see it.

Running this script, you should see a graphics window that looks like this:



We will be using gasp from here on to illustrate (pun intended) computer programming concepts and to add to our fun while learning. You can find out more about the GASP module by reading Appendix B.

Glossary

block A group of consecutive statements with the same indentation.

body The block of statements in a compound statement that follows the header.

boolean expression An expression that is either true or false.

boolean value There are exactly two boolean values: `True` and `False`. Boolean values result when a boolean expression is evaluated by the Python interepreter. They have type `bool`.

branch One of the possible paths of the flow of execution determined by conditional execution.

chained conditional A conditional branch with more than two possible flows of execution. In Python chained conditionals are written with `if ... elif ... else` statements.

comparison operator One of the operators that compares two values: ==, !=, >, <, >=, and <=.

condition The boolean expression in a conditional statement that determines which branch is executed.

conditional statement A statement that controls the flow of execution depending on some condition. In Python the keywords `if`, `elif`, and `else` are used for conditional statements.

logical operator One of the operators that combines boolean expressions: `and`, `or`, and `not`.

modulus operator An operator, denoted with a percent sign (`%`), that works on integers and yields the remainder when one number is divided by another.

nesting One program structure within another, such as a conditional statement inside a branch of another conditional statement.

prompt A visual cue that tells the user to input data.

type conversion An explicit statement that takes a value of one type and computes a corresponding value of another type.

wrapping code in a function The process of adding a function header and parameters to a sequence of program statements is often refered to as "wrapping the code in a function". This process is very useful whenever the program statements in question are going to be used multiple times.

Exercises

1. Try to evaluate the following numerical expressions in your head, then use the Python interpreter to check your results:

   (a) `>>> 5 % 2`

   (b) `>>> 9 % 5`

   (c) `>>> 15 % 12`

   (d) `>>> 12 % 15`

   (e) `>>> 6 % 6`

   (f) `>>> 0 % 7`

   (g) `>>> 7 % 0`

   What happened with the last example? Why? If you were able to correctly anticipate the computer's response in all but the last one, it is time to move on. If not, take time now to make up examples of your own. Explore the modulus operator until you are confident you understand how it works.

```
2. if x < y:
       print("%s is less than %s" % (x, y))
   elif x > y:
       print("%s is greater than %s" % (x, y))
   else:
       print("%s and %s are equal" % (x, y))
```

Wrap this code in a function called `compare(x, y)`. Call `compare` three times: one each where the first argument is less than, greater than, and equal to the second argument.

3. To better understand boolean expressions, it is helpful to construct truth tables. Two boolean expressions are *logically equivalent* if and only if they have the same truth table.

   The following Python script prints out the truth table for any boolean expression in two variables: p and q:

```
expression = raw_input("Enter a boolean expression in two variables, p and q:

print(" p      q      %s" % (expression))
length = len(" p      q      %s" % (expression))
print(length*"=")

for p in True, False:
    for q in True, False:
        print("%-7s %-7s %-7s" % (p, q, eval(expression)))
```

You will learn how this script works in later chapters. For now, you will use it to learn about boolean expressions. Copy this program to a file named `p_and_q.py`, then run it from the command line and give it: `p or q`, when prompted for a boolean expression. You should get the following output:

```
 p      q      p or q
====================
True    True    True
True    False   True
False   True    True
False   False   False
```

Now that we see how it works, let's wrap it in a function to make it easier to use:

```
def truth_table(expression):
    print(" p      q      %s" % (expression))
    length = len( " p      q      %s" % expression)
    print(length*"=")

    for p in True, False:
        for q in True, False:
            print("%-7s %-7s %-7s" % (p, q, eval(expression)))
```

We can import it into a Python shell and call `truth_table` with a string containing our boolean expression in p and q as an argument:

```
>>> from p_and_q import *
>>> truth_table("p or q")
p       q       p or q
====================
True    True    True
True    False   True
False   True    True
False   False   False
>>>
```

Use the `truth_table` functions with the following boolean expressions, recording the truth table produced each time:

(a) not(p or q)

(b) p and q

(c) not(p and q)

(d) not(p) or not(q)

(e) not(p) and not(q)

Which of these are logically equivalent?

4. Enter the following expressions into the Python shell:

```
True or False
True and False
not(False) and True
True or 7
False or 7
True and 0
False or 8
"happy" and "sad"
"happy" or "sad"
"" and "sad"
"happy" and ""
```

Analyze these results. What observations can you make about values of different types and logical operators? Can you write these observations in the form of simple *rules* about `and` and `or` expressions?

5.
```
if choice == 'a':
    function_a()
elif choice == 'b':
    function_b()
elif choice == 'c':
```

```
        function_c()
    else:
        print("Invalid choice.")
```

Wrap this code in a function called `dispatch(choice)`. Then define `function_a`, `function_b`, and `function_c` so that they print out a message saying they were called. For example:

```
def function_a():
    print("function_a was called...")
```

Put the four functions (`dispatch`, `function_a`, `function_b`, and `function_c` into a script named `ch04e05.py`. At the bottom of this script add a call to `dispatch('b')`. Your output should be:

```
function_b was called...
```

Finally, modify the script so that user can enter 'a', 'b', or 'c'. Test it by importing your script into the Python shell.

6. Write a function named `is_divisible_by_3` that takes a single integer as an argument and prints "This number is divisible by three." if the argument is evenly divisible by 3 and "This number is not divisible by three." otherwise.

   Now write a similar function named `is_divisible_by_5`.

7. Generalize the functions you wrote in the previous exercise into a function named `is_divisible_by_n(x, n)` that takes two integer arguments and prints out whether the first is divisible by the second. Save this in a file named `ch04e07.py`. Import it into a shell and try it out. A sample session might look like this:

```
>>> from ch04e07 import *
>>> is_divisible_by_n(20, 4)
Yes, 20 is divisible by 4
>>> is_divisible_by_n(21, 8)
No, 21 is not divisible by 8
```

8. What will be the output of the following?

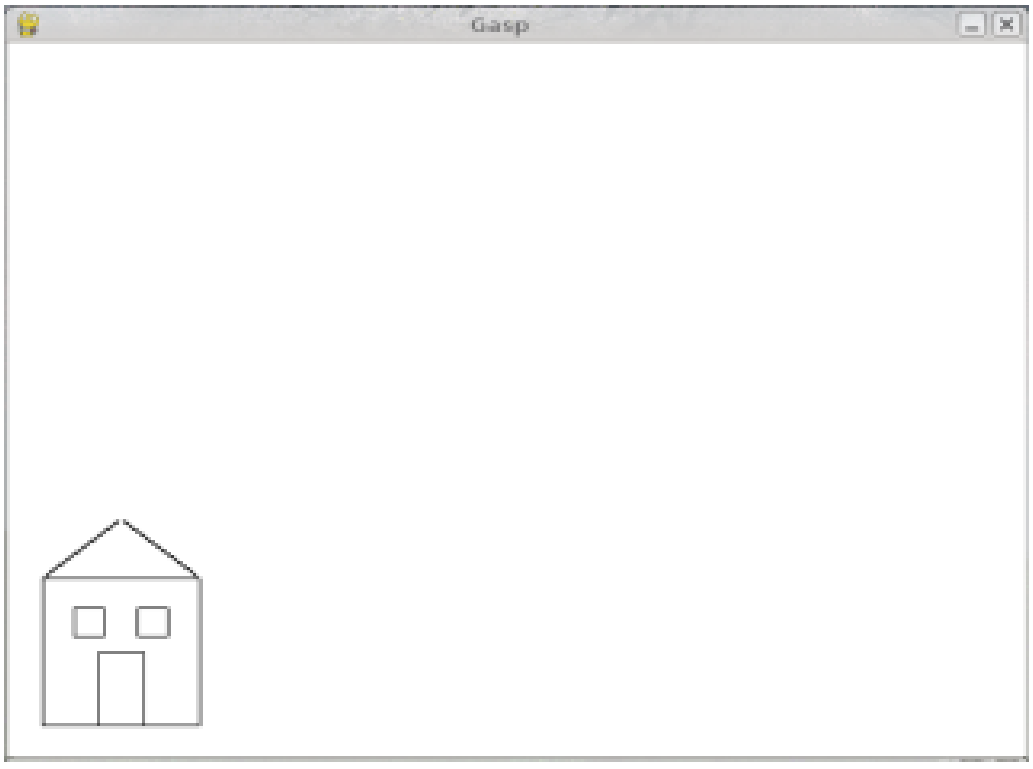
```
if "Ni!":
    print('We are the Knights who say, "Ni!"')
else:
    print("Stop it! No more of this!")

if 0:
    print("And now for something completely different...")
else:
    print("What's all this, then?")
```

Explain what happened and why it happened.

9. The following gasp script, in a file named `house.py`, draws a simple house on a gasp canvas:

```
from gasp import *          # import everything from the gasp library

begin_graphics()            # open the graphics canvas

Box((20, 20), 100, 100)     # the house
Box((55, 20), 30, 50)       # the door
Box((40, 80), 20, 20)       # the left window
Box((80, 80), 20, 20)       # the right window
Line((20, 120), (70, 160))  # the left roof
Line((70, 160), (120, 120)) # the right roof

update_when('key_pressed')  # keep the canvas open until a key is pressed
end_graphics()              # close the canvas (which would happen
                            # anyway, since the script ends here, but it
                            # is better to be explicit).
```

Run this script and confirm that you get a window that looks like this:



(a) Wrap the house code in a function named `draw_house()`.

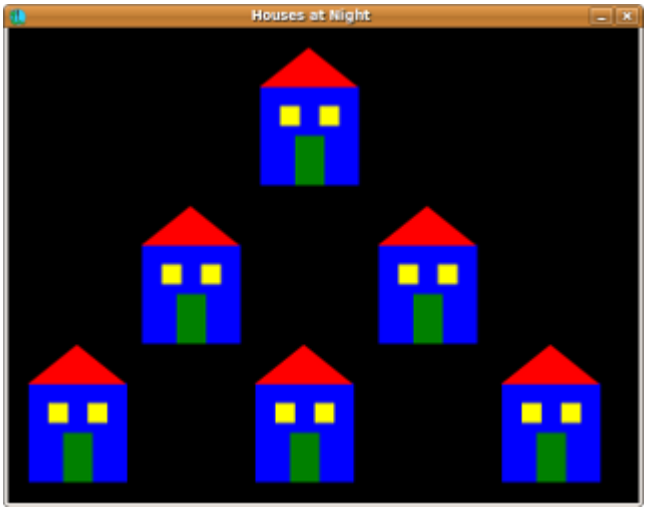
(b) Run the script now. Do you see a house? Why not?

(c) Add a call to `draw_house()` at the botton of the script so that the house returns to the screen.

(d) *Parameterize* the function with x and y parameters – the header should then become `def draw_house(x, y):`, so that you can pass in the location of the house on the canvas.

(e) Use `draw_house` to place five houses on the canvas in different locations.

10. *Exploration:* Read over Appendix B and write a script named `houses.py` that produces the following when run:



hint: You will need to use a `Polygon` for the roof instead of two `Lines` to get `filled=True` to work with it.

Fruitful functions

Return values

The built-in functions we have used, such as `abs`, `pow`, and `max`, have produced results. Calling each of these functions generates a value, which we usually assign to a variable or use as part of an expression.

```
biggest = max(3, 7, 2, 5)
x = abs(3 - 11) + 10
```

But so far, none of the functions we have written has returned a value.

In this chapter, we are going to write functions that return values, which we will call *fruitful functions*, for want of a better name. The first example is `area`, which returns the area of a circle with the given radius:

```
def area(radius):
    temp = 3.14159 * radius**2
    return temp
```

We have seen the `return` statement before, but in a fruitful function the `return` statement includes a **return value**. This statement means: Return immediately from this function and use the following expression as a return value. The expression provided can be arbitrarily complicated, so we could have written this function more concisely:

```
def area(radius):
    return 3.14159 * radius**2
```

On the other hand, **temporary variables** like `temp` often make debugging easier.

Sometimes it is useful to have multiple return statements, one in each branch of a conditional. We have already seen the built-in `abs`, now we see how to write our own:

```
def absolute_value(x):
    if x < 0:
        return -x
    else:
        return x
```

Since these `return` statements are in an alternative conditional, only one will be executed. As soon as one is executed, the function terminates without executing any subsequent statements.

Another way to write the above function is to leave out the `else` and just follow the `if` condition by the second `return` statement.

```
def absolute_value(x):
    if x < 0:
        return -x
    return x
```

Think about this version and convince yourself it works the same as the first one.

Code that appears after a `return` statement, or any other place the flow of execution can never reach, is called **dead code**.

In a fruitful function, it is a good idea to ensure that every possible path through the program hits a `return` statement. The following version of `absolute_value` fails to do this:

```
def absolute_value(x):
    if x < 0:
        return -x
    elif x > 0:
        return x
```

This version is not correct because if `x` happens to be 0, neither condition is true, and the function ends without hitting a `return` statement. In this case, the return value is a special value called **None**:

```
>>> print(absolute_value(0))
None
```

`None` is the unique value of a type called the `NoneType`:

```
>>> type(None)
```

All Python functions return `None` whenever they do not return another value.

Program development

At this point, you should be able to look at complete functions and tell what they do. Also, if you have been doing the exercises, you have written some small functions. As you write larger functions, you might start to have more difficulty, especially with runtime and semantic errors.

To deal with increasingly complex programs, we are going to suggest a technique called **incremental development**. The goal of incremental development is to avoid long debugging sessions by adding and testing only a small amount of code at a time.

As an example, suppose you want to find the distance between two points, given by the coordinates (x_1, y_1) and (x_2, y_2). By the Pythagorean theorem, the distance is:

$$distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The first step is to consider what a `distance` function should look like in Python. In other words, what are the inputs (parameters) and what is the output (return value)?

In this case, the two points are the inputs, which we can represent using four parameters. The return value is the distance, which is a floating-point value.

Already we can write an outline of the function:

```
def distance(x1, y1, x2, y2):
    return 0.0
```

Obviously, this version of the function doesn't compute distances; it always returns zero. But it is syntactically correct, and it will run, which means that we can test it before we make it more complicated.

To test the new function, we call it with sample values:

```
>>> distance(1, 2, 4, 6)
0.0
```

We chose these values so that the horizontal distance equals 3 and the vertical distance equals 4; that way, the result is 5 (the hypotenuse of a 3-4-5 triangle). When testing a function, it is useful to know the right answer.

At this point we have confirmed that the function is syntactically correct, and we can start adding lines of code. After each incremental change, we test the function again. If an error occurs at any point, we know where it must be — in the last line we added.

A logical first step in the computation is to find the differences x_2- x_1 and y_2- y_1. We will store those values in temporary variables named `dx` and `dy` and print them.

```
def distance(x1, y1, x2, y2):
    dx = x2 - x1
    dy = y2 - y1
    print("dx is", dx)
    print("dy is", dy)
    return 0.0
```

If the function is working, the outputs should be 3 and 4. If so, we know that the function is getting the right parameters and performing the first computation correctly. If not, there are only a few lines to check.

Next we compute the sum of squares of `dx` and `dy`:

```
def distance(x1, y1, x2, y2):
    dx = x2 - x1
    dy = y2 - y1
    dsquared = dx**2 + dy**2
    print("dsquared is: %s" % (dsquared))
    return 0.0
```

Notice that we removed the `print` statements we wrote in the previous step. Code like that is called **scaffolding** because it is helpful for building the program but is not part of the final product.

Again, we would run the program at this stage and check the output (which should be 25).

Finally, using the fractional exponent `0.5` to find the square root, we compute and return the result:

```
def distance(x1, y1, x2, y2):
    dx = x2 - x1
    dy = y2 - y1
    dsquared = dx**2 + dy**2
    result = dsquared**0.5
    return result
```

If that works correctly, you are done. Otherwise, you might want to print the value of `result` before the return statement.

When you start out, you should add only a line or two of code at a time. As you gain more experience, you might find yourself writing and debugging bigger chunks. Either way, the incremental development process can save you a lot of debugging time.

The key aspects of the process are:

1. Start with a working program and make small incremental changes. At any point, if there is an error, you will know exactly where it is.
2. Use temporary variables to hold intermediate values so you can output and check them.

3. Once the program is working, you might want to remove some of the scaffolding or consolidate multiple statements into compound expressions, but only if it does not make the program difficult to read.

Composition

As you should expect by now, you can call one function from within another. This ability is called **composition**.

As an example, we'll write a function that takes two points, the center of the circle and a point on the perimeter, and computes the area of the circle.

Assume that the center point is stored in the variables `xc` and `yc`, and the perimeter point is in `xp` and `yp`. The first step is to find the radius of the circle, which is the distance between the two points. Fortunately, we've just written a function, `distance`, that does just that, so now all we have to do is use it:

```
radius = distance(xc, yc, xp, yp)
```

The second step is to find the area of a circle with that radius and return it. Again we will use one of our earlier functions:

```
result = area(radius)
return result
```

Wrapping that up in a function, we get:

```
def area2(xc, yc, xp, yp):
    radius = distance(xc, yc, xp, yp)
    result = area(radius)
    return result
```

We called this function `area2` to distinguish it from the `area` function defined earlier. There can only be one function with a given name within a given module.

The temporary variables `radius` and `result` are useful for development and debugging, but once the program is working, we can make it more concise by composing the function calls:

```
def area2(xc, yc, xp, yp):
    return area(distance(xc, yc, xp, yp))
```

Boolean functions

Functions can return boolean values, which is often convenient for hiding complicated tests inside functions. For example:

```
def is_divisible(x, y):
    if x % y == 0:
        return True
    else:
        return False
```

The name of this function is `is_divisible`. It is common to give **boolean functions** names that sound like yes/no questions. `is_divisible` returns either `True` or `False` to indicate whether the `x` is or is not divisible by `y`.

We can make the function more concise by taking advantage of the fact that the condition of the `if` statement is itself a boolean expression. We can return it directly, avoiding the `if` statement altogether:

```
def is_divisible(x, y):
    return x % y == 0
```

This session shows the new function in action:

```
>>> is_divisible(6, 4)
False
>>> is_divisible(6, 3)
True
```

Boolean functions are often used in conditional statements:

```
if is_divisible(x, y):
    print("x is divisible by y")
else:
    print("x is not divisible by y")
```

It might be tempting to write something like:

```
if is_divisible(x, y) == True:
```

but the extra comparison is unnecessary.

The `function` type

A function is another type in Python, joining `int`, `float`, `str`, `bool`, and `NoneType`.

```
>>> def func():
...     return "function func was called..."
...
>>> type(func)
<type 'function'>
>>>
```

Just like the other types, functions can be passed as arguments to other functions:

```
def f(n):
    return 3*n - 6

def g(n):
    return 5*n + 2

def h(n):
    return -2*n + 17

def doto(value, func):
    return func(value)

print(doto(7, f))
print(doto(7, g))
print(doto(7, h))
```

`doto` is called three times. 7 is the argument for value each time, and the functions `f`, `g`, and `h` are passed in for `func` in turn. The output of this script is:

```
15
37
3
```

This example is a bit contrived, but we will see situations later where it is quite useful to pass a function to a function.

Programming with style

Readability is very important to programmers, since in practice programs are read and modified far more often than they are written. All the code examples in this book will be consistent with the *Python Enhancement Proposal 8* (PEP 8), a style guide developed by the Python community.

We'll have more to say about style as our programs become more complex, but a few pointers will be helpful already:

- use 4 spaces for indentation
- imports should go at the top of the file
- separate function definitions with two blank lines
- keep function definitions together
- keep top level statements, including function calls, together at the bottom of the program

Triple quoted strings

In addition to the single and double quoted strings we first saw in *Values and data types*, Python also has *triple quoted strings*:

```
>>> type("""This is a triple quoted string using 3 double quotes.""")
<type 'str'>
>>> type('''This triple quoted strings uses 3 single quotes.''')
<type 'str'>
>>>
```

Triple quoted strings can contain both single and double quotes inside them:

```
>>> print('''"Oh no", she exclaimed, "Ben's bike is broken!"''')
"Oh no", she exclaimed, "Ben's bike is broken!"
>>>
```

Finally, triple quoted strings can span multiple lines:

```
>>> message = """This message will
... span several
... lines."""
>>> print(message)
This message will
span several
lines.
>>>
```

Unit testing with `doctest`

It is a common best practice in software development these days to include automatic **unit testing** of source code. Unit testing provides a way to automatically verify that individual pieces of code, such as functions, are working properly. This makes it possible to change the implementation of a function at a later time and quickly test that it still does what it was intended to do.

Python has a built-in `doctest` module for easy unit testing. Doctests can be written within a triple quoted string on the *first line* of the body of a function or script. They consist of sample interpreter sessions with a series of inputs to a Python prompt followed by the expected output from the Python interpreter.

The `doctest` module automatically runs any statement begining with >>> and compares the following line with the output from the interpreter.

To see how this works, put the following in a script named `myfunctions.py`:

```
def is_divisible_by_2_or_5(n):
    """
      >>> is_divisible_by_2_or_5(8)
```

```
      True
    """

if __name__ == '__main__':
    import doctest
    doctest.testmod()
```

The last three lines are what make `doctest` run. Put them at the bottom of any file that includes doctests. We will explain how they work in Chapter 10 when we discuss modules.

Running the script will produce the following output:

```
$ python myfunctions.py
**********************************************************************
File "myfunctions.py", line 3, in __main__.is_divisible_by_2_or_5
Failed example:
    is_divisible_by_2_or_5(8)
Expected:
    True
Got nothing
**********************************************************************
1 items had failures:
   1 of   1 in __main__.is_divisible_by_2_or_5
***Test Failed*** 1 failures.
$
```

This is an example of a *failing test*. The test says: if you call `is_divisible_by_2_or_5(8)` the result should be `True`. Since `is_divisible_by_2_or_5` as written doesn't return anything at all, the test fails, and doctest tells us that it expected `True` but got nothing.

We can make this test pass by returning `True`:

```
def is_divisible_by_2_or_5(n):
    """
      >>> is_divisible_by_2_or_5(8)
      True
    """
    return True

if __name__ == '__main__':
    import doctest
    doctest.testmod()
```

If we run it now, there will be no output, which indicates that the test passed. Note again that the doctest string must be placed immediately after the function definition header in order to run.

To see more detailed out put, call the script with the `-v` command line option:

```
$ python myfunctions.py -v
Trying:
    is_divisible_by_2_or_5(8)
Expecting:
    True
ok
1 items had no tests:
    __main__
1 items passed all tests:
   1 tests in __main__.is_divisible_by_2_or_5
1 tests in 2 items.
1 passed and 0 failed.
Test passed.
$
```

While the test passed, our test suite is clearly inadequete, since `is_divisible_by_2_or_5` will now return `True` no matter what argument is passed to it. Here is a completed version with a more complete test suite and code that makes the tests pass:

```
def is_divisible_by_2_or_5(n):
    """
      >>> is_divisible_by_2_or_5(8)
      True
      >>> is_divisible_by_2_or_5(7)
      False
      >>> is_divisible_by_2_or_5(5)
      True
      >>> is_divisible_by_2_or_5(9)
      False
    """
    return n % 2 == 0 or n % 5 == 0

if __name__ == '__main__':
    import doctest
    doctest.testmod()
```

Run this script now with the `-v` command line option and see what you get.

Glossary

boolean function A function that returns a boolean value.

composition (of functions) Calling one function from within the body of another, or using the return value of one function as an argument to the call of another.

dead code Part of a program that can never be executed, often because it appears after a `return` statement.

fruitful function A function that yields a return value.

incremental development A program development plan intended to avoid debugging by adding and testing only a small amount of code at a time.

None A special Python value returned by functions that have no return statement, or a return statement without an argument. `None` is the only value of the type, `NoneType`.

return value The value provided as the result of a function call.

scaffolding Code that is used during program development but is not part of the final version.

temporary variable A variable used to store an intermediate value in a complex calculation.

unit testing An automatic procedure used to validate that individual units of code are working properly. Python has `doctest` built in for this purpose.

Exercises

All of the exercises below should be added to a file named `ch05.py` that contains the following at the bottom:

```
if __name__ == '__main__':
    import doctest
    doctest.testmod()
```

After completing each exercise in turn, run the program to confirm that the doctests for your new function pass.

1. Write a `compare` function that returns `1` if `a > b`, `0` if `a == b`, and `-1` if `a < b`.

```
    def compare(a, b):
        """
          >>> compare(5, 4)
          1
          >>> compare(7, 7)
          0
          >>> compare(2, 3)
          -1
          >>> compare(42, 1)
          1
        """
        #  Your function body should begin here.
```

   Fill in the body of the function so the doctests pass.

2. Use incremental development to write a function called `hypotenuse` that returns the length of the hypotenuse of a right triangle given the lengths of the two legs as parameters. Record each stage of the incremental development process as you go.

```
def hypotenuse(a, b):
    """
      >>> hypotenuse(3, 4)
      5.0
      >>> hypotenuse(12, 5)
      13.0
      >>> hypotenuse(7, 24)
      25.0
      >>> hypotenuse(9, 12)
      15.0
    """
```

When you are finished add your completed function with the doctests to `ch05.py` and confirm that the doctests pass.

3. Write a function `slope(x1, y1, x2, y2)` that returns the slope of the line through the points (x1, y1) and (x2, y2). Be sure your implementation of `slope` can pass the following doctests:

```
def slope(x1, y1, x2, y2):
    """
      >>> slope(5, 3, 4, 2)
      1.0
      >>> slope(1, 2, 3, 2)
      0.0
      >>> slope(1, 2, 3, 3)
      0.5
      >>> slope(2, 4, 1, 2)
      2.0
    """
```

Then a call to `slope` in a new function named `intercept(x1, y1, x2, y2)` that returns the y-intercept of the line through the points `(x1, y1)` and `(x2, y2)`.

```
def intercept(x1, y1, x2, y2):
    """
      >>> intercept(1, 6, 3, 12)
      3.0
      >>> intercept(6, 1, 1, 6)
      7.0
      >>> intercept(4, 6, 12, 8)
      5.0
    """
```

`intercept` should pass the doctests above.

4. Write a function called `is_even(n)` that takes an integer as an argument and returns `True` if the argument is an **even number** and `False` if it is **odd**.

Add your own doctests to this function.

5. Now write the function `is_odd(n)` that returns `True` when `n` is odd and `False` otherwise. Include doctests for this function as you write it.

   Finally, modify it so that it uses a call to `is_even` to determine if its argument is an odd integer.

6. Add the following function definition header and doctests to `ch05.py`:

```
def is_factor(f, n):
    """
      >>> is_factor(3, 12)
      True
      >>> is_factor(5, 12)
      False
      >>> is_factor(7, 14)
      True
      >>> is_factor(2, 14)
      True
      >>> is_factor(7, 15)
      False
    """
```

   Add a body to `is_factor` to make the doctests pass.

7. Add the following function definition header and doctests to `ch05.py`:

```
def is_multiple(m, n):
    """
      >>> is_multiple(12, 3)
      True
      >>> is_multiple(12, 4)
      True
      >>> is_multiple(12, 5)
      False
      >>> is_multiple(12, 6)
      True
      >>> is_multiple(12, 7)
      False
    """
```

   Add a body to `is_multiple` to make the doctests pass. Can you find a way to use `is_factor` in your definition of `is_multiple`?

8. Add the following function definition header and doctests to `ch05.py`:

```
def f2c(t):
    """
      >>> f2c(212)
      100
```

```
        >>> f2c(32)
        0
        >>> f2c(-40)
        -40
        >>> f2c(36)
        2
        >>> f2c(37)
        3
        >>> f2c(38)
        3
        >>> f2c(39)
        4
    """
```

Write a body for the function definition of `f2c` designed to return the integer value of the nearest degree Celsius for given tempurature in Fahrenheit. (*hint:* you may want to make use of the built-in function, `round`. Try printing `round.__doc__` in a Python shell and experimenting with round until you are comfortable with how it works.)

9. Add the following function definition header and doctests to `ch05.py`:

```
def c2f(t):
    """
        >>> c2f(0)
        32
        >>> c2f(100)
        212
        >>> c2f(-40)
        -40
        >>> c2f(12)
        54
        >>> c2f(18)
        64
        >>> c2f(-48)
        -54
    """
```

Add a function body for `c2f` to convert from Celsius to Fahrenheit.

Iteration

Multiple assignment

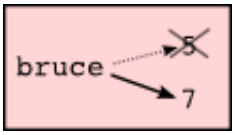
As you may have discovered, it is legal to make more than one assignment to the same variable. A new assignment makes an existing variable refer to a new value (and stop referring to the old

value).

```
bruce = 5
print(bruce)
bruce = 7
print(bruce)
```

The output of this program is `5` on one line and `7` on the next because the first time `bruce` is printed, its value is 5, and the second time, its value is 7.

Here is what **multiple assignment** looks like in a state diagram:



With multiple assignment it is especially important to distinguish between an assignment operation and a statement of equality. Because Python uses the equal sign (=) for assignment, it is tempting to interpret a statement like `a = b` as a statement of equality. It is not!

First, equality is symmetric and assignment is not. For example, in mathematics, if a = 7 then 7 = a. But in Python, the statement `a = 7` is legal and `7 = a` is not.

Furthermore, in mathematics, a statement of equality is always true. If a = b now, then a will always equal b. In Python, an assignment statement can make two variables equal, but they don't have to stay that way:

```
a = 5
b = a     # a and b are now equal
a = 3     # a and b are no longer equal
```

The third line changes the value of `a` but does not change the value of `b`, so they are no longer equal. (In some programming languages, a different symbol is used for assignment, such as <− or :=, to avoid confusion.)

Updating variables

One of the most common forms of multiple assignment is an update, where the new value of the variable depends on the old.

```
x = x + 1
```

This means get the current value of x, add one, and then update x with the new value.

If you try to update a variable that doesn't exist, you get an error, because Python evaluates the expression on the right side of the assignment operator before it assigns the resulting value to the name on the left:

```
>>> x = x + 1
Traceback (most recent call last):
  File "", line 1, in
NameError: name 'x' is not defined
```

Before you can update a variable, you have to **initialize** it, usually with a simple assignment:

```
>>> x = 0
>>> x = x + 1
>>>
```

Updating a variable by adding 1 is called an **increment**; subtracting 1 is called a **decrement**.

The `while` statement

Computers are often used to automate repetitive tasks. Repeating identical or similar tasks without making errors is something that computers do well and people do poorly.

Repeated execution of a set of statements is called **iteration**. Because iteration is so common, Python provides several language features to make it easier. The first feature we are going to look at is the `while` statement.

Here is a function called `countdown` that demonstrates the use of the `while` statement:

```
def countdown(n):
    while n > 0:
        print(n)
        n = n-1
    print("Blastoff!")
```

You can almost read the `while` statement as if it were English. It means, While `n` is greater than 0, continue displaying the value of `n` and then reducing the value of `n` by 1. When you get to 0, display the word `Blastoff!`

More formally, here is the flow of execution for a `while` statement:

1. Evaluate the condition, yielding `False` or `True`.
2. If the condition is false, exit the `while` statement and continue execution at the next statement (after the `while` loop code block).
3. If the condition is true, execute each of the statements in the body and then go back to step 1.

The body consists of all of the statements below the header with the same indentation (and sometimes called the `while` loop *code block*).

This type of flow is called a **loop** because the third step loops back around to the top. Notice that if the condition is false the first time through the loop, the statements inside the loop are never executed.

The body of the loop should change the value of one or more variables so that eventually the condition becomes false and the loop terminates. Otherwise the loop will repeat forever, which is called an **infinite loop**. An endless source of amusement for computer scientists is the observation that the directions on shampoo, Lather, rinse, repeat, are an infinite loop.

In the case of `countdown`, we can prove that the loop terminates because we know that the value of `n` is finite, and we can see that the value of `n` gets smaller each time through the loop, so eventually we have to get to 0. In other cases, it is not so easy to tell. Look at the following function, defined for all positive integers `n`:

```
def sequence(n):
    while n != 1:
        print(n)
        if n % 2 == 0:          # n is even
            n = n / 2
        else:                   # n is odd
            n = n * 3 + 1
```

The condition for this loop is `n != 1`, so the loop will continue until `n` is `1`, which will make the condition false.

Each time through the loop, the program outputs the value of `n` and then checks whether it is even or odd. If it is even, the value of `n` is divided by 2. If it is odd, the value is replaced by `n * 3 + 1`. For example, if the starting value (the argument passed to sequence) is 3, the resulting sequence is 3, 10, 5, 16, 8, 4, 2, 1.

Since `n` sometimes increases and sometimes decreases, there is no obvious proof that `n` will ever reach 1, or that the program terminates. For some particular values of `n`, we can prove termination. For example, if the starting value is a power of two, then the value of `n` will be even each time through the loop until it reaches 1. The previous example ends with such a sequence, starting with 16.

Particular values aside, the interesting question is whether we can prove that this program terminates for *all* values of `n`. So far, no one has been able to prove it *or* disprove it!

Tracing a program

To write effective computer programs a programmer needs to develop the ability to **trace** the execution of a computer program. Tracing involves becoming the computer and following the flow of execution through a sample program run, recording the state of all variables and any output the program generates after each instruction is executed.

To understand this process, let's trace the call to `sequence(3)` from the previous section. At the start of the trace, we have a local variable, `n` (the parameter), with an initial value of 3. Since 3 is not equal to 1, the `while` loop body is executed. 3 is printed and `3 % 2 == 0` is evaluated. Since it evaluates to `False`, the `else` branch is executed and `3 * 3 + 1` is evaluated and assigned to `n`.

To keep track of all this as you hand trace a program, make a column heading on a piece of paper for each variable created as the program runs and another one for output. Our trace so far would look something like this:

```
n                 output
--                ------
3                 3
10
```

Since `10 != 1` evaluates to `True`, the loop body is again executed, and 10 is printed. `10 % 2 == 0` is true, so the `if` branch is executed and `n` becomes 5. By the end of the trace we have:

```
n                 output
--                ------
3                 3
10                10
5                 5
16                16
8                 8
4                 4
2                 2
1
```

Tracing can be a bit tedious and error prone (that's why we get computers to do this stuff in the first place!), but it is an essential skill for a programmer to have. From this trace we can learn a lot about the way our code works. We can observe that as soon as n becomes a power of 2, for example, the program will require $\log_2(n)$ executions of the loop body to complete. We can also see that the final 1 will not be printed as output.

Counting digits

The following function counts the number of decimal digits in a positive integer expressed in decimal format:

```
def num_digits(n):
    count = 0
    while n > 0:
        count = count + 1
        n = n / 10
    return count
```

A call to `num_digits(710)` will return `3`. Trace the execution of this function call to convince yourself that it works.

This function demonstrates another pattern of computation called a **counter** or **accumulator**. The variable `count` is initialized to 0 and then incremented each time the loop body is executed. When the loop exits, `count` contains the result – the total number of times the loop body was executed, which is the same as the number of digits.

If we wanted to only count digits that are either 0 or 5, adding a conditional before incrementing the counter will do the trick:

```
def num_zero_and_five_digits(n):
    count = 0
    while n > 0:
        digit = n % 10
        if digit == 0 or digit == 5:
            count = count + 1
        n = n / 10
    return count
```

Confirm that `num_zero_and_five_digits(1055030250)` returns 7.

Abbreviated assignment

Incrementing a variable is so common that Python provides an abbreviated syntax for it:

```
>>> count = 0
>>> count += 1
>>> count
1
>>> count += 1
>>> count
2
>>>
```

`count += 1` is an abreviation for `count = count + 1`. The increment value does not have to be 1:

```
>>> n = 2
>>> n += 5
>>> n
7
>>>
```

There are also abbreviations for `-=`, `*=`, `/=`, and `%=`:

```
>>> n = 2
>>> n *= 5
>>> n
10
>>> n -= 4
>>> n
6
>>> n /= 2
>>> n
3
```

```
>>> n %= 2
>>> n
1
```

Tables

One of the things loops are good for is generating tabular data. Before computers were readily available, people had to calculate logarithms, sines and cosines, and other mathematical functions by hand. To make that easier, mathematics books contained long tables listing the values of these functions. Creating the tables was slow and boring, and they tended to be full of errors.

When computers appeared on the scene, one of the initial reactions was, This is great! We can use the computers to generate the tables, so there will be no errors. That turned out to be true (mostly) but shortsighted. Soon thereafter, computers and calculators were so pervasive that the tables became obsolete.

Well, almost. For some operations, computers use tables of values to get an approximate answer and then perform computations to improve the approximation. In some cases, there have been errors in the underlying tables, most famously in the table the Intel Pentium used to perform floating-point division.

Although a log table is not as useful as it once was, it still makes a good example of iteration. The following program outputs a sequence of values in the left column and 2 raised to the power of that value in the right column:

```
x = 1
while x < 13:
    print("%d\t%d" % (x, 2**x))
    x += 1
```

The string `'\t'` represents a **tab character**. The backslash character in `'\t'` indicates the beginning of an **escape sequence**. Escape sequences are used to represent invisible characters like tabs and newlines. The sequence `\n` represents a **newline**.

NOTE: if you are confused by the `%d` string formatting, see str_formatting in Section 7.13.

An escape sequence can appear anywhere in a string; in this example, the tab escape sequence is the only thing in the string. How do you think you represent a backslash in a string?

As characters and strings are displayed on the screen, an invisible marker called the **cursor** keeps track of where the next character will go. After a `print` statement, the cursor normally goes to the beginning of the next line.

The tab character shifts the cursor to the right until it reaches one of the tab stops. Tabs are useful for making columns of text line up, as in the output of the previous program:

```
1       2
2       4
```

```
3       8
4       16
5       32
6       64
7       128
8       256
9       512
10      1024
11      2048
12      4096
```

Because of the tab characters between the columns, the position of the second column does not depend on the number of digits in the first column.

Two-dimensional tables

In this next example, we'd like to use a loop to print multiple elements on the same line. Unfortunately, the `print` statement as we have been using it does not allow us to print without also printing a blank line. For example:

```
x = 1
while x <= 6:
    print("%d " % (x))
    x += 1
```

This produces the following output, with each of the numbers on the same line:

```
1
2
3
4
5
6
```

We would like a solution that allows all of those numbers on the same line. One way to do this is to *accumulate* a string, then print out the final string.

In the example below, `output` is initialized to the empty string. Then, each time through the `while` loop, we add the value of `x` (converted to a string) plus a space.

```
x = 1
output = ""
while x <= 6:
  output += str(x) + " "
  x += 1
print(output)
```

This produces the following output, with each of the numbers on the same line:

```
1 2 3 4 5 6
```

Now that you can print multiple items on the same line, let's continue with building a two-dimensional table.

A two-dimensional table is a table where you read the value at the intersection of a row and a column. A multiplication table is a good example. Let's say you want to print a multiplication table for the values from 1 to 6. A good way to start is to write a loop that prints the multiples of 2, all on one line.

```
i = 1
output = ""
while i <= 6:
  output += str(2*i) + "\t"
  i += 1
print(output)
```

The first line initializes a variable named `i`, which acts as a counter or **loop variable**. The second line initializes our string accumulator variable. As the loop executes, the value of `i` increases from 1 to 6. When `i` is 7, the loop terminates. Each time through the loop, it add the value of `2 * i`, followed by a tab character, to the output string.

After the loop completes, the `print` statement displays the output:

```
2       4       6       8       10      12
```

So far, so good. The next step is to **encapsulate** and **generalize**.

Encapsulation and generalization

Encapsulation is the process of wrapping a piece of code in a function, allowing you to take advantage of all the things functions are good for. You have already seen two examples of encapsulation: `print_parity` in chapter 4; and `is_divisible` in chapter 5.

Generalization means taking something specific, such as printing the multiples of 2, and making it more general, such as printing the multiples of any integer.

This function encapsulates the previous loop and generalizes it to print multiples of `n`:

```
def print_multiples(n):
  i = 1
  output = ""
  while i <= 6:
    output += str(n*i) + "\t"
    i += 1
  print(output)
```

To encapsulate, all we had to do was add the first line, which declares the name of the function and the parameter list. To generalize, all we had to do was replace the value 2 with the parameter `n`.

If we call this function with the argument 2 (i.e., `print_multiples(2)`), we get the same output as before. With the argument 3 (i.e., `print_multiples(3)`), the output is:

```
3       6       9       12      15      18
```

With the argument 4, the output is:

```
4       8       12      16      20      24
```

By now you can probably guess how to print a multiplication table — by calling `print_multiples` repeatedly with different arguments. In fact, we can use another loop:

```
i = 1
while i <= 6:
    print_multiples(i)
    i += 1
```

The output of this program is a multiplication table from 1 to 6:

```
1       2       3       4       5       6
2       4       6       8       10      12
3       6       9       12      15      18
4       8       12      16      20      24
5       10      15      20      25      30
6       12      18      24      30      36
```

More encapsulation

To demonstrate encapsulation *again*, let's take the code from the last section and wrap it up in a function:

```
def print_mult_table():
    i = 1
    while i <= 6:
        print_multiples(i)
        i += 1
```

This process is a common **development plan**. We develop code by writing lines of code outside any function, or typing them in to the interpreter. When we get the code working, we extract it and wrap it up in a function.

This development plan is particularly useful if you don't know how to divide the program into functions when you start writing. This approach lets you *design as you go*.

Local variables

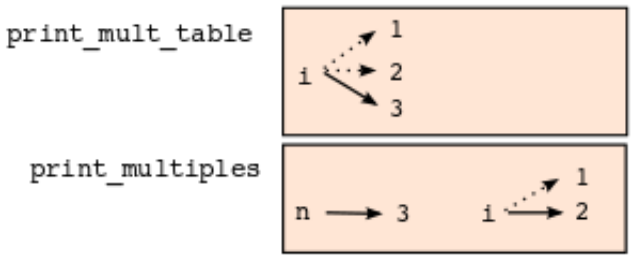
You might be wondering how we can use the same variable, `i`, in both `print_multiples` and `print_mult_table`. Doesn't it cause problems when one of the functions changes the value

of the variable?

The answer is *no*, because the `i` in `print_multiples` and the `i` in `print_mult_table` are *not* the same variable.

Variables created inside a function definition are local; you can't access a local variable from outside its home function. That means you are free to have multiple variables with the same name as long as they are not in the same function.

The stack diagram for this program shows that the two variables named `i` are not the same variable. They can refer to different values, and changing one does not affect the other.



The value of `i` in `print_mult_table` goes from 1 to 6. In the diagram it happens to be 3. The next time through the loop it will be 4. Each time through the loop, `print_mult_table` calls `print_multiples` with the current value of `i` as an argument. That value gets assigned to the parameter `n`.

Inside `print_multiples`, the value of `i` goes from 1 to 6. In the diagram, it happens to be 2. Changing this variable has no effect on the value of `i` in `print_mult_table`.

It is common and perfectly legal to have different local variables with the same name. In particular, names like `i` and `j` are used frequently as loop variables. If you avoid using them in one function just because you used them somewhere else, you will probably make the program harder to read.

More generalization

As another example of generalization, imagine you wanted a program that would print a multiplication table of any size, not just the six-by-six table. You could add a parameter to `print_mult_table`:

```
def print_mult_table(high):
    i = 1
    while i <= high:
        print_multiples(i)
        i += 1
```

We replaced the value 6 with the parameter `high`. If we call `print_mult_table` with the argument 7, it displays:

```
1       2       3       4       5       6
2       4       6       8       10      12
```

```
3       6       9       12      15      18
4       8       12      16      20      24
5       10      15      20      25      30
6       12      18      24      30      36
7       14      21      28      35      42
```

This is fine, except that we probably want the table to be square — with the same number of rows and columns. To do that, we add another parameter to `print_multiples` to specify how many columns the table should have.

Just to be annoying, we call this parameter `high`, demonstrating that different functions can have parameters with the same name (just like local variables). Here's the whole program:

```
def print_multiples(n, high):
  i = 1
  output = ""
  while i <= high:
    output += str(n*i) + "\t"
    i += 1
  print(output)

def print_mult_table(high):
  i = 1
  while i <= high:
    print_multiples(i,high)
    i += 1
```

Notice that when we added a new parameter, we had to change the first line of the function (the function heading), and we also had to change the place where the function is called in `print_mult_table`.

As expected, this program (i.e., `print_mult_table(7)`) generates a square seven-by-seven table:

```
1       2       3       4       5       6       7
2       4       6       8       10      12      14
3       6       9       12      15      18      21
4       8       12      16      20      24      28
5       10      15      20      25      30      35
6       12      18      24      30      36      42
7       14      21      28      35      42      49
```

Calling it with a different argument (e.g., 12), generates a different square table:

```
1       2       3       4       5       6       7       8       9       10      11
2       4       6       8       10      12      14      16      18      20      22
3       6       9       12      15      18      21      24      27      30      33
4       8       12      16      20      24      28      32      36      40      44
5       10      15      20      25      30      35      40      45      50      55
```

```
6       12      18      24      30      36      42      48      54      60      66
7       14      21      28      35      42      49      56      63      70      77
8       16      24      32      40      48      56      64      72      80      88
9       18      27      36      45      54      63      72      81      90      99
10      20      30      40      50      60      70      80      90      100     110
11      22      33      44      55      66      77      88      99      110     121
12      24      36      48      60      72      84      96      108     120     132
```

When you generalize a function appropriately, you often get a program with capabilities you didn't plan. For example, you might notice that, because ab = ba, all the entries in the table appear twice. You could save ink by printing only half the table. To do that, you only have to change one line of `print_mult_table`. Change

```
print_multiples(i, high)
```

to

```
print_multiples(i, i)
```

and you get:

```
1
2       4
3       6       9
4       8       12      16
5       10      15      20      25
6       12      18      24      30      36
7       14      21      28      35      42      49
```

Functions

A few times now, we have mentioned all the things functions are good for. By now, you might be wondering what exactly those things are. Here are some of them:

1. Giving a name to a sequence of statements makes your program easier to read and debug.
2. Dividing a long program into functions allows you to separate parts of the program, debug them in isolation, and then compose them into a whole.
3. Functions facilitate the use of iteration.
4. Well-designed functions are often useful for many programs. Once you write and debug one, you can reuse it.

Newton's Method

Loops are often used in programs that compute numerical results by starting with an approximate answer and iteratively improving it.

For example, one way of computing square roots is Newton's method. Suppose that you want to know the square root of `n`. If you start with almost any approximation, you can compute a better approximation with the following formula:

```
better =  (approx + n/approx)/2
```

By repeatedly applying this formula until the better approximation is equal to the previous one, we can write a function for computing the square root:

```
def sqrt(n):
    approx = n/2.0
    better = (approx + n/approx)/2.0
    while better != approx:
        approx = better
        better = (approx + n/approx)/2.0
    return approx
```

Try calling this function with `25` as an argument to confirm that it returns `5.0`.

Algorithms

Newton's method is an example of an **algorithm**: it is a mechanical process for solving a category of problems (in this case, computing square roots).

It is not easy to define an algorithm. It might help to start with something that is not an algorithm. When you learned to multiply single-digit numbers, you probably memorized the multiplication table. In effect, you memorized 100 specific solutions. That kind of knowledge is not algorithmic.

But if you were lazy, you probably cheated by learning a few tricks. For example, to find the product of n and 9, you can write n - 1 as the first digit and 10 - n as the second digit. This trick is a general solution for multiplying any single-digit number by 9. That's an algorithm!

Similarly, the techniques you learned for addition with carrying, subtraction with borrowing, and long division are all algorithms. One of the characteristics of algorithms is that they do not require any intelligence to carry out. They are mechanical processes in which each step follows from the last according to a simple set of rules.

In our opinion, it is embarrassing that humans spend so much time in school learning to execute algorithms that, quite literally, require no intelligence.

On the other hand, the process of designing algorithms is interesting, intellectually challenging, and a central part of what we call programming.

Some of the things that people do naturally, without difficulty or conscious thought, are the hardest to express algorithmically. Understanding natural language is a good example. We all do it, but so far no one has been able to explain *how* we do it, at least not in the form of an algorithm.

Glossary

algorithm A step-by-step process for solving a category of problems.

body The statements inside a loop.

counter A variable used to count something, usually initialized to zero and incremented in the body of a loop.

cursor An invisible marker that keeps track of where the next character will be printed.

decrement Decrease by 1.

development plan A process for developing a program. In this chapter, we demonstrated a style of development based on developing code to do simple, specific things and then encapsulating and generalizing.

encapsulate To divide a large complex program into components (like functions) and isolate the components from each other (by using local variables, for example).

escape sequence An escape character, \, followed by one or more printable characters used to designate a nonprintable character.

generalize To replace something unnecessarily specific (like a constant value) with something appropriately general (like a variable or parameter). Generalization makes code more versatile, more likely to be reused, and sometimes even easier to write.

increment Both as a noun and as a verb, increment means to increase by 1.

infinite loop A loop in which the terminating condition is never satisfied.

initialization (of a variable) To initialize a variable is to give it an initial value, usually in the context of multiple assignment. Since in Python variables don't exist until they are assigned values, they are initialized when they are created. In other programming languages this is not the case, and variables can be created without being initialized, in which case they have either default or *garbage* values.

iteration Repeated execution of a set of programming statements.

loop A statement or group of statements that execute repeatedly until a terminating condition is satisfied.

loop variable A variable used as part of the terminating condition of a loop.

multiple assignment Making more than one assignment to the same variable during the execution of a program.

newline A special character that causes the cursor to move to the beginning of the next line.

tab A special character that causes the cursor to move to the next tab stop on the current line.

trace To follow the flow of execution of a program by hand, recording the change of state of the variables and any output produced.

Exercises

1. Write a single string that:

```
produces
this
output.
```

2. Add a print statement to the `sqrt` function defined in section 6.14 that prints out `better` each time it is calculated. Call your modified function with 25 as an argument and record the results.

3. Trace the execution of the last version of `print_mult_table` and figure out how it works.

4. Write a function `print_triangular_numbers(n)` that prints out the first n triangular numbers. A call to `print_triangular_numbers(5)` would produce the following output:

```
1        1
2        3
3        6
4        10
5        15
```

(*hint: use a web search to find out what a triangular number is.*)

5. Open a file named `ch06.py` and add the following:

```
if __name__ == '__main__':
    import doctest
    doctest.testmod()
```

Write a function, `is_prime`, which takes a single integral argument and returns `True` when the argument is a **prime number** and `False` otherwise. Add doctests to your function as you develop it.

6. What will `num_digits(0)` return? Modify it to return `1` for this case. Why does a call to `num_digits(-24)` result in an infinite loop (*hint: -1/10 evaluates to -1*)? Modify `num_digits` so that it works correctly with any integer value. Add the following to the `ch06.py` file you created in the previous exercise:

```
def num_digits(n):
    """
```

```
        >>> num_digits(12345)
        5
        >>> num_digits(0)
        1
        >>> num_digits(-12345)
        5
    """
```

Add your function body to `num_digits` and confirm that it passes the doctests.

7. Add the following to the `ch06.py`:

```
def num_even_digits(n):
    """
      >>> num_even_digits(123456)
      3
      >>> num_even_digits(2468)
      4
      >>> num_even_digits(1357)
      0
      >>> num_even_digits(2)
      1
      >>> num_even_digits(20)
      2
    """
```

Write a body for `num_even_digits` so that it works as expected.

8. Add the following to `ch06.py`:

```
def print_digits(n):
    """
      >>> print_digits(13789)
      9 8 7 3 1
      >>> print_digits(39874613)
      3 1 6 4 7 8 9 3
      >>> print_digits(213141)
      1 4 1 3 1 2
    """
```

Write a body for `print_digits` so that it passes the given doctests.

9. Write a function `sum_of_squares_of_digits` that computes the sum of the squares of the digits of an integer passed to it. For example, `sum_of_squares_of_digits(987)` should return 194, since `9**2 + 8**2 + 7**2 == 81 + 64 + 49 == 194`.

```
def sum_of_squares_of_digits(n):
    """
      >>> sum_of_squares_of_digits(1)
      1
```

```
        >>> sum_of_squares_of_digits(9)
        81
        >>> sum_of_squares_of_digits(11)
        2
        >>> sum_of_squares_of_digits(121)
        6
        >>> sum_of_squares_of_digits(987)
        194
    """
```

Check your solution against the doctests above.

Strings

A compound data type

So far we have seen five types: `int`, `float`, `bool`, `NoneType` and `str`. Strings are qualitatively different from the other four because they are made up of smaller pieces — **characters**.

Types that comprise smaller pieces are called **compound data types**. Depending on what we are doing, we may want to treat a compound data type as a single thing, or we may want to access its parts. This ambiguity is useful.

The **bracket operator** selects a single character from a string:

```
>>> fruit = "banana"
>>> letter = fruit[1]
>>> print(letter)
```

The expression `fruit[1]` selects character number 1 from `fruit`. The variable `letter` refers to the result. When we display `letter`, we get a surprise:

```
a
```

The first letter of `"banana"` is not `a`, unless you are a computer scientist. For perverse reasons, computer scientists always start counting from zero. The 0th letter (zero-eth) of `"banana"` is `b`. The 1th letter (one-eth) is `a`, and the 2th (two-eth) letter is `n`.

If you want the zero-eth letter of a string, you just put 0, or any expression with the value 0, in the brackets:

```
>>> letter = fruit[0]
>>> print(letter)
b
```

The expression in brackets is called an **index**. An index specifies a member of an ordered set, in this case the set of characters in the string. The index *indicates* which one you want, hence the

name. It can be any integer expression.

Length

The `len` function returns the number of characters in a string:

```
>>> fruit = "banana"
>>> len(fruit)
6
```

To get the last letter of a string, you might be tempted to try something like this:

```
length = len(fruit)
last = fruit[length]        # ERROR!
```

That won't work. It causes the runtime error `IndexError: string index out of range`. The reason is that there is no 6th letter in `"banana"`. Since we started counting at zero, the six letters are numbered 0 to 5. To get the last character, we have to subtract 1 from `length`:

```
length = len(fruit)
last = fruit[length-1]
```

Alternatively, we can use **negative indices**, which count backward from the end of the string. The expression `fruit[-1]` yields the last letter, `fruit[-2]` yields the second to last, and so on.

Traversal and the `for` loop

A lot of computations involve processing a string one character at a time. Often they start at the beginning, select each character in turn, do something to it, and continue until the end. This pattern of processing is called a **traversal**. One way to encode a traversal is with a `while` statement:

```
index = 0
while index < len(fruit):
    letter = fruit[index]
    print(letter)
    index += 1
```

This loop traverses the string and displays each letter on a line by itself. The loop condition is `index < len(fruit)`, so when `index` is equal to the length of the string, the condition is false, and the body of the loop is not executed. The last character accessed is the one with the index `len(fruit)-1`, which is the last character in the string.

Using an index to traverse a set of values is so common that Python provides an alternative, simpler syntax — the `for` loop:

```
for char in fruit:
    print(char)
```

Each time through the loop, the next character in the string is assigned to the variable `char`. The loop continues until no characters are left.

The following example shows how to use concatenation and a `for` loop to generate an abecedarian series. Abecedarian refers to a series or list in which the elements appear in alphabetical order. For example, in Robert McCloskey's book *Make Way for Ducklings*, the names of the ducklings are Jack, Kack, Lack, Mack, Nack, Ouack, Pack, and Quack. This loop outputs these names in order:

```
prefixes = "JKLMNOPQ"
suffix = "ack"

for letter in prefixes:
    print(letter + suffix)
```

The output of this program is:

```
Jack
Kack
Lack
Mack
Nack
Oack
Pack
Qack
```

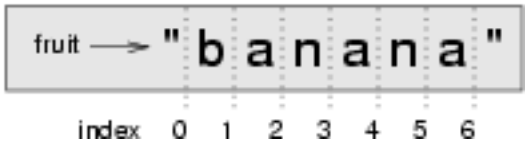
Of course, that's not quite right because Ouack and Quack are misspelled. You'll fix this as an exercise below.

String slices

A substring of a string is called a **slice**. Selecting a slice is similar to selecting a character:

```
>>> s = "Peter, Paul, and Mary"
>>> print(s[0:5])
Peter
>>> print(s[7:11])
Paul
>>> print(s[17:21])
Mary
```

The operator `[n:m]` returns the part of the string from the n-eth character to the m-eth character, including the first but excluding the last. This behavior is counterintuitive; it makes more sense if you imagine the indices pointing *between* the characters, as in the following diagram:



If you omit the first index (before the colon), the slice starts at the beginning of the string. If you omit the second index, the slice goes to the end of the string. Thus:

```
>>> fruit = "banana"
>>> fruit[:3]
'ban'
>>> fruit[3:]
'ana'
```

What do you think `s[:]` means?

String comparison

The comparison operators work on strings. To see if two strings are equal:

```
if word == "banana":
    print("Yes, we have no bananas!")
```

Other comparison operations are useful for putting words in lexigraphical order:

```
if word < "banana":
    print("Your word, " + word + ", comes before banana.")
elif word > "banana":
    print("Your word, " + word + ", comes after banana.")
else:
    print("Yes, we have no bananas!")
```

This is similar to the alphabetical order you would use with a dictionary, except that all the uppercase letters come before all the lowercase letters. As a result:

```
Your word, Zebra, comes before banana.
```

A common way to address this problem is to convert strings to a standard format, such as all lowercase, before performing the comparison. A more difficult problem is making the program realize that zebras are not fruit.

Strings are immutable

It is tempting to use the `[]` operator on the left side of an assignment, with the intention of changing a character in a string. For example:

```
greeting = "Hello, world!"
greeting[0] = 'J'             # ERROR!
print(greeting)
```

Instead of producing the output `Jello, world!`, this code produces the runtime error `TypeError: 'str' object doesn't support item assignment`.

Strings are **immutable**, which means you can't change an existing string. The best you can do is create a new string that is a variation on the original:

```
greeting = "Hello, world!"
new_greeting = 'J' + greeting[1:]
print(new_greeting)
```

The solution here is to concatenate a new first letter onto a slice of `greeting`. This operation has no effect on the original string.

The `in` operator

The `in` operator tests if one string is a substring of another:

```
>>> 'p' in 'apple'
True
>>> 'i' in 'apple'
False
>>> 'ap' in 'apple'
True
>>> 'pa' in 'apple'
False
```

Note that a string is a substring of itself:

```
>>> 'a' in 'a'
True
>>> 'apple' in 'apple'
True
```

Combining the `in` operator with string concatenation using `+`, we can write a function that removes all the vowels from a string:

```
def remove_vowels(s):
    vowels = "aeiouAEIOU"
    s_without_vowels = ""
    for letter in s:
        if letter not in vowels:
            s_without_vowels += letter
    return s_without_vowels
```

Test this function to confirm that it does what we wanted it to do.

A `find` function

What does the following function do?

```
def find(strng, ch):
    index = 0
    while index < len(strng):
        if strng[index] == ch:
            return index
        index += 1
    return -1
```

In a sense, `find` is the opposite of the `[]` operator. Instead of taking an index and extracting the corresponding character, it takes a character and finds the index where that character appears. If the character is not found, the function returns `-1`.

This is the first example we have seen of a `return` statement inside a loop. If `strng[index] == ch`, the function returns immediately, breaking out of the loop prematurely.

If the character doesn't appear in the string, then the program exits the loop normally and returns `-1`.

This pattern of computation is sometimes called a eureka traversal because as soon as we find what we are looking for, we can cry Eureka! and stop looking.

Looping and counting

The following program counts the number of times the letter `a` appears in a string, and is another example of the counter pattern introduced in *Counting digits*:

```
fruit = "banana"
count = 0
for char in fruit:
    if char == 'a':
        count += 1
print(count)
```

Optional parameters

To find the locations of the second or third occurence of a character in a string, we can modify the `find` function, adding a third parameter for the starting postion in the search string:

```
def find2(strng, ch, start):
    index = start
    while index < len(strng):
        if strng[index] == ch:
            return index
        index += 1
    return -1
```

The call `find2('banana', 'a', 2)` now returns `3`, the index of the first occurance of 'a' in 'banana' after index 2. What does `find2('banana', 'n', 3)` return? If you said, 4, there is a good chance you understand how `find2` works.

Better still, we can combine `find` and `find2` using an **optional parameter**:

```
def find(strng, ch, start=0):
    index = start
    while index < len(strng):
        if strng[index] == ch:
            return index
        index += 1
    return -1
```

The call `find('banana', 'a', 2)` to this version of `find` behaves just like `find2`, while in the call `find('banana', 'a')`, `start` will be set to the **default value** of `0`.

If we add another optional parameter to `find` we can use it to search either forward or backward:

```
def find(strng, ch, start=0, step=1):
    index = start
    while 0 <= index < len(strng):
        if strng[index] == ch:
            return index
        index += step
    return -1
```

Passing in a value of `len(strng)-1` for start and `-1` for `step` will make `find` search from the end of the string toward the beginning. Note that we needed to check a lower bound for `index` in the while loop as well as an upper bound to accomodate this change.

The `string` module

The `string` module contains useful functions that manipulate strings. As usual, we have to import the module before we can use it:

```
>>> import string
```

To see what is inside it, use the `dir` function with the module name as an argument.

```
>>> dir(string)
```

which will return the list of items inside the string module:

```
['Template', '_TemplateMetaclass', '__builtins__', '__doc__',
'__file__', '__name__', '_float', '_idmap', '_idmapL', '_int',
'_long', '_multimap', '_re', 'ascii_letters', 'ascii_lowercase',
'ascii_uppercase', 'atof', 'atof_error', 'atoi', 'atoi_error',
'atol', 'atol_error', 'capitalize', 'capwords', 'center',
'count', 'digits', 'expandtabs', 'find', 'hexdigits', 'index',
'index_error', 'join', 'joinfields', 'letters', 'ljust',
'lower', 'lowercase', 'lstrip', 'maketrans', 'octdigits',
'printable', 'punctuation', 'replace', 'rfind', 'rindex',
'rjust', 'rsplit', 'rstrip', 'split', 'splitfields', 'strip',
'swapcase', 'translate', 'upper', 'uppercase', 'whitespace',
'zfill']
```

To find out more about an item in this list, we can use the `type` command. We need to specify the module name followed by the item using **dot notation**.

```
>>> type(string.digits)
<type 'str'>
>>> type(string.find)
<type 'function'>
```

Since `string.digits` is a string, we can print it to see what it contains:

```
>>> print(string.digits)
0123456789
```

Not surprisingly, it contains each of the decimal digits.

`string.find` is a function which does much the same thing as the function we wrote. To find out more about it, we can print out its **docstring**, `__doc__`, which contains documentation on the function:

```
>>> print(string.find.__doc__)
find(s, sub [,start [,end]]) -> in

    Return the lowest index in s where substring sub is found,
    such that sub is contained within s[start,end].  Optional
    arguments start and end are interpreted as in slice notation.

    Return -1 on failure.
```

The parameters in square brackets are optional parameters. We can use `string.find` much as we did our own `find`:

```
>>> fruit = "banana"
>>> index = string.find(fruit, "a")
>>> print(index)
1
```

This example demonstrates one of the benefits of modules — they help avoid collisions between the names of built-in functions and user-defined functions. By using dot notation we can specify which version of `find` we want.

Actually, `string.find` is more general than our version. it can find substrings, not just characters:

```
>>> string.find("banana", "na")
2
```

Like ours, it takes an additional argument that specifies the index at which it should start:

```
>>> string.find("banana", "na", 3)
4
```

Unlike ours, its second optional parameter specifies the index at which the search should end:

```
>>> string.find("bob", "b", 1, 2)
-1
```

In this example, the search fails because the letter *b* does not appear in the index range from `1` to `2` (not including `2`).

Character classification

It is often helpful to examine a character and test whether it is upper- or lowercase, or whether it is a character or a digit. The `string` module provides several constants that are useful for these purposes. One of these, `string.digits`, we have already seen.

The string `string.lowercase` contains all of the letters that the system considers to be lowercase. Similarly, `string.uppercase` contains all of the uppercase letters. Try the following and see what you get:

```
print(string.lowercase)
print(string.uppercase)
print(string.digits)
```

We can use these constants and `find` to classify characters. For example, if `find(lowercase, ch)` returns a value other than `-1`, then `ch` must be lowercase:

```
def is_lower(ch):
    return string.find(string.lowercase, ch) != -1
```

Alternatively, we can take advantage of the `in` operator:

```
def is_lower(ch):
    return ch in string.lowercase
```

As yet another alternative, we can use the comparison operator:

```
def is_lower(ch):
    return 'a' <= ch <= 'z'
```

If `ch` is between *a* and *z*, it must be a lowercase letter.

Another constant defined in the `string` module may surprise you when you print it:

```
>>> print(string.whitespace)
```

Whitespace characters move the cursor without printing anything. They create the white space between visible characters (at least on white paper). The constant `string.whitespace` contains all the whitespace characters, including space, tab (`\t`), and newline (`\n`).

There are other useful functions in the `string` module, but this book isn't intended to be a reference manual. On the other hand, the *Python Library Reference* is. Along with a wealth of other documentation, it's available from the Python website, http://www.python.org.

String formatting

The most concise and powerful way to format a string in Python is to use the *string formatting operator*, `%`, together with Python's string formatting operations. To see how this works, let's start with a few examples:

```
>>> "His name is %s."  % "Arthur"
'His name is Arthur.'
>>> name = "Alice"
>>> age = 10
>>> "I am %s and I am %d years old." % (name, age)
'I am Alice and I am 10 years old.'
>>> n1 = 4
>>> n2 = 5
>>> "2**10 = %d and %d * %d = %f" % (2**10, n1, n2, n1 * n2)
'2**10 = 1024 and 4 * 5 = 20.000000'
>>>
```

The syntax for the string formatting operation looks like this:

```
"<FORMAT>" % (<VALUES>)
```

It begins with a *format* string which contains a sequence of characters and *conversion specifications*. Conversion specifications always start with a `%`. In the previous examples we saw three conversion specifications: `%s`, `%d`, and `%f`. Following the format string is a single `%` and then a

sequence of values, *one per conversion specification*, seperated by commas and enclosed in parentheses. The parentheses are optional if there is only a single value. The conversion specifications indicate where in the formatted string the values should be placed and in some cases how the values should be converted to strings.

In the first example above, there is a single conversion specification, `%s`, which indicates a string. The single value, `"Arthur"`, maps to it, and is not enclosed in parenthesis.

In the second example, `name` has string value, `"Alice"`, and `age` has integer value, `10`. These map to the two converstion specifications, `%s` and `%d`. The `d` in the second converstion specification indicates that the value is a decimal (base 10) integer.

In the third example, variables `n1` and `n2` have integer values `4` and `5`, respectively. There are four conversion specifications in the format string: three `%d`'s and a `%f`. The `f` indicates that the value should be represented as a floating point number. The four values that map to the four conversion specifications are: `2**10`, `n1`, `n2`, and `n1 * n2`.

`s`, `d`, and `f` are all the conversion types we will need for this book. To see a complete list, see the String Formatting Operations section of the Python Library Reference.

We can also use *padding* to specify the minimum number of characters a value should occupy when it is formatted. If the formatted value is too short extra blank space characters will be added. For example:

```
>>> "%6s" % "hi"
'    hi'
>>> "%-6s" % "hi"
'hi    '
>>> "%6s" % "hi there, pythonista!"
'hi there, pythonista!'
```

The numbers in the conversion spefications indicate the minimum size of the resulting string. The - in the second example tells the formatter to put any necessary padding to the right. This is also called *left-justification* because the value ends up on the left side of the formatted string. The final example shows that these numbers really are specifying a *minimum* width; the string we supplied was longer than six characters, but python still prints the entire string, not just the first six characters, `"hi the"`.

Padding is useful if we want to display data in neatly aligned columns. Without string formatting we might try to do something like this:

```
i = 1
print("i\ti**2\ti**3\ti**5\ti**10\ti**20")
while i <= 10:
    print("%d\t%d\t%d\t%d\t%d\t%d" % (i, i**2, i**3, i**5, i**10, i**20))
    i += 1
```

This program prints out a table of various powers of the numbers from 1 to 10. In its current form it relies on the tab character (`\t`) to align the columns of values, but this breaks down when the values in the table get larger than the 8 character tab width:

```
i        i**2     i**3     i**5     i**10    i**20
1        1        1        1        1        1
2        4        8        32       1024     1048576
3        9        27       243      59049    3486784401
4        16       64       1024     1048576          1099511627776
5        25       125      3125     9765625          95367431640625
6        36       216      7776     60466176         3656158440062976
7        49       343      16807    282475249        79792266297612001
8        64       512      32768    1073741824       1152921504606846976
9        81       729      59049    3486784401       12157665459056928801
10       100      1000     100000   10000000000      100000000000000000000
```

One possible solution would be to change the tab width, but the first column already has more space than it needs. The best solution would be to set the width of each column independently. We can use padding to accomplish this:

```
i = 1
print("%-4s%-5s%-6s%-8s%-13s%s" % ('i', 'i**2', 'i**3', 'i**5', 'i**10', 'i**20'))
while i <= 10:
    print("%-4s%-5s%-6s%-8s%-13s%s" % (i, i**2, i**3, i**5, i**10, i**20))
    i += 1
```

Running this code produces the following output:

```
i   i**2 i**3  i**5    i**10        i**20
1   1    1     1       1            1
2   4    8     32      1024         1048576
3   9    27    243     59049        3486784401
4   16   64    1024    1048576      1099511627776
5   25   125   3125    9765625      95367431640625
6   36   216   7776    60466176     3656158440062976
7   49   343   16807   282475249    79792266297612001
8   64   512   32768   1073741824   1152921504606846976
9   81   729   59049   3486784401   12157665459056928801
10  100  1000  100000  10000000000  100000000000000000000
```

Here we have specified the width of each column by choosing padding values big enough to accommodate all the numbers in that column. Notice that the specifier for the last column is just `%s`. Can you explain why that is?

Finally, when formatting floats with `%f`, you can specify the precsion — i.e. the number of decimal places:

```
>>> "%.3f" % 123.456789
'123.457'
>>> "%.3f" % 123.4
'123.400'
```

As you can see from the first example, python will round the float appropriately. You can also

combine all three of these options, specifying the padding, the justification, and the precision all at once:

```
>>> "My number is %-10.3f" % 123.456789
'My number is 123.457    '
```

Summary and First Exercises

This chapter introduced a lot of new ideas. The following summary and set of exercises may prove helpful in remembering what you learned:

indexing ([]) Access a single character in a string using its position (starting from 0). Example: `'This'[2]` evaluates to `'i'`.

length function (len) Returns the number of characters in a string. Example: `len('happy')` evaluates to `5`.

for loop traversal (for) *Traversing* a string means accessing each character in the string, one at a time. For example, the following for loop:

```
for letter in 'Example':
    print(2 * letter)
```

will print each letter of the string doubled (e.g. `EE`), each on its own line.

slicing ([:]) A *slice* is a substring of a string. Example: `'bananas and cream'[3:6]` evaluates to `ana` (so does `'bananas and cream'[1:4]`).

string comparison (>, <, >=, <=, ==) The comparision operators work with strings, evaluating according to lexigraphical order. Examples: `'apple' < 'banana'` evaluates to `True`. `'Zeta' < 'Appricot'` evaluates to `False`. `'Zebra' <= 'aardvark'` evaluates to `True` because all upper case letters precede lower case letters.

in operator (in) The `in` operator tests whether one character or string is contained inside another string. Examples: `'heck' in "I'll be checking for you."` evaluates to `True`. `'cheese' in "I'll be checking for you."` evaluates to `False`.

First Exercises

1. Write the Python interpreter's evaluation to each of the following expressions:

```
>>> 'Python'[1]
```

```
>>> "Strings are sequences of characters."[5]
```

```
>>> len("wonderful")
```

```
>>> 'Mystery'[:4]
```

```
>>> 'p' in 'Pinapple'
```

```
>>> 'apple' in 'Pinapple'
```

```
>>> 'pear' in 'Pinapple'
```

```
>>> 'apple' > 'pinapple'
```

```
>>> 'pinapple' < 'Peach'
```

2. Write Python code to make each of the following doctests pass:

```
"""
  >>> type(fruit)
  <type 'str'>
  >>> len(fruit)
  8
  >>> fruit[:3]
  'ram'
"""
```

```
"""
  >>> group = "John, Paul, George, and Ringo"
  >>> group[12:x]
  'George'
  >>> group[n:m]
  'Paul'
  >>> group[:r]
  'John'
  >>> group[s:]
  'Ringo'
"""
```

```
"""
  >>> len(s)
  8
  >>> s[4:6] == 'on'
  True
"""
```

Glossary

compound data type A data type in which the values are made up of components, or elements, that are themselves values.

default value The value given to an optional parameter if no argument for it is provided in the function call.

docstring A string constant on the first line of a function or module definition (and as we will see later, in class and method definitions as well). Docstrings provide a convenient way to associate documentation with code. Docstrings are also used by the `doctest` module for automated testing.

dot notation Use of the **dot operator**, `.`, to access functions inside a module.

immutable A compound data type whose elements cannot be assigned new values.

index A variable or value used to select a member of an ordered set, such as a character from a string.

optional parameter A parameter written in a function header with an assignment to a default value which it will receive if no corresponding argument is given for it in the function call.

slice A part of a string (substring) specified by a range of indices. More generally, a subsequence of any sequence type in Python can be created using the slice operator (`sequence[start:stop]`).

traverse To iterate through the elements of a set, performing a similar operation on each.

whitespace Any of the characters that move the cursor without printing visible characters. The constant `string.whitespace` contains all the white-space characters.

Exercises

1. Modify:

```
prefixes = "JKLMNOPQ"
suffix = "ack"

for letter in prefixes:
    print(letter + suffix)
```

so that `Ouack` and `Quack` are spelled correctly.

2. Encapsulate

```
fruit = "banana"
count = 0
for char in fruit:
    if char == 'a':
        count += 1
print(count)
```

in a function named `count_letters`, and generalize it so that it accepts the string and the letter as arguments.

3. Now rewrite the `count_letters` function so that instead of traversing the string, it repeatedly calls `find` (the version from *Optional parameters*), with the optional third parameter to locate new occurences of the letter being counted.
4. Which version of `is_lower` do you think will be fastest? Can you think of other reasons besides speed to prefer one version or the other?
5. Create a file named `stringtools.py` and put the following in it:

```
def reverse(s):
    """
      >>> reverse('happy')
      'yppah'
      >>> reverse('Python')
      'nohtyP'
      >>> reverse("")
      ''
      >>> reverse("P")
      'P'
    """

if __name__ == '__main__':
    import doctest
    doctest.testmod()
```

   Add a function body to `reverse` to make the doctests pass.

6. Add `mirror` to `stringtools.py`.

```
def mirror(s):
    """
      >>> mirror("good")
      'gooddoog'
      >>> mirror("yes")
      'yessey'
      >>> mirror('Python')
      'PythonnohtyP'
      >>> mirror("")
      ''
      >>> mirror("a")
      'aa'
    """
```

   Write a function body for it that will make it work as indicated by the doctests.

7. Include `remove_letter` in `stringtools.py`.

```
def remove_letter(letter, strng):
    """
      >>> remove_letter('a', 'apple')
```

```
        'pple'
        >>> remove_letter('a', 'banana')
        'bnn'
        >>> remove_letter('z', 'banana')
        'banana'
        >>> remove_letter('i', 'Mississippi')
        'Mssssppp'
    """
```

Write a function body for it that will make it work as indicated by the doctests.

8. Finally, add bodies to each of the following functions, one at a time

```
def is_palindrome(s):
    """
      >>> is_palindrome('abba')
      True
      >>> is_palindrome('abab')
      False
      >>> is_palindrome('tenet')
      True
      >>> is_palindrome('banana')
      False
      >>> is_palindrome('straw warts')
      True
    """
```

```
def count(sub, s):
    """
      >>> count('is', 'Mississippi')
      2
      >>> count('an', 'banana')
      2
      >>> count('ana', 'banana')
      2
      >>> count('nana', 'banana')
      1
      >>> count('nanan', 'banana')
      0
    """
```

```
def remove(sub, s):
    """
      >>> remove('an', 'banana')
      'bana'
      >>> remove('cyc', 'bicycle')
      'bile'
      >>> remove('iss', 'Mississippi')
```

```
        'Missippi'
        >>> remove('egg', 'bicycle')
        'bicycle'
    """
```

```
def remove_all(sub, s):
    """
      >>> remove_all('an', 'banana')
      'ba'
      >>> remove_all('cyc', 'bicycle')
      'bile'
      >>> remove_all('iss', 'Mississippi')
      'Mippi'
      >>> remove_all('eggs', 'bicycle')
      'bicycle'
    """
```

until all the doctests pass.

9. Try each of the following formatted string operations in a Python shell and record the results:
   (a) “%s %d %f” % (5, 5, 5)
   (b) “%-.2f” % 3
   (c) “%-10.2f%-10.2f” % (7, 1.0/2)
   (d) print(” $%5.2fn $%5.2fn $%5.2f” % (3, 4.5, 11.2))
10. The following formatted strings have errors. Fix them:
   (a) “%s %s %s %s” % (‘this’, ‘that’, ‘something’)
   (b) “%s %s %s” % (‘yes’, ‘no’, ‘up’, ‘down’)
   (c) “%d %f %f” % (3, 3, ‘three’)

Case Study: Catch

Getting started

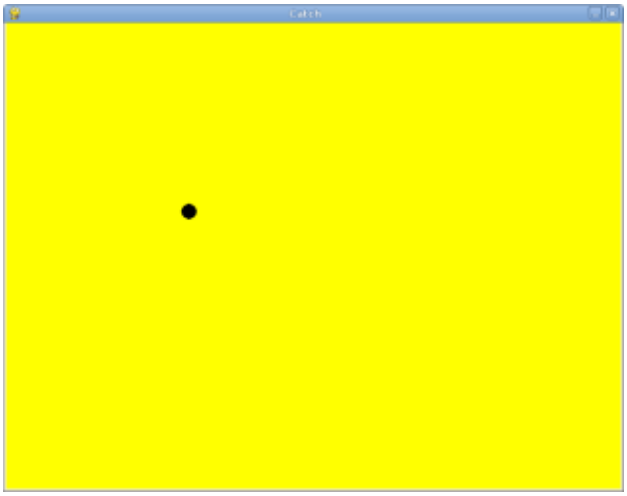
In our first case study we will build a small video game using the facilities in the GASP package. The game will shoot a ball across a window from left to right and you will manipulate a mitt at the right side of the window to catch it.

Using `while` to move a ball

`while` statements can be used with `gasp` to add motion to a program. The following program moves a black ball across an 800 x 600 pixel graphics canvas. Add this to a file named `pitch.py`:

```
from gasp import *

begin_graphics(800, 600, title="Catch", background=color.YELLOW)
set_speed(120)

ball_x = 10
ball_y = 300
ball = Circle((ball_x, ball_y), 10, filled=True)
dx = 4
dy = 1

while ball_x < 810:
    ball_x += dx
    ball_y += dy
    move_to(ball, (ball_x, ball_y))
    update_when('next_tick')

end_graphics()
```

As the ball moves across the screen, you will see a graphics window that looks like this:



Trace the first few iterations of this program to be sure you see what is happening to the variables `x` and `y`.

Some new things to learn about GASP from this example:

- `begin_graphics` can take arguments for width, height, title, and background color of the graphics canvas.
- `set_speed` can takes a frame rate in frames per second.

- Adding `filled=True` to `Circle(...)` makes the resulting circle solid.
- `ball = Circle` stores the circle (we will talk later about what a circle actually is) in a variable named `ball` so that it can be referenced later.
- The `move_to` function in GASP allows a programmer to pass in a shape (the ball in this case) and a location, and moves the shape to that location.
- The `update_when` function is used to delay the action in a gasp program until a specified event occurs. The event `'next_tick'` waits until the next frame, for an amount of time determined by the frame rate set with `set_speed`. Other valid arguments for `update_when` are `'key_pressed'` and `'mouse_clicked'`.

Varying the pitches

To make our game more interesting, we want to be able to vary the speed and direction of the ball. GASP has a function, `random_between(low, high)`, that returns a **random** integer between `low` and `high`. To see how this works, run the following program:

```
from gasp import *

i = 0
while i < 10:
    print(random_between(-5, 5))
    i += 1
```

Each time the function is called a more or less random integer is chosen between -5 and 5. When we ran this program we got:

```
-2
-1
-4
1
-2
3
-5
-3
4
-5
```

You will probably get a different sequence of numbers.

Let's use `random_between` to vary the direction of the ball. Replace the line in `pitch.py` that assigns `1` to `y`:

```
dy = 1
```

with an assignment to a random number between -4 and 4:

```
dy = random_between(-4, 4)
```

Making the ball bounce

Running this new version of the program, you will notice that ball frequently goes off either the top or bottom edges of the screen before it completes its journey. To prevent this, let's make the ball bounce off the edges by changing the sign of `dy` and sending the ball back in the opposite verticle direction.

Add the following as the first line of the body of the while loop in `pitch.py`:

```
if ball_y >= 590 or ball_y <= 10:
    dy *= -1
```

Run the program several times to see how it behaves.

The `break` statement

The **break** statement is used to immediately leave the body of a loop. The following program implements a simple guessing game:

```
from gasp import *

number = random_between(1, 1000)
guesses = 1
guess = int(raw_input("Guess the number between 1 and 1000: "))

while guess != number:
    if guess > number:
        print("Too high!")
    else:
        print("Too low!")
    guess = int(raw_input("Guess the number between 1 and 1000: "))
    guesses += 1

print("\n\nCongratulations, you got it in %d guesses!\n\n" % (guesses))
```

Using a `break` statement, we can rewrite this program to eliminate the duplication of the `input` statement:

```
from gasp import *

number = random_between(1, 1000)
guesses = 0
```

```
while True:
    guess = int(raw_input("Guess the number between 1 and 1000: "))
    guesses += 1
    if guess > number:
        print("Too high!")
    elif guess < number:
        print("Too low!")
    else:
        print("\n\nCongratulations, you got it in %d guesses!\n\n" % (guesses))
        break
```

This program makes use of the mathematical law of **trichotomy** (given real numbers a and b, a > b, a < b, or a = b). While both versions of the program are 15 lines long, it could be argued that the logic in the second version is clearer.

Put this program in a file named `guess.py`.

Responding to the keyboard

The following program creates a circle (or mitt) which responds to keyboard input. Pressing the `j` or `k` keys moves the mitt up and down, respectively. Add this to a file named `mitt.py`:

```
from gasp import *

begin_graphics(800, 600, title="Catch", background=color.YELLOW)
set_speed(120)

mitt_x = 780
mitt_y = 300
mitt = Circle((mitt_x, mitt_y), 20)

while True:
    if key_pressed('k') and mitt_y <= 580:
        mitt_y += 5
    elif key_pressed('j') and mitt_y >= 20:
        mitt_y -= 5

    if key_pressed('escape'):
        break

    move_to(mitt, (mitt_x, mitt_y))
    update_when('next_tick')

end_graphics()
```

Run `mitt.py`, pressing `j` and `k` to move up and down the screen.

Checking for collisions

The following program moves two balls toward each other from opposite sides of the screen. When they collide , both balls disappear and the program ends:

```
from gasp import *

def distance(x1, y1, x2, y2):
    return ((x2 - x1)**2 + (y2 - y1)**2)**0.5

begin_graphics(800, 600, title="Catch", background=color.YELLOW)
set_speed(120)

ball1_x = 10
ball1_y = 300
ball1 = Circle((ball1_x, ball1_y), 10, filled=True)
ball1_dx = 4

ball2_x = 790
ball2_y = 300
ball2 = Circle((ball2_x, ball2_y), 10)
ball2_dx = -4

while ball1_x < 810:
    ball1_x += ball1_dx
    ball2_x += ball2_dx
    move_to(ball1, (ball1_x, ball1_y))
    move_to(ball2, (ball2_x, ball2_y))
    if distance(ball1_x, ball1_y, ball2_x, ball2_y) <= 20:
        remove_from_screen(ball1)
        remove_from_screen(ball2)
        break
    update_when('next_tick')

sleep(1)
end_graphics()
```

Put this program in a file named `collide.py` and run it.

Putting the pieces together

In order to combine the moving ball, moving mitt, and collision detection, we need a single `while` loop that does each of these things in turn:

```
from gasp import *
```

```
def distance(x1, y1, x2, y2):
    return ((x2 - x1)**2 + (y2 - y1)**2)**0.5

begin_graphics(800, 600, title="Catch", background=color.YELLOW)
set_speed(120)

ball_x = 10
ball_y = 300
ball = Circle((ball_x, ball_y), 10, filled=True)
dx = 4
dy = random_between(-4, 4)

mitt_x = 780
mitt_y = 300
mitt = Circle((mitt_x, mitt_y), 20)

while True:
    # move the ball
    if ball_y >= 590 or ball_y <= 10:
        dy *= -1
    ball_x += dx
    if ball_x > 810:              # the ball has gone off the screen
        break
    ball_y += dy
    move_to(ball, (ball_x, ball_y))

    # check on the mitt
    if key_pressed('k') and mitt_y <= 580:
        mitt_y += 5
    elif key_pressed('j') and mitt_y >= 20:
        mitt_y -= 5

    if key_pressed('escape'):
        break

    move_to(mitt, (mitt_x, mitt_y))

    if distance(ball_x, ball_y, mitt_x, mitt_y) <= 30:  # ball is caught
        remove_from_screen(ball)
        break

    update_when('next_tick')

end_graphics()
```

Put this program in a file named `catch.py` and run it several times. Be sure to catch the ball on some runs and miss it on others.

Displaying text

This program displays scores for both a player and the computer on the graphics screen. It generates a random number of 0 or 1 (like flipping a coin) and adds a point to the player if the value is 1 and to the computer if it is not. It then updates the display on the screen.

```
from gasp import *

begin_graphics(800, 600, title="Catch", background=color.YELLOW)
set_speed(120)

player_score = 0
comp_score = 0

player = Text("Player: %d Points" % player_score, (10, 570), size=24)
computer = Text("Computer: %d Points" % comp_score, (640, 570), size=24)

while player_score < 5 and comp_score < 5:
    sleep(1)
    winner = random_between(0, 1)
    if winner:
        player_score += 1
        remove_from_screen(player)
        player = Text("Player: %d Points" % player_score, (10, 570), size=24)
    else:
        comp_score += 1
        remove_from_screen(computer)
        computer = Text("Computer: %d Points" % comp_score, (640, 570), size=24)

if player_score == 5:
    Text("Player Wins!", (340, 290), size=32)
else:
    Text("Computer Wins!", (340, 290), size=32)

sleep(4)

end_graphics()
```

Put this program in a file named `scores.py` and run it.

We can now modify `catch.py` to diplay the winner. Immediately after the `if ball_x > 810:` conditional, add the following:

```
Text("Computer Wins!", (340, 290), size=32)
sleep(2)
```

It is left as an excercise to display when the player wins.

Abstraction

Our program is getting a bit complex. To make matters worse, we are about to increase its complexity. The next stage of development requires a **nested loop**. The outer loop will handle repeating rounds of play until either the player or the computer reaches a winning score. The inner loop will be the one we already have, which plays a single round, moving the ball and mitt, and determining if a catch or a miss has occured.

Research suggests there are clear limits to our ability to process cognitive tasks (see George A. Miller's The Magical Number Seven, Plus or Minus Two: Some Limits on our Capacity for Processing Information). The more complex a program becomes, the more difficult it is for even an experienced programmer to develop and maintain.

To handle increasing complexity, we can wrap groups of related statements in functions, using **abstraction** to hide program details. This allows us to mentally treat a group of programming statements as a single concept, freeing up mental bandwidth for further tasks. The ability to use abstraction is one of the most powerful ideas in computer programming.

Here is a completed version of `catch.py`:

```
from gasp import *

COMPUTER_WINS = 1
PLAYER_WINS = 0
QUIT = -1


def distance(x1, y1, x2, y2):
    return ((x2 - x1)**2 + (y2 - y1)**2)**0.5


def play_round():
    ball_x = 10
    ball_y = random_between(20, 280)
    ball = Circle((ball_x, ball_y), 10, filled=True)
    dx = 4
    dy = random_between(-5, 5)

    mitt_x = 780
    mitt_y = random_between(20, 280)
    mitt = Circle((mitt_x, mitt_y), 20)

    while True:
        if ball_y >= 590 or ball_y <= 10:
            dy *= -1
        ball_x += dx
        ball_y += dy
        if ball_x >= 810:
```

```
            remove_from_screen(ball)
            remove_from_screen(mitt)
            return COMPUTER_WINS
        move_to(ball, (ball_x, ball_y))

        if key_pressed('k') and mitt_y <= 580:
            mitt_y += 5
        elif key_pressed('j') and mitt_y >= 20:
            mitt_y -= 5

        if key_pressed('escape'):
            return QUIT

        move_to(mitt, (mitt_x, mitt_y))

        if distance(ball_x, ball_y, mitt_x, mitt_y) <= 30:
            remove_from_screen(ball)
            remove_from_screen(mitt)
            return PLAYER_WINS

        update_when('next_tick')


def play_game():
    player_score = 0
    comp_score = 0

    while True:
        pmsg = Text("Player: %d Points" % player_score, (10, 570), size=24)
        cmsg = Text("Computer: %d Points" % comp_score, (640, 570), size=24)
        sleep(3)
        remove_from_screen(pmsg)
        remove_from_screen(cmsg)

        result = play_round()

        if result == PLAYER_WINS:
            player_score += 1
        elif result == COMPUTER_WINS:
            comp_score += 1
        else:
            return QUIT

        if player_score == 5:
            return PLAYER_WINS
        elif comp_score == 5:
            return COMPUTER_WINS
```

```
begin_graphics(800, 600, title="Catch", background=color.YELLOW)
set_speed(120)

result = play_game()

if result == PLAYER_WINS:
    Text("Player Wins!", (340, 290), size=32)
elif result == COMPUTER_WINS:
    Text("Computer Wins!", (340, 290), size=32)

sleep(4)

end_graphics()
```

Some new things to learn from this example:

- Following good organizational practices makes programs easier to read. Use the following organization in your programs:
  - imports
  - global constants
  - function definitions
  - main body of the program
- Symbolic **constants** like `COMPUTER_WINS`, `PLAYER_WINS`, and `QUIT` can be used to enhance readability of the program. It is customary to name constants with all capital letters. In Python it is up to the programmer to never assign a new value to a constant , since the language does not provide an easy way to enforce this (many other programming languages do).
- We took the version of the program developed in section 8.8 and wrapped it in a function named `play_round()`. `play_round` makes use of the constants defined at the top of the program. It is much easier to remember `COMPUTER_WINS` than it is the arbitrary numeric value assigned to it.
- A new function, `play_game()`, creates variables for `player_score` and `comp_score`. Using a `while` loop, it repeatedly calls `play_round`, checking the result of each call and updating the score appropriately. Finally, when either the player or computer reach 5 points, `play_game` returns the winner to the main body of the program, which then displays the winner and then quits.
- There are two variables named `result`—one in the `play_game` function and one in the main body of the program. While they have the same name, they are in different *namespaces*, and bear no relation to each other. Each function creates its own namespace, and names defined within the body of the function are not visible to code outside the function body.

Namespaces will be discussed in greater detail in the next chapter.

Glossary

abstraction *Generalization* by reducing the information content of a concept. Functions in Python can be used to group a number of program statements with a single name, abstracting out the details and making the program easier to understand.

constant A numerical value that does not change during the execution of a program. It is conventional to use names with all uppercase letters to repesent constants, though Python programs rely on the discipline of the programmers to enforce this, since there is no language mechanism to support true constants in Python.

nested loop A loop inside the body of another loop.

random Having no specific pattern. Unpredictable. Computers are designed to be predicatable, and it is not possible to get a truly random value from a computer. Certain functions produce sequences of values that appear as if they were random, and it is these *psuedorandom* values that we get from Python.

trichotomy Given any real numbers a and b, exactly one of the following relations holds: $a < b$, $a > b$, or $a = b$. Thus when you can establish that two of the relations are false, you can assume the remaining one is true.

Exercises

1. What happens when you press the key while running `mitt.py`? List the two lines from the program that produce this behavior and explain how they work.
2. What is the name of the counter variable in `guess.py`? With a proper strategy, the maximum number of guesses required to arrive at the correct number should be 11. What is this strategy?
3. What happens when the mitt in `mitt.py` gets to the top or bottom of the graphics window? List the lines from the program that control this behavior and explain *in detail* how they work.
4. Change the value of `ball1_dx` in `collide.py` to 2. How does the program behave differently? Now change `ball1_dx` back to 4 and set `ball2_dx` to -2. Explain *in detail* how these changes effect the behavior of the program.
5. Comment out (put a # in front of the statement) the `break` statement in `collide.py`. Do you notice any change in the behavior of the program? Now also comment out the `remove_from_screen(ball1)` statement. What happens now? Experiment with commenting and uncommenting the two `remove_from_screen` statements and the `break` statement until you can describe *specifically* how these statements work together to produce the desired behavior in the program.

6. Where can you add the lines

```
Text("Player Wins!", (340, 290), size=32)
sleep(2)
```

to the version of `catch.py` in section 8.8 so that the program displays this message when the ball is caught?

7. Trace the flow of execution in the final version of `catch.py` when you press the `escape` key during the execution of `play_round`. What happens when you press this key? Why?

8. List the main body of the final version of `catch.py`. Describe *in detail* what each line of code does. Which statement calls the function that starts the game?

9. Identify the function responsible for displaying the ball and the mitt. What other operations are provided by this function?

10. Which function keeps track of the score? Is this also the function that displays the score? Justify your answer by discussing specific parts of the code which implement these operations.

Project: `pong.py`

Pong was one of the first commercial video games. With a capital P it is a registered trademark, but pong is used to refer any of the table tennis like paddle and ball video games.

`catch.py` already contains all the programming tools we need to develop our own version of pong. Incrementally changing `catch.py` into `pong.py` is the goal of this project, which you will accomplish by completing the following series of exercises:

1. Copy `catch.py` to `pong1.py` and change the ball into a paddle by using `Box` instead of the `Circle`. You can look at Appendix A for more information on `Box`. Make the adjustments needed to keep the paddle on the screen.

2. Copy `pong1.py` to `pong2.py`. Replace the `distance` function with a boolean function `hit(bx, by, r, px, py, h)` that returns `True` when the vertical coordinate of the ball (`by`) is between the bottom and top of the paddle, and the horizontal location of the ball (`bx`) is less than or equal to the radius (`r`) away from the front of the paddle. Use `hit` to determine when the ball hits the paddle, and make the ball bounce back in the opposite horizontal direction when `hit` returns `True`. Your completed function should pass these doctests:

```
def hit(bx, by, r, px, py, h):
    """
      >>> hit(760, 100, 10, 780, 100, 100)
      False
      >>> hit(770, 100, 10, 780, 100, 100)
      True
      >>> hit(770, 200, 10, 780, 100, 100)
```

```
            True
            >>> hit(770, 210, 10, 780, 100, 100)
            False
        """
```

Finally, change the scoring logic to give the player a point when the ball goes off the screen on the left.

3. Copy `pong2.py` to `pong3.py`. Add a new paddle on the left side of the screen which moves up when `'a'` is pressed and down when `'s'` is pressed. Change the starting point for the ball to the center of the screen, (400, 300), and make it randomly move to the left or right at the start of each round.

Lists

A **list** is an ordered set of values, where each value is identified by an index. The values that make up a list are called its **elements**. Lists are similar to strings, which are ordered sets of characters, except that the elements of a list can have any type. Lists and strings — and other things that behave like ordered sets — are called **sequences**.

List values

There are several ways to create a new list; the simplest is to enclose the elements in square brackets (`[` and `]`):

```
[10, 20, 30, 40]
["spam", "bungee", "swallow"]
```

The first example is a list of four integers. The second is a list of three strings. The elements of a list don't have to be the same type. The following list contains a string, a float, an integer, and (mirabile dictu) another list:

```
["hello", 2.0, 5, [10, 20]]
```

A list within another list is said to be **nested**.

Finally, there is a special list that contains no elements. It is called the empty list, and is denoted `[]`.

Like numeric 0 values and the empty string, the empty list is false in a boolean expression:

```
>>> if []:
...     print('This is true.')
... else:
...     print('This is false.')
...
```

```
This is false.
>>>
```

With all these ways to create lists, it would be disappointing if we couldn't assign list values to variables or pass lists as parameters to functions. We can:

```
>>> vocabulary = ["ameliorate", "castigate", "defenestrate"]
>>> numbers = [17, 123]
>>> empty = []
>>> print(vocabulary)
['ameliorate', 'castigate', 'defenestrate']
>>> print(numbers)
[17, 123]
>>> print(empty)
[]
```

Accessing elements

The syntax for accessing the elements of a list is the same as the syntax for accessing the characters of a string—the bracket operator (`[]` – not to be confused with an empty list). The expression inside the brackets specifies the index. Remember that the indices start at 0:

```
>>> print(numbers[0])
17
```

Any integer expression can be used as an index:

```
>>> numbers[9-8]
123
>>> numbers[1.0]
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: list indices must be integers
```

If you try to read or write an element that does not exist, you get a runtime error:

```
>>> numbers[2]
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
IndexError: list index out of range
```

If an index has a negative value, it counts backward from the end of the list:

```
>>> numbers[-1]
123
>>> numbers[-2]
17
>>> numbers[-3]
```

```
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
IndexError: list index out of range
```

`numbers[-1]` is the last element of the list, `numbers[-2]` is the second to last, and `numbers[-3]` doesn't exist.

It is common to use a loop variable as a list index.

```
horsemen = ["war", "famine", "pestilence", "death"]

i = 0
while i < 4:
    print(horsemen[i])
    i += 1
```

This `while` loop counts from 0 to 4. When the loop variable `i` is 4, the condition fails and the loop terminates. So the body of the loop is only executed when `i` is 0, 1, 2, and 3.

Each time through the loop, the variable `i` is used as an index into the list, printing the `i`-eth element. This pattern of computation is called a **list traversal**.

List length

The function `len` returns the length of a list, which is equal to the number of its elements. It is a good idea to use this value as the upper bound of a loop instead of a constant. That way, if the size of the list changes, you won't have to go through the program changing all the loops; they will work correctly for any size list:

```
horsemen = ["war", "famine", "pestilence", "death"]

i = 0
num = len(horsemen)
while i < num:
    print(horsemen[i])
    i += 1
```

The last time the body of the loop is executed, `i` is `len(horsemen) - 1`, which is the index of the last element. When `i` is equal to `len(horsemen)`, the condition fails and the body is not executed, which is a good thing, because `len(horsemen)` is not a legal index.

Although a list can contain another list, the nested list still counts as a single element. The length of this list is 4:

```
['spam!', 1, ['Brie', 'Roquefort', 'Pol le Veq'], [1, 2, 3]]
```

List membership

`in` is a boolean operator that tests membership in a sequence. We used it previously with strings, but it also works with lists and other sequences:

```
>>> horsemen = ['war', 'famine', 'pestilence', 'death']
>>> 'pestilence' in horsemen
True
>>> 'debauchery' in horsemen
False
```

Since pestilence is a member of the `horsemen` list, the `in` operator returns `True`. Since debauchery is not in the list, `in` returns `False`.

We can use the `not` in combination with `in` to test whether an element is not a member of a list:

```
>>> 'debauchery' not in horsemen
True
```

List operations

The + operator concatenates lists:

```
>>> a = [1, 2, 3]
>>> b = [4, 5, 6]
>>> c = a + b
>>> print(c)
[1, 2, 3, 4, 5, 6]
```

Similarly, the $\star$ operator repeats a list a given number of times:

```
>>> [0] * 4
[0, 0, 0, 0]
>>> [1, 2, 3] * 3
[1, 2, 3, 1, 2, 3, 1, 2, 3]
```

The first example repeats `[0]` four times. The second example repeats the list `[1, 2, 3]` three times.

List slices

The slice operations we saw with strings also work on lists:

```
>>> a_list = ['a', 'b', 'c', 'd', 'e', 'f']
>>> a_list[1:3]
['b', 'c']
>>> a_list[:4]
```

```
['a', 'b', 'c', 'd']
>>> a_list[3:]
['d', 'e', 'f']
>>> a_list[:]
['a', 'b', 'c', 'd', 'e', 'f']
```

The `range` function

Lists that contain consecutive integers are common, so Python provides a simple way to create them:

```
>>> range(1, 5)
[1, 2, 3, 4]
```

The `range` function takes two arguments and returns a list that contains all the integers from the first to the second, including the first but *not the second*.

There are two other forms of `range`. With a single argument, it creates a list that starts at 0:

```
>>> range(10)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

If there is a third argument, it specifies the space between successive values, which is called the **step size**. This example counts from 1 to 10 by steps of 2:

```
>>> range(1, 10, 2)
[1, 3, 5, 7, 9]
```

If the step size is negative, then `start` must be greater than `stop`

```
>>> range(20, 4, -5)
[20, 15, 10, 5]
```

or the result will be an empty list.

```
>>> range(10, 20, -5)
[]
```

Lists are mutable

Unlike strings, lists are **mutable**, which means we can change their elements. Using the bracket operator on the left side of an assignment, we can update one of the elements:

```
>>> fruit = ["banana", "apple", "quince"]
>>> fruit[0] = "pear"
>>> fruit[-1] = "orange"
```

```
>>> print(fruit)
['pear', 'apple', 'orange']
```

The bracket operator applied to a list can appear anywhere in an expression. When it appears on the left side of an assignment, it changes one of the elements in the list, so the first element of `fruit` has been changed from `'banana'` to `'pear'`, and the last from `'quince'` to `'orange'`. An assignment to an element of a list is called **item assignment**. Item assignment does not work for strings:

```
>>> my_string = 'TEST'
>>> my_string[2] = 'X'
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: 'str' object does not support item assignment
```

but it does for lists:

```
>>> my_list = ['T', 'E', 'S', 'T']
>>> my_list[2] = 'X'
>>> my_list
['T', 'E', 'X', 'T']
```

With the slice operator we can update several elements at once:

```
>>> a_list = ['a', 'b', 'c', 'd', 'e', 'f']
>>> a_list[1:3] = ['x', 'y']
>>> print(a_list)
['a', 'x', 'y', 'd', 'e', 'f']
```

We can also remove elements from a list by assigning the empty list to them:

```
>>> a_list = ['a', 'b', 'c', 'd', 'e', 'f']
>>> a_list[1:3] = []
>>> print(a_list)
['a', 'd', 'e', 'f']
```

And we can add elements to a list by squeezing them into an empty slice at the desired location:

```
>>> a_list = ['a', 'd', 'f']
>>> a_list[1:1] = ['b', 'c']
>>> print(a_list)
['a', 'b', 'c', 'd', 'f']
>>> a_list[4:4] = ['e']
>>> print(a_list)
['a', 'b', 'c', 'd', 'e', 'f']
```

List deletion

Using slices to delete list elements can be awkward, and therefore error-prone. Python provides an alternative that is more readable.

`del` removes an element from a list:

```
>>> a = ['one', 'two', 'three']
>>> del a[1]
>>> a
['one', 'three']
```

As you might expect, `del` handles negative indices and causes a runtime error if the index is out of range.

You can use a slice as an index for `del`:

```
>>> a_list = ['a', 'b', 'c', 'd', 'e', 'f']
>>> del a_list[1:5]
>>> print(a_list)
['a', 'f']
```

As usual, slices select all the elements up to, but not including, the second index.

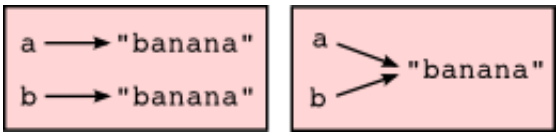
Objects and values

If we execute these assignment statements,

```
a = "banana"
b = "banana"
```

we know that `a` and `b` will refer to a string with the letters `"banana"`. But we don't know yet whether they point to the *same* string.

There are two possible states:



In one case, `a` and `b` refer to two different things that have the same value. In the second case, they refer to the same thing. These things have names — they are called **objects**. An object is something a variable can refer to.

We can test whether two names have the same value using ==:

```
>>> a == b
True
```

We can test whether two names refer to the same object using the *is* operator:

```
>>> a is b
True
```

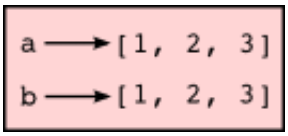
This tells us that both `a` and `b` refer to the same object, and that it is the second of the two state diagrams that describes the relationship.

Since strings are *immutable*, Python optimizes resources by making two names that refer to the same string value refer to the same object.

This is not the case with lists:

```
>>> a = [1, 2, 3]
>>> b = [1, 2, 3]
>>> a == b
True
>>> a is b
False
```

The state diagram here looks like this:



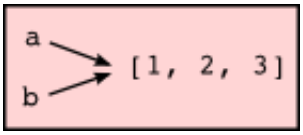
`a` and `b` have the same value but do not refer to the same object.

Aliasing

Since variables refer to objects, if we assign one variable to another, both variables refer to the same object:

```
>>> a = [1, 2, 3]
>>> b = a
>>> a is b
True
```

In this case, the state diagram looks like this:



Because the same list has two different names, `a` and `b`, we say that it is **aliased**. Changes made with one alias affect the other:

```
>>> b[0] = 5
>>> print(a)
[5, 2, 3]
```

Although this behavior can be useful, it is sometimes unexpected or undesirable. In general, it is safer to avoid aliasing when you are working with mutable objects. Of course, for immutable objects, there's no problem. That's why Python is free to alias strings when it sees an opportunity to economize.

Cloning lists

If we want to modify a list and also keep a copy of the original, we need to be able to make a copy of the list itself, not just the reference. This process is sometimes called **cloning**, to avoid the ambiguity of the word copy.

The easiest way to clone a list is to use the slice operator:

```
>>> a = [1, 2, 3]
>>> b = a[:]
>>> print(b)
[1, 2, 3]
```

Taking any slice of `a` creates a new list. In this case the slice happens to consist of the whole list.

Now we are free to make changes to `b` without worrying about `a`:

```
>>> b[0] = 5
>>> print(a)
[1, 2, 3]
```

Lists and `for` loops

The `for` loop also works with lists. The generalized syntax of a `for` loop is:

```
for VARIABLE in LIST:
    BODY
```

This statement is equivalent to:

```
i = 0
while i < len(LIST):
    VARIABLE = LIST[i]
    BODY
    i += 1
```

The `for` loop is more concise because we can eliminate the loop variable, `i`. Here is an equivalent to the `while` loop from the *Accessing elements* section written with a `for` loop.

```
for horseman in horsemen:
    print(horseman)
```

It almost reads like English: For (every) horseman in (the list of) horsemen, print (the name of the) horseman.

Any list expression can be used in a `for` loop:

```
for number in range(20):
    if number % 3 == 0:
        print(number)

for fruit in ["banana", "apple", "quince"]:
    print("I like to eat " + fruit + "s!")
```

The first example prints all the multiples of 3 between 0 and 19. The second example expresses enthusiasm for various fruits.

Since lists are mutable, it is often desirable to traverse a list, modifying each of its elements. The following squares all the numbers from `1` to `5`:

```
numbers = [1, 2, 3, 4, 5]

for index in range(len(numbers)):
    numbers[index] = numbers[index]**2
```

Take a moment to think about `range(len(numbers))` until you understand how it works. We are interested here in both the *value* and its *index* within the list, so that we can assign a new value to it.

This pattern is common enough that Python provides a nicer way to implement it:

```
numbers = [1, 2, 3, 4, 5]

for index, value in enumerate(numbers):
    numbers[index] = value**2
```

`enumerate` generates both the index and the value associated with it during the list traversal. Try this next example to see more clearly how `enumerate` works:

```
>>> for index, value in enumerate(['banana', 'apple', 'pear', 'quince']):
...     print("%d %s" % (index, value))
...
0 banana
1 apple
2 pear
3 quince
>>>
```

List parameters

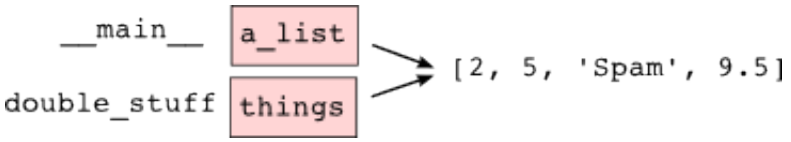
Passing a list as an argument actually passes a reference to the list, not a copy of the list. Since lists are mutable changes made to the parameter change the argument as well. For example, the function below takes a list as an argument and multiplies each element in the list by 2:

```
def double_stuff(a_list):
    for index, value in enumerate(a_list):
        a_list[index] = 2 * value
```

If we put `double_stuff` in a file named `ch09.py`, we can test it out like this:

```
>>> from ch09 import double_stuff
>>> things = [2, 5, 'Spam', 9.5]
>>> double_stuff(things)
>>> things
[4, 10, 'SpamSpam', 19.0]
>>>
```

The parameter `a_list` and the variable `things` are aliases for the same object. The state diagram looks like this:



Since the list object is shared by two frames, we drew it between them.

If a function modifies a list parameter, the caller sees the change.

Pure functions and modifiers

Functions which take lists as arguments and change them during execution are called **modifiers** and the changes they make are called **side effects**.

A **pure function** does not produce side effects. It communicates with the calling program only through parameters, which it does not modify, and a return value. Here is `double_stuff` written as a pure function:

```
def double_stuff(a_list):
    new_list = []
    for value in a_list:
        new_list += [2 * value]
    return new_list
```

This version of `double_stuff` does not change its arguments:

```
>>> from ch09 import double_stuff
>>> things = [2, 5, 'Spam', 9.5]
>>> double_stuff(things)
[4, 10, 'SpamSpam', 19.0]
>>> things
[2, 5, 'Spam', 9.5]
>>>
```

To use the pure function version of `double_stuff` to modify `things`, you would assign the return value back to `things`:

```
>>> things = double_stuff(things)
>>> things
[4, 10, 'SpamSpam', 19.0]
>>>
```

Which is better?

Anything that can be done with modifiers can also be done with pure functions. In fact, some programming languages only allow pure functions. There is some evidence that programs that use pure functions are faster to develop and less error-prone than programs that use modifiers. Nevertheless, modifiers are convenient at times, and in some cases, functional programs are less efficient.

In general, we recommend that you write pure functions whenever it is reasonable to do so and resort to modifiers only if there is a compelling advantage. This approach might be called a *functional programming style*.

Nested lists

A nested list is a list that appears as an element in another list. In this list, the element with index 3 is a nested list:

```
>>> nested = ["hello", 2.0, 5, [10, 20]]
```

If we print `nested[3]`, we get `[10, 20]`. To extract an element from the nested list, we can proceed in two steps:

```
>>> elem = nested[3]
>>> elem[0]
10
```

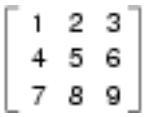
Or we can combine them:

```
>>> nested[3][1]
20
```

Bracket operators evaluate from left to right, so this expression gets the three-eth element of `nested` and extracts the one-eth element from it.

Matrices

Nested lists are often used to represent matrices. For example, the matrix:



might be represented as:

```
>>> matrix = [[1, 2, 3], [4, 5, 6], [7, 8, 9]]
```

`matrix` is a list with three elements, where each element is a row of the matrix. We can select an entire row from the matrix in the usual way:

```
>>> matrix[1]
[4, 5, 6]
```

Or we can extract a single element from the matrix using the double-index form:

```
>>> matrix[1][1]
5
```

The first index selects the row, and the second index selects the column. Although this way of representing matrices is common, it is not the only possibility. A small variation is to use a list of columns instead of a list of rows. Later we will see a more radical alternative using a dictionary.

Test-driven development (TDD)

Test-driven development (TDD) is a software development practice which arrives at a desired feature through a series of small, iterative steps motivated by automated tests which are *written first* that express increasing refinements of the desired feature.

Doctest enables us to easily demonstrate TDD. Let's say we want a function which creates a `rows` by `columns` matrix given arguments for `rows` and `columns`.

We first setup a test for this function in a file named `matrices.py`:

```
def make_matrix(rows, columns):
    """
      >>> make_matrix(3, 5)
      [[0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0]]
    """
```

```
if __name__ == '__main__':
    import doctest
    doctest.testmod()
```

Running this returns in a failing test:

```
**********************************************************************
File "matrices.py", line 3, in __main__.make_matrix
Failed example:
    make_matrix(3, 5)
Expected:
    [[0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0]]
Got nothing
**********************************************************************
1 items had failures:
   1 of   1 in __main__.make_matrix
***Test Failed*** 1 failures.
```

The test fails because the body of the function contains only a single triple quoted string and no return statement, so it returns `None`. Our test indicates that we wanted it to return a matrix with 3 rows of 5 columns of zeros.

The rule in using TDD is to use the *simplest thing that works* in writing a solution to pass the test, so in this case we can simply return the desired result:

```
def make_matrix(rows, columns):
    """
      >>> make_matrix(3, 5)
      [[0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0]]
    """
    return [[0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0]]
```

Running this now the test passes, but our current implementation of `make_matrix` always returns the same result, which is clearly not what we intended. To fix this, we first motivate our improvement by adding a test:

```
def make_matrix(rows, columns):
    """
      >>> make_matrix(3, 5)
      [[0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0]]
      >>> make_matrix(4, 2)
      [[0, 0], [0, 0], [0, 0], [0, 0]]
    """
    return [[0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0]]
```

which as we expect fails:

```
**********************************************************************
File "matrices.py", line 5, in __main__.make_matrix
```

```
Failed example:
    make_matrix(4, 2)
Expected:
    [[0, 0], [0, 0], [0, 0], [0, 0]]
Got:
    [[0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0]]
**********************************************************************
1 items had failures:
   1 of   2 in __main__.make_matrix
***Test Failed*** 1 failures.
```

This technique is called *test-driven* because code should only be written when there is a failing test to make pass. Motivated by the failing test, we can now produce a more general solution:

```
def make_matrix(rows, columns):
    """
      >>> make_matrix(3, 5)
      [[0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0]]
      >>> make_matrix(4, 2)
      [[0, 0], [0, 0], [0, 0], [0, 0]]
    """
    return [[0] * columns] * rows
```

This solution appears to work, and we may think we are finished, but when we use the new function later we discover a bug:

```
>>> from matrices import *
>>> m = make_matrix(4, 3)
>>> m
[[0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 0]]
>>> m[1][2] = 7
>>> m
[[0, 0, 7], [0, 0, 7], [0, 0, 7], [0, 0, 7]]
>>>
```

We wanted to assign the element in the second row and the third column the value 7, instead, *all* elements in the third column are 7!

Upon reflection, we realize that in our current solution, each row is an *alias* of the other rows. This is definitely not what we intended, so we set about fixing the problem, *first by writing a failing test*:

```
def make_matrix(rows, columns):
    """
      >>> make_matrix(3, 5)
      [[0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0]]
      >>> make_matrix(4, 2)
      [[0, 0], [0, 0], [0, 0], [0, 0]]
      >>> m = make_matrix(4, 2)
```

```
      >>> m[1][1] = 7
      >>> m
      [[0, 0], [0, 7], [0, 0], [0, 0]]
    """
    return [[0] * columns] * rows
```

With a failing test to fix, we are now driven to a better solution:

```
def make_matrix(rows, columns):
    """
      >>> make_matrix(3, 5)
      [[0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0]]
      >>> make_matrix(4, 2)
      [[0, 0], [0, 0], [0, 0], [0, 0]]
      >>> m = make_matrix(4, 2)
      >>> m[1][1] = 7
      >>> m
      [[0, 0], [0, 7], [0, 0], [0, 0]]
    """
    matrix = []
    for row in range(rows):
        matrix += [[0] * columns]
    return matrix
```

Using TDD has several benefits to our software development process. It:

- helps us think concretely about the problem we are trying to solve *before* we attempt to solve it.
- encourages breaking down complex problems into smaller, simpler problems and working our way toward a solution of the larger problem step-by-step.
- assures that we have a well developed automated test suite for our software, facilitating later additions and improvements.

Strings and lists

Python has a command called `list` that takes a sequence type as an argument and creates a list out of its elements.

```
>>> list("Crunchy Frog")
['C', 'r', 'u', 'n', 'c', 'h', 'y', ' ', 'F', 'r', 'o', 'g']
```

There is also a `str` command that takes any Python value as an argument and returns a string representation of it.

```
>>> str(5)
'5'
```

```
>>> str(None)
'None'
>>> str(list("nope"))
"['n', 'o', 'p', 'e']"
```

As we can see from the last example, `str` can't be used to join a list of characters together. To do this we could use the `join` function in the `string` module:

```
>>> import string
>>> char_list = list("Frog")
>>> char_list
['F', 'r', 'o', 'g']
>>> string.join(char_list, '')
'Frog'
```

Two of the most useful functions in the `string` module involve lists of strings. The `split` function breaks a string into a list of words. By default, any number of whitespace characters is considered a word boundary:

```
>>> import string
>>> song = "The rain in Spain..."
>>> string.split(song)
['The', 'rain', 'in', 'Spain...']
```

An optional argument called a **delimiter** can be used to specify which characters to use as word boundaries. The following example uses the string `ai` as the delimiter:

```
>>> string.split(song, 'ai')
['The r', 'n in Sp', 'n...']
```

Notice that the delimiter doesn't appear in the list.

`string.join` is the inverse of `string.split`. It takes two arguments: a list of strings and a *separator* which will be placed between each element in the list in the resultant string.

```
>>> import string
>>> words = ['crunchy', 'raw', 'unboned', 'real', 'dead', 'frog']
>>> string.join(words, ' ')
'crunchy raw unboned real dead frog'
>>> string.join(words, '**')
'crunchy**raw**unboned**real**dead**frog'
```

Glossary

aliases Multiple variables that contain references to the same object.

clone To create a new object that has the same value as an existing object. Copying a reference to an object creates an alias but doesn't clone the object.

delimiter A character or string used to indicate where a string should be split.

element One of the values in a list (or other sequence). The bracket operator selects elements of a list.

index An integer variable or value that indicates an element of a list.

list A named collection of objects, where each object is identified by an index.

list traversal The sequential accessing of each element in a list.

modifier A function which changes its arguments inside the function body. Only mutable types can be changed by modifiers.

mutable type A data type in which the elements can be modified. All mutable types are compound types. Lists are mutable data types; strings are not.

nested list A list that is an element of another list.

object A thing to which a variable can refer.

pure function A function which has no side effects. Pure functions only make changes to the calling program through their return values.

sequence Any of the data types that consist of an ordered set of elements, with each element identified by an index.

side effect A change in the state of a program made by calling a function that is not a result of reading the return value from the function. Side effects can only be produced by modifiers.

step size The interval between successive elements of a linear sequence. The third (and optional argument) to the `range` function is called the step size. If not specified, it defaults to 1.

test-driven development (TDD) A software development practice which arrives at a desired feature through a series of small, iterative steps motivated by automated tests which are *written first* that express increasing refinements of the desired feature. (see the Wikipedia article on Test-driven development for more information.)

Exercises

1. Write a loop that traverses:

```
['spam!', 1, ['Brie', 'Roquefort', 'Pol le Veq'], [1, 2, 3]]
```

and prints the length of each element. What happens if you send an integer to `len`? Change `1` to `'one'` and run your solution again.

2. Open a file named `ch09e02.py` and with the following content:

```
#  Add your doctests here:
"""
"""
```

```
    # Write your Python code here:


    if __name__ == '__main__':
        import doctest
        doctest.testmod()
```

Add each of the following sets of doctests to the docstring at the top of the file and write Python code to make the doctests pass.

```
    """
      >>> a_list[3]
      42
      >>> a_list[6]
      'Ni!'
      >>> len(a_list)
      8
    """
```

```
    """
      >>> b_list[1:]
      ['Stills', 'Nash']
      >>> group = b_list + c_list
      >>> group[-1]
      'Young'
    """
```

```
    """
      >>> 'war' in mystery_list
      False
      >>> 'peace' in mystery_list
      True
      >>> 'justice' in mystery_list
      True
      >>> 'oppression' in mystery_list
      False
      >>> 'equality' in mystery_list
      True
    """
```

```
    """
      >>> range(a, b, c)
      [5, 9, 13, 17]
    """
```

Only add one set of doctests at a time. The next set of doctests should not be added until the previous set pass.

3. What is the Python interpreter's response to the following?

```
>>> range(10, 0, -2)
```

   The three arguments to the *range* function are *start*, *stop*, and *step*, respectively. In this example, `start` is greater than `stop`. What happens if `start < stop` and `step < 0`? Write a rule for the relationships among `start`, `stop`, and `step`.

4. Draw a state diagram for `a` and `b` before and after the third line of the following python code is executed:

```
a = [1, 2, 3]
b = a[:]
b[0] = 5
```

5. What will be the output of the following program?

```
this = ['I', 'am', 'not', 'a', 'crook']
that = ['I', 'am', 'not', 'a', 'crook']
print("Test 1: %s" % (this is that))
that = this
print("Test 2: %s" % (this is that))
```

   Provide a *detailed* explaination of the results.

6. Open a file named `ch09e06.py` and use the same procedure as in exercise 2 to make the following doctests pass:

```
"""
  >>> 13 in junk
  True
  >>> del junk[4]
  >>> junk
  [3, 7, 9, 10, 17, 21, 24, 27]
  >>> del junk[a:b]
  >>> junk
  [3, 7, 27]
"""
```

```
"""
  >>> nlist[2][1]
  0
  >>> nlist[0][2]
  17
  >>> nlist[1][1]
  5
"""
```

```
"""
  >>> import string
```

```
    >>> string.split(message, '??')
    ['this', 'and', 'that']
"""
```

7. Lists can be used to represent mathematical *vectors*. In this exercise and several that follow you will write functions to perform standard operations on vectors. Create a file named `vectors.py` and write Python code to make the doctests for each function pass.

   Write a function `add_vectors(u, v)` that takes two lists of numbers of the same length, and returns a new list containing the sums of the corresponding elements of each.

```
def add_vectors(u, v):
    """
      >>> add_vectors([1, 0], [1, 1])
      [2, 1]
      >>> add_vectors([1, 2], [1, 4])
      [2, 6]
      >>> add_vectors([1, 2, 1], [1, 4, 3])
      [2, 6, 4]
      >>> add_vectors([11, 0, -4, 5], [2, -4, 17, 0])
      [13, -4, 13, 5]
    """
```

   `add_vectors` should pass the doctests above.

8. Write a function `scalar_mult(s, v)` that takes a number, `s`, and a list, `v` and returns the scalar multiple of `v` by `s`.

```
def scalar_mult(s, v):
    """
      >>> scalar_mult(5, [1, 2])
      [5, 10]
      >>> scalar_mult(3, [1, 0, -1])
      [3, 0, -3]
      >>> scalar_mult(7, [3, 0, 5, 11, 2])
      [21, 0, 35, 77, 14]
    """
```

9. Write a function `dot_product(u, v)` that takes two lists of numbers of the same length, and returns the sum of the products of the corresponding elements of each (the dot_product).

```
def dot_product(u, v):
    """
      >>> dot_product([1, 1], [1, 1])
      2
      >>> dot_product([1, 2], [1, 4])
      9
      >>> dot_product([1, 2, 1], [1, 4, 3])
      12
```

```
        >>> dot_product([2, 0, -1, 1], [1, 5, 2, 0])
        0
    """
```

Verify that `dot_product` passes the doctests above.

10. *Extra challenge for the mathematically inclined*: Write a function `cross_product(u, v)` that takes two lists of numbers of length 3 and returns their cross product. You should write your own doctests and use the test driven development process described in the chapter.

11. Create a new module named `matrices.py` and add the following two functions introduced in the section on test-driven development:

```
def add_row(matrix):
    """
      >>> m = [[0, 0], [0, 0]]
      >>> add_row(m)
      [[0, 0], [0, 0], [0, 0]]
      >>> n = [[3, 2, 5], [1, 4, 7]]
      >>> add_row(n)
      [[3, 2, 5], [1, 4, 7], [0, 0, 0]]
      >>> n
      [[3, 2, 5], [1, 4, 7]]
    """
```

```
def add_column(matrix):
    """
      >>> m = [[0, 0], [0, 0]]
      >>> add_column(m)
      [[0, 0, 0], [0, 0, 0]]
      >>> n = [[3, 2], [5, 1], [4, 7]]
      >>> add_column(n)
      [[3, 2, 0], [5, 1, 0], [4, 7, 0]]
      >>> n
      [[3, 2], [5, 1], [4, 7]]
    """
```

Your new functions should pass the doctests. Note that the last doctest in each function assures that `add_row` and `add_column` are pure functions. (*hint:* Python has a `copy` module with a function named `deepcopy` that could make your task easier here. We will talk more about `deepcopy` in chapter 13, but google python copy module if you would like to try it now.)

12. Write a function `add_matrices(m1, m2)` that adds `m1` and `m2` and returns a new matrix containing their sum. You can assume that `m1` and `m2` are the same size. You add two matrices by adding their corresponding values.

```
def add_matrices(m1, m2):
    """
```

```
        >>> a = [[1, 2], [3, 4]]
        >>> b = [[2, 2], [2, 2]]
        >>> add_matrices(a, b)
        [[3, 4], [5, 6]]
        >>> c = [[8, 2], [3, 4], [5, 7]]
        >>> d = [[3, 2], [9, 2], [10, 12]]
        >>> add_matrices(c, d)
        [[11, 4], [12, 6], [15, 19]]
        >>> c
        [[8, 2], [3, 4], [5, 7]]
        >>> d
        [[3, 2], [9, 2], [10, 12]]
    """
```

Add your new function to `matrices.py` and be sure it passes the doctests above. The last two doctests confirm that `add_matrices` is a pure function.

13. Write a function `scalar_mult(s, m)` that multiplies a matrix, `m`, by a scalar, `s`.

```
def scalar_mult(s, m):
    """
        >>> a = [[1, 2], [3, 4]]
        >>> scalar_mult(3, a)
        [[3, 6], [9, 12]]
        >>> b = [[3, 5, 7], [1, 1, 1], [0, 2, 0], [2, 2, 3]]
        >>> scalar_mult(10, b)
        [[30, 50, 70], [10, 10, 10], [0, 20, 0], [20, 20, 30]]
        >>> b
        [[3, 5, 7], [1, 1, 1], [0, 2, 0], [2, 2, 3]]
    """
```

Add your new function to `matrices.py` and be sure it passes the doctests above.

14. Write functions `row_times_column` and `matrix_mult`:

```
def row_times_column(m1, row, m2, column):
    """
        >>> row_times_column([[1, 2], [3, 4]], 0, [[5, 6], [7, 8]], 0)
        19
        >>> row_times_column([[1, 2], [3, 4]], 0, [[5, 6], [7, 8]], 1)
        22
        >>> row_times_column([[1, 2], [3, 4]], 1, [[5, 6], [7, 8]], 0)
        43
        >>> row_times_column([[1, 2], [3, 4]], 1, [[5, 6], [7, 8]], 1)
        50
    """
```

```
def matrix_mult(m1, m2):
    """
```

```
        >>> matrix_mult([[1, 2], [3,  4]], [[5, 6], [7, 8]])
        [[19, 22], [43, 50]]
        >>> matrix_mult([[1, 2, 3], [4,  5, 6]], [[7, 8], [9, 1], [2, 3]])
        [[31, 19], [85, 55]]
        >>> matrix_mult([[7, 8], [9, 1], [2, 3]], [[1, 2, 3], [4, 5, 6]])
        [[39, 54, 69], [13, 23, 33], [14, 19, 24]]
    """
```

Add your new functions to `matrices.py` and be sure it passes the doctests above.

15. Create a new module named `numberlists.py` and add the following functions to the module:

```
def only_evens(numbers):
    """
      >>> only_evens([1, 3, 4, 6, 7, 8])
      [4, 6, 8]
      >>> only_evens([2, 4, 6, 8, 10, 11, 0])
      [2, 4, 6, 8, 10, 0]
      >>> only_evens([1, 3, 5, 7, 9, 11])
      []
      >>> only_evens([4, 0, -1, 2, 6, 7, -4])
      [4, 0, 2, 6, -4]
      >>> nums = [1, 2, 3, 4]
      >>> only_evens(nums)
      [2, 4]
      >>> nums
      [1, 2, 3, 4]
    """
```

```
def only_odds(numbers):
    """
      >>> only_odds([1, 3, 4, 6, 7, 8])
      [1, 3, 7]
      >>> only_odds([2, 4, 6, 8, 10, 11, 0])
      [11]
      >>> only_odds([1, 3, 5, 7, 9, 11])
      [1, 3, 5, 7, 9, 11]
      >>> only_odds([4, 0, -1, 2, 6, 7, -4])
      [-1, 7]
      >>> nums = [1, 2, 3, 4]
      >>> only_odds(nums)
      [1, 3]
      >>> nums
      [1, 2, 3, 4]
    """
```

Be sure these new functions pass the doctests.

16. Add a function `multiples_of(num, numlist)` to `numberlists.py` that takes an integer (`num`), and a list of integers (`numlist`) as arguments and returns a list of those integers in `numlist` that are multiples of `num`. Add your own doctests and use TDD to develope this function.

17. Given:

```
import string

song = "The rain in Spain..."
```

Describe the relationship between `string.join(string.split(song))` and `song`. Are they the same for all strings? When would they be different?

18. Write a function `replace(s, old, new)` that replaces all occurences of `old` with `new` in a string `s`.

```
def replace(s, old, new):
    """
      >>> replace('Mississippi', 'i', 'I')
      'MIssIssIppI'
      >>> s = 'I love spom!  Spom is my favorite food.  Spom, spom, spom, yum!
      >>> replace(s, 'om', 'am')
      'I love spam!  Spam is my favorite food.  Spam, spam, spam, yum!'
      >>> replace(s, 'o', 'a')
      'I lave spam!  Spam is my favarite faad.  Spam, spam, spam, yum!'
    """
```

Your solution should pass the doctests above. *Hint*: use `string.split` and `string.join`.

Modules and files

Modules

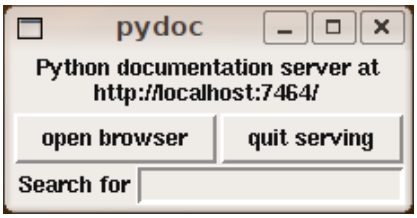
A **module** is a file containing Python definitions and statements intended for use in other Python programs. There are many Python modules that come with Python as part of the **standard library**. We have seen two of these already, the `doctest` module and the `string` module.

pydoc

You can use **pydoc** to search through the Python libraries installed on your system. At the **command prompt** type the following:

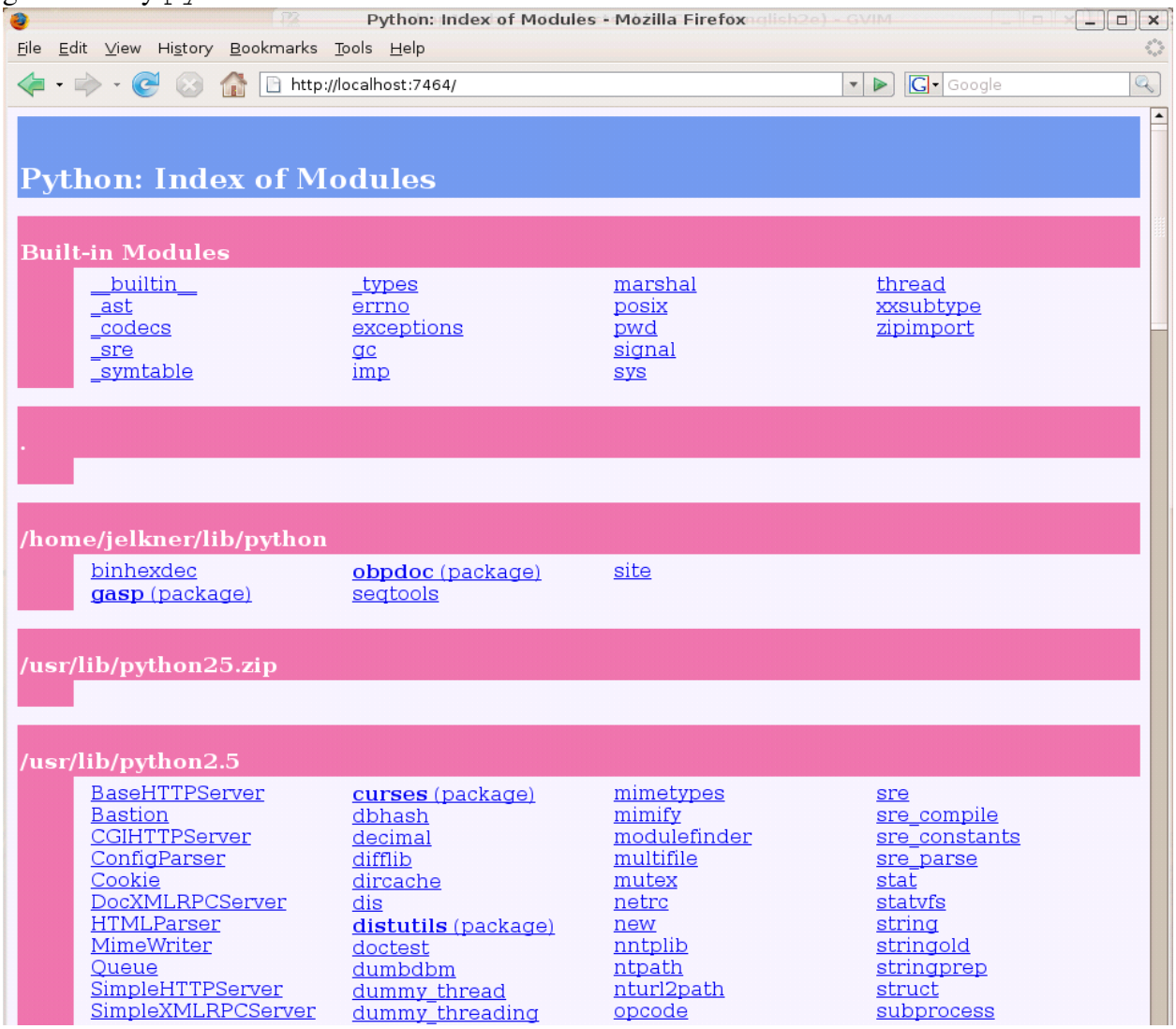
```
$ pydoc -g
```

and the following will appear:



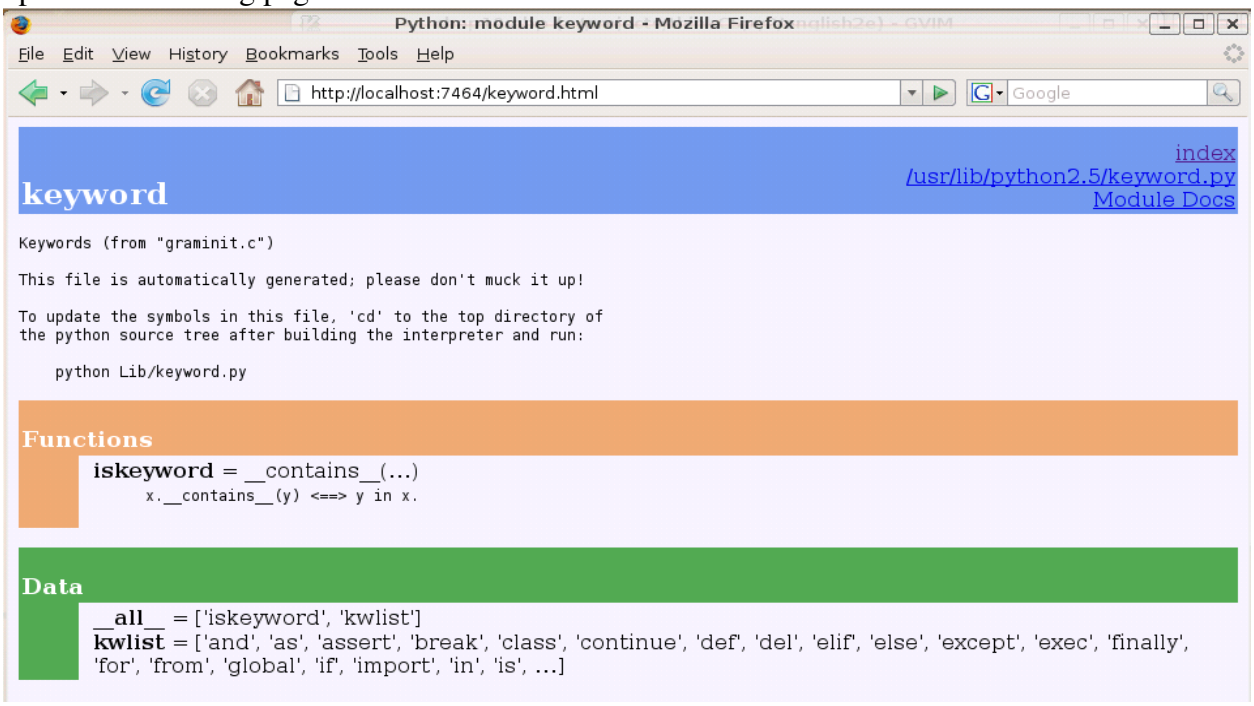
(*note*: see exercise 2 if you get an error)

Click on the open browser button to launch a web browser window containing the documentation generated by `pydoc`:



This is a listing of all the python libraries found by Python on your system. Clicking on a module name opens a new page with documenation for that module. Clicking `keyword`, for example,

opens the following page:



Documentation for most modules contains three color coded sections:

- *Classes* in pink
- *Functions* in orange
- *Data* in green

Classes will be discussed in later chapters, but for now we can use pydoc to see the functions and data contained within modules.

The `keyword` module contains a single function, `iskeyword`, which as its name suggests is a boolean function that returns `True` if a string passed to it is a keyword:

```
>>> from keyword import *
>>> iskeyword('for')
True
>>> iskeyword('all')
False
>>>
```

The data item, `kwlist` contains a list of all the current keywords in Python:

```
>>> from keyword import *
>>> print(kwlist)
['and', 'as', 'assert', 'break', 'class', 'continue', 'def', 'del', 'elif',
'else', 'except', 'exec', 'finally', 'for', 'from', 'global', 'if', 'import',
'in', 'is', 'lambda', 'not', 'or', 'pass', 'print', 'raise', 'return', 'try',
'while', 'with', 'yield']
```

```
>>>
```

We encourage you to use `pydoc` to explore the extensive libraries that come with Python. There are so many treasures to discover!

Creating modules

All we need to create a module is a text file with a `.py` extension on the filename:

```
#  seqtools.py
#
def remove_at(pos, seq):
    return seq[:pos] + seq[pos+1:]
```

We can now use our module in both scripts and the Python shell. To do so, we must first *import* the module. There are two ways to do this:

```
>>> from seqtools import remove_at
>>> s = "A string!"
>>> remove_at(4, s)
'A sting!'
```

and:

```
>>> import seqtools
>>> s = "A string!"
>>> seqtools.remove_at(4, s)
'A sting!'
```

In the first example, `remove_at` is called just like the functions we have seen previously. In the second example the name of the module and a dot (.) are written before the function name.

Notice that in either case we do not include the `.py` file extension when importing. Python expects the file names of Python modules to end in `.py`, so the file extention is not included in the **import statement**.

The use of modules makes it possible to break up very large programs into manageable sized parts, and to keep related parts together.

Namespaces

A **namespace** is a syntactic container which permits the same name to be used in different modules or functions (and as we will see soon, in classes and methods).

Each module determines its own namespace, so we can use the same name in multiple modules without causing an identification problem.

```
# module1.py

question = "What is the meaning of life, the Universe, and everything?"
answer = 42
```

```
# module2.py

question = "What is your quest?"
answer = "To seek the holy grail."
```

We can now import both modules and access `question` and `answer` in each:

```
>>> import module1
>>> import module2
>>> print(module1.question)
What is the meaning of life, the Universe, and everything?
>>> print(module2.question)
What is your quest?
>>> print(module1.answer)
42
>>> print(module2.answer)
To seek the holy grail.
>>>
```

If we had used `from module1 import *` and `from module2 import *` instead, we would have a **naming collision** and would not be able to access `question` and `answer` from `module1`.

Functions also have their own namespace:

```
def f():
    n = 7
    print("printing n inside of f: %d" % (n))

def g():
    n = 42
    print("printing n inside of g: %d" % (n))

n = 11
print("printing n before calling f: %d" % (n))
f()
print("printing n after calling f: %d" % (n))
g()
print("printing n after calling g: %d" % (n))
```

Running this program produces the following output:

```
printing n before calling f: 11
printing n inside of f: 7
printing n after calling f: 11
printing n inside of g: 42
printing n after calling g: 11
```

The three `n`'s here do not collide since they are each in a different namespace.

Namespaces permit several programmers to work on the same project without having naming collisions.

Attributes and the dot operator

Variables defined inside a module are called **attributes** of the module. They are accessed by using the **dot operator** (`.`). The `question` attribute of `module1` and `module2` are accessed using `module1.question` and `module2.question`.

Modules contain functions as well as attributes, and the dot operator is used to access them in the same way. `seqtools.remove_at` refers to the `remove_at` function in the `seqtools` module.

In Chapter 7 we introduced the `find` function from the `string` module. The `string` module contains many other useful functions:

```
>>> import string
>>> string.capitalize('maryland')
'Maryland'
>>> string.capwords("what's all this, then, amen?")
"What's All This, Then, Amen?"
>>> string.center('How to Center Text Using Python', 70)
'                    How to Center Text Using Python                     '
>>> string.upper('angola')
'ANGOLA'
>>>
```

You should use pydoc to browse the other functions and attributes in the string module.

String and list methods

As the Python language developed, most of functions from the `string` module have also been added as **methods** of string objects. A method acts much like a function, but the syntax for calling it is a bit different:

```
>>> 'maryland'.capitalize()
'Maryland'
>>> "what's all this, then, amen?".title()
```

```
"What'S All This, Then, Amen?"
>>> 'How to Center Text Using Python'.center(70)
'                    How to Center Text Using Python                     '
>>> 'angola'.upper()
'ANGOLA'
>>>
```

String methods are built into string objects, and they are *invoked* (called) by following the object with the dot operator and the method name.

We will be learning how to create our own objects with their own methods in later chapters. For now we will only be using methods that come with Python's built-in objects.

The dot operator can also be used to access built-in methods of list objects:

```
>>> mylist = []
>>> mylist.append(5)
>>> mylist.append(27)
>>> mylist.append(3)
>>> mylist.append(12)
>>> mylist
[5, 27, 3, 12]
>>>
```

`append` is a list method which adds the argument passed to it to the end of the list. Continuing with this example, we show several other list methods:

```
>>> mylist.insert(1, 12)
>>> mylist
[5, 12, 27, 3, 12]
>>> mylist.count(12)
2
>>> mylist.extend([5, 9, 5, 11])
>>> mylist
[5, 12, 27, 3, 12, 5, 9, 5, 11])
>>> mylist.index(9)
6
>>> mylist.count(5)
3
>>> mylist.reverse()
>>> mylist
[11, 5, 9, 5, 12, 3, 27, 12, 5]
>>> mylist.sort()
>>> mylist
[3, 5, 5, 5, 9, 11, 12, 12, 27]
>>> mylist.remove(12)
>>> mylist
[3, 5, 5, 5, 9, 11, 12, 27]
>>>
```

Experiment with the list methods in this example until you feel confident that you understand how they work.

Reading and writing text files

While a program is running, its data is stored in *random access memory* (RAM). RAM is fast and inexpensive, but it is also **volatile**, which means that when the program ends, or the computer shuts down, data in RAM disappears. To make data available the next time you turn on your computer and start your program, you have to write it to a **non-volatile** storage medium, such as a hard drive, usb drive, or CD-RW.

Data on non-volatile storage media is stored in named locations on the media called **files**. By reading and writing files, programs can save information between program runs.

Working with files is a lot like working with a notebook. To use a notebook, you have to open it. When you're done, you have to close it. While the notebook is open, you can either write in it or read from it. In either case, you know where you are in the notebook. You can read the whole notebook in its natural order or you can skip around.

All of this applies to files as well. To open a file, you specify its name and indicate whether you want to read or write.

Opening a file creates a file object. In this example, the variable `myfile` refers to the new file object.

```
>>> myfile = open('test.dat', 'w')
>>> print(myfile)
<open file 'test.dat', mode 'w' at 0x2aaaaab80cd8>
```

The open function takes two arguments. The first is the name of the file, and the second is the **mode**. Mode `'w'` means that we are opening the file for writing.

If there is no file named `test.dat`, it will be created. If there already is one, it will be replaced by the file we are writing.

When we print the file object, we see the name of the file, the mode, and the location of the object.

To put data in the file we invoke the `write` method on the file object:

```
>>> myfile.write("Now is the time")
>>> myfile.write("to close the file")
```

Closing the file tells the system that we are done writing and makes the file available for reading:

```
>>> myfile.close()
```

Now we can open the file again, this time for reading, and read the contents into a string. This time, the mode argument is `'r'` for reading:

```
>>> myfile = open('test.dat', 'r')
```

If we try to open a file that doesn't exist, we get an error:

```
>>> myfile = open('test.cat', 'r')
IOError: [Errno 2] No such file or directory: 'test.cat'
```

Not surprisingly, the `read` method reads data from the file. With no arguments, it reads the entire contents of the file into a single string:

```
>>> text = myfile.read()
>>> print(text)
Now is the timeto close the file
```

There is no space between time and to because we did not write a space between the strings.

`read` can also take an argument that indicates how many characters to read:

```
>>> myfile = open('test.dat', 'r')
>>> print(myfile.read(5))
Now i
```

If not enough characters are left in the file, `read` returns the remaining characters. When we get to the end of the file, `read` returns the empty string:

```
>>> print(myfile.read(1000006))
s the timeto close the file
>>> print(myfile.read())

>>>
```

The following function copies a file, reading and writing up to fifty characters at a time. The first argument is the name of the original file; the second is the name of the new file:

```
def copy_file(oldfile, newfile):
    infile = open(oldfile, 'r')
    outfile = open(newfile, 'w')
    while True:
        text = infile.read(50)
        if text == "":
            break
        outfile.write(text)
    infile.close()
    outfile.close()
    return
```

This functions continues looping, reading 50 characters from `infile` and writing the same 50 charaters to `outfile` until the end of `infile` is reached, at which point `text` is empty and the `break` statement is executed.

Text files

A **text file** is a file that contains printable characters and whitespace, organized into lines separated by newline characters. Since Python is specifically designed to process text files, it provides methods that make the job easy.

To demonstrate, we'll create a text file with three lines of text separated by newlines:

```
>>> outfile = open("test.dat","w")
>>> outfile.write("line one\nline two\nline three\n")
>>> outfile.close()
```

The `readline` method reads all the characters up to and including the next newline character:

```
>>> infile = open("test.dat","r")
>>> print(infile.readline())
line one

>>>
```

`readlines` returns all of the remaining lines as a list of strings:

```
>>> print(infile.readlines())
['line two\012', 'line three\012']
```

In this case, the output is in list format, which means that the strings appear with quotation marks and the newline character appears as the escape sequence `\\012`.

At the end of the file, `readline` returns the empty string and `readlines` returns the empty list:

```
>>> print(infile.readline())

>>> print(infile.readlines())
[]
```

The following is an example of a line-processing program. `filter` makes a copy of `oldfile`, omitting any lines that begin with `#`:

```
def filter(oldfile, newfile):
    infile = open(oldfile, 'r')
    outfile = open(newfile, 'w')
    while True:
        text = infile.readline()
        if text == "":
            break
        if text[0] == '#':
            continue
        outfile.write(text)
    infile.close()
```

```
    outfile.close()
    return
```

The **continue statement** ends the current iteration of the loop, but continues looping. The flow of execution moves to the top of the loop, checks the condition, and proceeds accordingly.

Thus, if `text` is the empty string, the loop exits. If the first character of `text` is a hash mark, the flow of execution goes to the top of the loop. Only if both conditions fail do we copy `text` into the new file.

Directories

Files on non-volatile storage media are organized by a set of rules known as a **file system**. File systems are made up of files and **directories**, which are containers for both files and other directories.

When you create a new file by opening it and writing, the new file goes in the current directory (wherever you were when you ran the program). Similarly, when you open a file for reading, Python looks for it in the current directory.

If you want to open a file somewhere else, you have to specify the **path** to the file, which is the name of the directory (or folder) where the file is located:

```
>>> wordsfile = open('/usr/share/dict/words', 'r')
>>> wordlist = wordsfile.readlines()
>>> print(wordlist[:6])
['\n', 'A\n', "A's\n", 'AOL\n', "AOL's\n", 'Aachen\n']
```

This example opens a file named `words` that resides in a directory named `dict`, which resides in `share`, which resides in `usr`, which resides in the top-level directory of the system, called `/`. It then reads in each line into a list using `readlines`, and prints out the first 5 elements from that list.

You cannot use `/` as part of a filename; it is reserved as a **delimiter** between directory and filenames.

The file `/usr/share/dict/words` should exist on unix based systems, and contains a list of words in alphabetical order.

Counting Letters

The `ord` function returns the integer representation of a character:

```
>>> ord('a')
97
>>> ord('A')
```

```
65
>>>
```

This example explains why `'Apple' < 'apple'` evaluates to `True`.

The `chr` function is the inverse of `ord`. It takes an integer as an argument and returns its character representation:

```
>>> for i in range(65, 71):
...     print(chr(i))
...
A
B
C
D
E
F
>>>
```

The following program, `countletters.py` counts the number of times each character occurs in the book Alice in Wonderland:

```
#
# countletters.py
#

def display(i):
    if i == 10: return 'LF'
    if i == 13: return 'CR'
    if i == 32: return 'SPACE'
    return chr(i)

infile = open('alice_in_wonderland.txt', 'r')
text = infile.read()
infile.close()

counts = 128 * [0]

for letter in text:
    counts[ord(letter)] += 1

outfile = open('alice_counts.dat', 'w')
outfile.write("%-12s%s\n" % ("Character", "Count"))
outfile.write("================\n")

for i in range(len(counts)):
    if counts[i]:
        outfile.write("%-12s%d\n" % (display(i), counts[i]))
```

```
outfile.close()
```

Run this program and look at the output file it generates using a text editor. You will be asked to analyze the program in the exercises below.

The sys module and argv

The sys module contains functions and variables which provide access to the *environment* in which the python interpreter runs.

The following example shows the values of a few of these variables on one of our systems:

```
>>> import sys
>>> sys.platform
'linux2'
>>> sys.path
['', '/home/jelkner/lib/python', '/usr/lib/python25.zip', '/usr/lib/python2.5',
'/usr/lib/python2.5/plat-linux2', '/usr/lib/python2.5/lib-tk',
'/usr/lib/python2.5/lib-dynload', '/usr/local/lib/python2.5/site-packages',
'/usr/lib/python2.5/site-packages', '/usr/lib/python2.5/site-packages/Numeric',
'/usr/lib/python2.5/site-packages/gst-0.10',
'/var/lib/python-support/python2.5', '/usr/lib/python2.5/site-packages/gtk-2.0',
'/var/lib/python-support/python2.5/gtk-2.0']
>>> sys.version
'2.5.1 (r251:54863, Mar  7 2008, 04:10:12) \n[GCC 4.1.3 20070929 (prerelease)
(Ubuntu 4.1.2-16ubuntu2)]'
>>>
```

Starting **Jython** on the same machine produces different values for the same variables:

```
>>> import sys
>>> sys.platform
'java1.6.0_03'
>>> sys.path
['', '/home/jelkner/.', '/usr/share/jython/Lib', '/usr/share/jython/Lib-cpython']
>>> sys.version
'2.1'
>>>
```

The results will be different on your machine of course.

The argv variable holds a list of strings read in from the **command line** when a Python script is run. These **command line arguments** can be used to pass information into a program at the same time it is invoked.

```
#
# demo_argv.py
```

```
#
import sys

print(sys.argv)
```

Running this program from the unix command prompt demonstrates how `sys.argv` works:

```
$ python demo_argv.py this and that 1 2 3
['demo_argv.py', 'this', 'and', 'that', '1', '2', '3']
$
```

`argv` is a list of strings. Notice that the first element is the name of the program. Arguments are separated by white space, and separated into a list in the same way that `string.split` operates. If you want an argument with white space in it, use quotes:

```
$ python demo_argv.py "this and" that "1 2" 3
['demo_argv.py', 'this and', 'that', '1 2', '3']
$
```

With `argv` we can write useful programs that take their input directly from the command line. For example, here is a program that finds the sum of a series of numbers:

```
#
# sum.py
#
from sys import argv

nums = argv[1:]

for index, value in enumerate(nums):
    nums[index] = float(value)

print(sum(nums))
```

In this program we use the `from <module> import <attribute>` style of importing, so `argv` is brought into the module's main namespace.

We can now run the program from the command prompt like this:

```
$ python sum.py 3 4 5 11
23.0
$ python sum.py 3.5 5 11 100
119.5
```

You are asked to write similar programs as exercises.

Glossary

argv `argv` is short for *argument vector* and is a variable in the `sys` module which stores a list of command line arguments passed to a program at run time.

attribute A variable defined inside a module (or class or instance – as we will see later). Module attributes are accessed by using the **dot operator** (`.`).

command line The sequence of characters read into the *command interpreter* in a *command line interface* (see the Wikipedia article on command line interface for more information).

command line argument A value passed to a program along with the program's invocation at the *command prompt* of a command line interface (CLI).

command prompt A string displayed by a command line interface indicating that commands can be entered.

continue statement A statement that causes the current iteration of a loop to end. The flow of execution goes to the top of the loop, evaluates the condition, and proceeds accordingly.

delimiter A sequence of one or more characters used to specify the boundary between separate parts of text.

directory A named collection of files, also called a folder. Directories can contain files and other directories, which are refered to as *subdirectories* of the directory that contains them.

dot operator The dot operator (`.`) permits access to attributes and functions of a module (or attributes and methods of a class or instance – as we will see later).

file A named entity, usually stored on a hard drive, floppy disk, or CD-ROM, that contains a stream of characters.

file system A method for naming, accessing, and organizing files and the data they contain.

import statement A statement which makes the objects contained in a module available for use within another module. There are two forms for the import statement. Using a hypothetical module named `mymod` containing functions `f1` and `f2`, and variables `v1` and `v2`, examples of these two forms include:

```
import mymod
```

and

```
from mymod import f1, f2, v1, v2
```

The second form brings the imported objects into the namespace of the importing module, while the first form preserves a seperate namespace for the imported module, requiring `mymod.v1` to access the `v1` variable.

Jython An implementation of the Python programming language written in Java. (see the Jython home page at http://www.jython.org for more information.)

method Function-like attribute of an object. Methods are *invoked* (called) on an object using the dot operator. For example:

```
>>> s = "this is a string."
>>> s.upper()
'THIS IS A STRING.'
>>>
```

We say that the method, `upper` is invoked on the string, `s`. `s` is implicitly the first argument to `upper`.

mode A distinct method of operation within a computer program. Files in Python can be opened in one of three modes: read (`'r'`), write (`'w'`), and append (`'a'`).

module A file containing Python definitions and statements intended for use in other Python programs. The contents of a module are made available to the other program by using the `import` statement.

namespace A syntactic container providing a context for names so that the same name can reside in different namespaces without ambiguity. In Python, modules, classes, functions and methods all form namespaces.

naming collision A situation in which two or more names in a given namespace cannot be unambiguously resolved. Using

```
import string
```

instead of

```
from string import *
```

prevents naming collisions.

non-volatile memory Memory that can maintain its state without power. Hard drives, flash drives, and rewritable compact disks (CD-RW) are each examples of non-volatile memory.

path The name and location of a file within a file system. For example:

```
/usr/share/dict/words
```

indicates a file named `words` found in the `dict` subdirectory of the `share` subdirectory of the `usr` directory.

pydoc A documentation generator that comes with the Python standard library.

standard library A library is a collection of software used as tools in the development of other software. The standard library of a programming language is the set of such tools that are distributed with the core programming language. Python comes with an extensive standard library.

text file A file that contains printable characters organized into lines separated by newline characters.

volatile memory Memory which requires an electrical current to maintain state. The *main memory* or RAM of a computer is volatile. Information stored in RAM is lost when the computer is turned off.

Exercises

1. Complete the following:
   - Start the pydoc server with the command `pydoc -g` at the command prompt.
   - Click on the open browser button in the pydoc tk window.
   - Find the `calendar` module and click on it.
   - While looking at the *Functions* section, try out the following in a Python shell:

```
>>> import calendar
>>> year = calendar.calendar(2008)
>>> print(year)                          # What happens here?
```

   - Experiment with `calendar.isleap`. What does it expect as an argument? What does it return as a result? What kind of a function is this?

   Make detailed notes about what you learned from this exercise.

2. If you don't have `Tkinter` installed on your computer, then `pydoc -g` will return an error, since the graphics window that it opens requires `Tkinter`. An alternative is to start the web server directly:

```
$ pydoc -p 7464
```

   This starts the pydoc web server on port 7464. Now point your web browser at:

```
http://localhost:7464
```

   and you will be able to browse the Python libraries installed on your system. Use this approach to start `pydoc` and take a look at the `math` module.

   (a) How many functions are there in the `math` module?

   (b) What does `math.ceil` do? What about `math.floor`? (*hint:* both `floor` and `ceil` expect floating point arguments.)

   (c) Describe how we have been computing the same value as `math.sqrt` without using the `math` module.

   (d) What are the two data constants in the `math` module?

   Record detailed notes of your investigation in this exercise.

3. Use `pydoc` to investigate the `copy` module. What does `deepcopy` do? In which exercises from last chapter would `deepcopy` have come in handy?

4. Create a module named `mymodule1.py`. Add attributes `myage` set to your current age, and `year` set to the current year. Create another module named `mymodule2.py`. Add attributes `myage` set to 0, and `year` set to the year you were born. Now create a file named `namespace_test.py`. Import both of the modules above and write the following statement:

```
print((mymodule2.myage - mymodule1.myage) == (mymodule2.year - mymodule1.year))
```

   When you will run `namespace_test.py` you will see either `True` or `False` as output depending on whether or not you've already had your birthday this year.

5. Add the following statement to `mymodule1.py`, `mymodule2.py`, and `namespace_test.py` from the previous exercise:

```
print("My name is %s" % (__name__))
```

   Run `namespace_test.py`. What happens? Why? Now add the following to the bottom of `mymodule1.py`:

```
if __name__ == '__main__':
    print("This won't run if I'm imported.")
```

   Run `mymodule1.py` and `namespace_test.py` again. In which case do you see the new print statement?

6. In a Python shell try the following:

```
>>> import this
```

   What does Tim Peters have to say about namespaces?

7. Use `pydoc` to find and test three other functions from the `string` module. Record your findings.

8. Rewrite `matrix_mult` from the last chapter using what you have learned about list methods.

9. The `dir` function, which we first saw in Chapter 7, prints out a list of the *attributes* of an object passed to it as an argument. In other words, `dir` returns the contents of the *namespace* of its argument. Use `dir(str)` and `dir(list)` to find at least three string and list methods which have not been introduced in the examples in the chapter. You should ignore anything that begins with double underscore (__) for the time being. Be sure to make detailed notes of your findings, including names of the new methods and examples of their use. (*hint:* Print the docstring of a function you want to explore. For example, to find out how `str.join` works, `print(str.join.__doc__)`)

10. Give the Python interpreter's response to each of the following from a continuous interpreter session:

```
>>> s = "If we took the bones out, it wouldn't be crunchy, would it?"
>>> s.split()
```

```
>>> type(s.split())
```

```
>>> s.split('o')
```

```
>>> s.split('i')
```

```
>>> '0'.join(s.split('o'))
```

Be sure you understand why you get each result. Then apply what you have learned to fill in the body of the function below using the `split` and `join` methods of `str` objects:

```
def myreplace(old, new, s):
    """
    Replace all occurences of old with new in the string s.

      >>> myreplace(',', ';', 'this, that, and, some, other, thing')
     'this; that; and; some; other; thing'
     >>> myreplace(' ', '**', 'Words will now be separated by stars.')
     'Words**will**now**be**separated**by**stars.'
    """
```

Your solution should pass all doctests.

11. Create a module named `wordtools.py` with the following at the bottom:

```
if __name__ == '__main__':
    import doctest
    doctest.testmod()
```

Explain how this statement makes both using and testing this module convenient. What will be the value of `__name__` when `wordtools.py` is imported from another module? What will it be when it is run as a main program? In which case will the doctests run? Now add bodies to each of the following functions to make the doctests pass:

```
def cleanword(word):
    """
      >>> cleanword('what?')
      'what'
      >>> cleanword('"now!"')
      'now'
      >>> cleanword('?+="word!,@$()"')
      'word'
    """
```

```
    def has_dashdash(s):
        """
          >>> has_dashdash('distance--but')
          True
          >>> has_dashdash('several')
          False
          >>> has_dashdash('critters')
          False
          >>> has_dashdash('spoke--fancy')
          True
          >>> has_dashdash('yo-yo')
          False
        """
```

```
    def extract_words(s):
        """
          >>> extract_words('Now is the time!  "Now", is the time? Yes, now.')
          ['now', 'is', 'the', 'time', 'now', 'is', 'the', 'time', 'yes', 'now']
          >>> extract_words('she tried to curtsey as she spoke--fancy')
          ['she', 'tried', 'to', 'curtsey', 'as', 'she', 'spoke', 'fancy']
        """
```

```
    def wordcount(word, wordlist):
        """
          >>> wordcount('now', ['now', 'is', 'time', 'is', 'now', 'is', 'is'])
          ['now', 2]
          >>> wordcount('is', ['now', 'is', 'time', 'is', 'now', 'is', 'the', 'is'
          ['is', 4]
          >>> wordcount('time', ['now', 'is', 'time', 'is', 'now', 'is', 'is'])
          ['time', 1]
          >>> wordcount('frog', ['now', 'is', 'time', 'is', 'now', 'is', 'is'])
          ['frog', 0]
        """
```

```
    def wordset(wordlist):
        """
          >>> wordset(['now', 'is', 'time', 'is', 'now', 'is', 'is'])
          ['is', 'now', 'time']
          >>> wordset(['I', 'a', 'a', 'is', 'a', 'is', 'I', 'am'])
          ['I', 'a', 'am', 'is']
          >>> wordset(['or', 'a', 'am', 'is', 'are', 'be', 'but', 'am'])
          ['a', 'am', 'are', 'be', 'but', 'is', 'or']
        """
```

```
    def longestword(wordset):
        """
          >>> longestword(['a', 'apple', 'pear', 'grape'])
          5
```

```
    >>> longestword(['a', 'am', 'I', 'be'])
    2
    >>> longestword(['this', 'that', 'supercalifragilisticexpialidocious'])
    34
    """
```

Save this module so you can use the tools it contains in your programs.

12. unsorted_fruits.txt contains a list of 26 fruits, each one with a name that begins with a different letter of the alphabet. Write a program named `sort_fruits.py` that reads in the fruits from `unsorted_fruits.txt` and writes them out in alphabetical order to a file named `sorted_fruits.txt`.

13. Answer the following questions about `countletters.py`:

    - Explain in detail what the three lines do:

```
infile = open('alice_in_wonderland.txt', 'r')
text = infile.read()
infile.close()
```

      What would `type(text)` return after these lines have been executed?

    - What does the expression `128 * [0]` evaluate to? Read about ASCII in Wikipedia and explain why you think the variable, `counts` is assigned to `128 * [0]` in light of what you read.

    - What does

```
for letter in text:
    counts[ord(letter)] += 1
```

      do to `counts`?

    - Explain the purpose of the `display` function. Why does it check for values `10`, `13`, and `32`? What is special about those values?

    - Describe in detail what the lines

```
outfile = open('alice_counts.dat', 'w')
outfile.write("%-12s%s\n" % ("Character", "Count"))
outfile.write("================\n")
```

      do. What will be in `alice_counts.dat` when they finish executing?

    - Finally, explain in detail what

```
for i in range(len(counts)):
    if counts[i]:
        outfile.write("%-12s%d\n" % (display(i), counts[i]))
```

      does. What is the purpose of `if counts[i]`?

14. Write a program named `mean.py` that takes a sequence of numbers on the command line and returns the mean of their values.

```
$ python mean.py 3 4
3.5
$ python mean.py 3 4 5
4.0
$ python mean.py 11 15 94.5 22
35.625
```

A session of your program running on the same input should produce the same output as the sample session above.

15. Write a program named `median.py` that takes a sequence of numbers on the command line and returns the median of their values.

```
$ python median.py 3 7 11
7
$ python median.py 19 85 121
85
$ python median.py 11 15 16 22
15.5
```

A session of your program running on the same input should produce the same output as the sample session above.

16. Modify the `countletters.py` program so that it takes the file to open as a command line argument. How will you handle the naming of the output file?

Recursion and exceptions

Tuples and mutability

So far, you have seen two compound types: strings, which are made up of characters; and lists, which are made up of elements of any type. One of the differences we noted is that the elements of a list can be modified, but the characters in a string cannot. In other words, strings are **immutable** and lists are **mutable**.

A **tuple**, like a list, is a sequence of items of any type. Unlike lists, however, tuples are immutable. Syntactically, a tuple is a comma-separated sequence of values:

```
>>> tup = 2, 4, 6, 8, 10
```

Although it is not necessary, it is conventional to enclose tuples in parentheses:

```
>>> tup = (2, 4, 6, 8, 10)
```

To create a tuple with a single element, we have to include the final comma:

```
>>> tup = (5,)
>>> type(tup)
<type 'tuple'>
```

Without the comma, Python treats `(5)` as an integer in parentheses:

```
>>> tup = (5)
>>> type(tup)
<type 'int'>
```

Syntax issues aside, tuples support the same sequence operations as strings and lists. The index operator selects an element from a tuple.

```
>>> tup = ('a', 'b', 'c', 'd', 'e')
>>> tup[0]
'a'
```

And the slice operator selects a range of elements.

```
>>> tup[1:3]
('b', 'c')
```

But if we try to use item assignment to modify one of the elements of the tuple, we get an error:

```
>>> tup[0] = 'X'
TypeError: 'tuple' object does not support item assignment
```

Of course, even if we can't modify the elements of a tuple, we can replace it with a different tuple:

```
>>> tup = ('X',) + tup[1:]
>>> tup
('X', 'b', 'c', 'd', 'e')
```

Alternatively, we could first convert it to a list, modify it, and convert it back into a tuple:

```
>>> tup = ('X', 'b', 'c', 'd', 'e')
>>> tup = list(tup)
>>> tup
['X', 'b', 'c', 'd', 'e']
>>> tup[0] = 'a'
>>> tup = tuple(tup)
>>> tup
('a', 'b', 'c', 'd', 'e')
```

Tuple assignment

Once in a while, it is useful to swap the values of two variables. With conventional assignment statements, we have to use a temporary variable. For example, to swap `a` and `b`:

```
temp = a
a = b
b = temp
```

If we have to do this often, this approach becomes cumbersome. Python provides a form of **tuple assignment** that solves this problem neatly:

```
a, b = b, a
```

The left side is a tuple of variables; the right side is a tuple of values. Each value is assigned to its respective variable. All the expressions on the right side are evaluated before any of the assignments. This feature makes tuple assignment quite versatile.

Naturally, the number of variables on the left and the number of values on the right have to be the same:

```
>>> a, b, c, d = 1, 2, 3
ValueError: need more than 3 values to unpack
```

Tuples as return values

Functions can return tuples as return values. For example, we could write a function that swaps two parameters:

```
def swap(x, y):
    return y, x
```

Then we can assign the return value to a tuple with two variables:

```
a, b = swap(a, b)
```

In this case, there is no great advantage in making `swap` a function. In fact, there is a danger in trying to encapsulate `swap`, which is the following tempting mistake:

```
def swap(x, y):       # incorrect version
    x, y = y, x
```

If we call this function like this:

```
swap(a, b)
```

then `a` and `x` are aliases for the same value. Changing `x` inside `swap` makes `x` refer to a different value, but it has no effect on `a` in `__main__`. Similarly, changing `y` has no effect on `b`.

This function runs without producing an error message, but it doesn't do what we intended. This is an example of a semantic error.

Pure functions and modifiers revisited

In *Pure functions and modifiers* we discussed *pure functions* and *modifiers* as related to lists. Since tuples are immutable we can not write modifiers on them.

Here is a modifier that inserts a new value into the middle of a list:

```
#
# seqtools.py
#

def insert_in_middle(val, lst):
    middle = len(lst)/2
    lst[middle:middle] = [val]
```

We can run it to see that it works:

```
>>> from seqtools import *
>>> my_list = ['a', 'b', 'd', 'e']
>>> insert_in_middle('c', my_list)
>>> my_list
['a', 'b', 'c', 'd', 'e']
```

If we try to use it with a tuple, however, we get an error:

```
>>> my_tuple = ('a', 'b', 'd', 'e')
>>> insert_in_middle('c', my_tuple)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "seqtools.py", line 7, in insert_in_middle
    lst[middle:middle] = [val]
TypeError: 'tuple' object does not support item assignment
>>>
```

The problem is that tuples are immutable, and don't support slice assignment. A simple solution to this problem is to make `insert_in_middle` a pure function:

```
def insert_in_middle(val, tup):
    middle = len(tup)/2
    return tup[:middle] + (val,) + tup[middle:]
```

This version now works for tuples, but not for lists or strings. If we want a version that works for all sequence types, we need a way to encapsulate our value into the correct sequence type. A small helper function does the trick:

```
def encapsulate(val, seq):
    if type(seq) == type(""):
        return str(val)
    if type(seq) == type([]):
        return [val]
    return (val,)
```

Now we can write `insert_in_middle` to work with each of the built-in sequence types:

```
def insert_in_middle(val, seq):
    middle = len(seq)/2
    return seq[:middle] + encapsulate(val, seq) + seq[middle:]
```

The last two versions of `insert_in_middle` are pure functions. They don't have any side effects. Adding `encapsulate` and the last version of `insert_in_middle` to the `seqtools.py` module, we can test it:

```
>>> from seqtools import *
>>> my_string = 'abde'
>>> my_list = ['a', 'b', 'd', 'e']
>>> my_tuple = ('a', 'b', 'd', 'e')
>>> insert_in_middle('c', my_string)
'abcde'
>>> insert_in_middle('c', my_list)
['a', 'b', 'c', 'd', 'e']
>>> insert_in_middle('c', my_tuple)
('a', 'b', 'c', 'd', 'e')
>>> my_string
'abde'
```

The values of `my_string`, `my_list`, and `my_tuple` are not changed. If we want to use `insert_in_middle` to change them, we have to assign the value returned by the function call back to the variable:

```
>>> my_string = insert_in_middle('c', my_string)
>>> my_string
'abcde'
```

Recursive data structures

All of the Python data types we have seen can be grouped inside lists and tuples in a variety of ways. Lists and tuples can also be nested, providing myriad possibilities for organizing data. The organization of data for the purpose of making it easier to use is called a **data structure**.

It's election time and we are helping to compute the votes as they come in. Votes arriving from individual wards, precincts, municipalities, counties, and states are sometimes reported as a sum

total of votes and sometimes as a list of subtotals of votes. After considering how best to store the tallies, we decide to use a *nested number list*, which we define as follows:

A *nested number list* is a list whose elements are either:

1. numbers
2. nested number lists

Notice that the term, nested number list is used in its own definition. **Recursive definitions** like this are quite common in mathematics and computer science. They provide a concise and powerful way to describe **recursive data structures** that are partially composed of smaller and simpler instances of themselves. The definition is not circular, since at some point we will reach a list that does not have any lists as elements.

Now suppose our job is to write a function that will sum all of the values in a nested number list. Python has a built-in function which finds the sum of a sequence of numbers:

```
>>> sum([1, 2, 8])
11
>>> sum((3, 5, 8.5))
16.5
>>>
```

For our *nested number list*, however, `sum` will not work:

```
>>> sum([1, 2, [11, 13], 8])
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: unsupported operand type(s) for +: 'int' and 'list'
>>>
```

The problem is that the third element of this list, `[11, 13]`, is itself a list, which can not be added to `1`, `2`, and `8`.

Recursion

To sum all the numbers in our recursive nested number list we need to traverse the list, visiting each of the elements within its nested structure, adding any numeric elements to our sum, and *repeating this process* with any elements which are lists.

Modern programming languages generally support **recursion**, which means that functions can *call themselves* within their definitions. Thanks to recursion, the Python code needed to sum the values of a nested number list is surprisingly short:

```
def recursive_sum(nested_num_list):
    sum = 0
    for element in nested_num_list:
        if type(element) == type([]):
```

```
            sum = sum + recursive_sum(element)
        else:
            sum = sum + element
    return sum
```

The body of `recursive_sum` consists mainly of a `for` loop that traverses `nested_num_list`. If `element` is a numerical value (the `else` branch), it is simply added to `sum`. If `element` is a list, then `recursive_sum` is called again, with the element as an argument. The statement inside the function definition in which the function calls itself is known as the **recursive call**.

Recursion is truly one of the most beautiful and elegant tools in computer science.

A slightly more complicated problem is finding the largest value in our nested number list:

```
def recursive_max(nested_num_list):
    """
      >>> recursive_max([2, 9, [1, 13], 8, 6])
      13
      >>> recursive_max([2, [[100, 7], 90], [1, 13], 8, 6])
      100
      >>> recursive_max([2, [[13, 7], 90], [1, 100], 8, 6])
      100
      >>> recursive_max([[[13, 7], 90], 2, [1, 100], 8, 6])
      100
    """
    largest = nested_num_list[0]
    while type(largest) == type([]):
        largest = largest[0]

    for element in nested_num_list:
        if type(element) == type([]):
            max_of_elem = recursive_max(element)
            if largest < max_of_elem:
                largest = max_of_elem
        else:                               # element is not a list
            if largest < element:
                largest = element

    return largest
```

Doctests are included to provide examples of `recursive_max` at work.

The added twist to this problem is finding a numerical value for initializing `largest`. We can't just use `nested_num_list[0]`, since that my be either a number or a list. To solve this problem we use a while loop that assigns `largest` to the first numerical value no matter how deeply it is nested.

The two examples above each have a **base case** which does not lead to a recursive call: the case

where the element is a number and not a list. Without a base case, you have **infinite recursion**, and your program will not work. Python stops after reaching a maximum recursion depth and returns a runtime error.

Write the following in a file named `infinite_recursion.py`:

```
#
# infinite_recursion.py
#
def recursion_depth(number):
    print("Recursion depth number %d." % (number))
    recursion_depth(number + 1)

recursion_depth(0)
```

At the unix command prompt in the same directory in which you saved your program, type the following:

```
python infinite_recursion.py
```

After watching the messages flash by, you will be presented with the end of a long traceback that ends in with the following:

```
  ...
  File "infinite_recursion.py", line 3, in recursion_depth
    recursion_depth(number + 1)
RuntimeError: maximum recursion depth exceeded
```

We would certainly never want something like this to happen to a user of one of our programs, so before finishing the recursion discussion, let's see how errors like this are handled in Python.

Exceptions

Whenever a runtime error occurs, it creates an **exception**. The program stops running at this point and Python prints out the traceback, which ends with the exception that occured.

For example, dividing by zero creates an exception:

```
>>> print(55/0)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
ZeroDivisionError: integer division or modulo by zero
>>>
```

So does accessing a nonexistent list item:

```
>>> a = []
>>> print(a[5])
Traceback (most recent call last):
```

```
  File "<stdin>", line 1, in <module>
IndexError: list index out of range
>>>
```

Or trying to make an item assignment on a string:

```
>>> S = "Swarthnore"
>>> S[6] = 'm'
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: 'str' object does not support item assignment
>>>
```

Or trying to convert something to an integer that can't be converted:

```
>>> age = int(raw_input("How old are you? "))
How old are you? pony
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
ValueError: invalid literal for int() with base 10: 'pony'
>>>
```

In each case, the error message on the last line has two parts: the type of error before the colon, and specifics about the error after the colon.

Sometimes we want to execute an operation that might cause an exception, but we don't want the program to stop. We can **handle the exception** using the `try` and `except` statements.

For the age input above, we want to *try* to convert the age to an integer, and handle the exception if we can't convert it:

```
age = raw_input("How old are you? ")
try:
  age = int(age)
except ValueError:
  print("Please enter a number...")
```

As another example, we might prompt the user for the name of a file and then try to open it. If the file doesn't exist, we don't want the program to crash; we want to handle the exception:

```
filename = raw_input('Enter a file name: ')
try:
    f = open (filename, "r")
except:
    print('There is no file named % s' % (filename))
```

The `try` statement executes the statements in the first block. If no exceptions occur, it ignores the `except` statement. If any exception occurs, it executes the statements in the `except` branch and then continues.

We can encapsulate this capability in a function: `exists` takes a filename and returns true if the file exists, false if it doesn't:

```
def exists(filename):
    try:
        f = open(filename)
        f.close()
        return True
    except:
        return False
```

You can use multiple `except` blocks to handle different kinds of exceptions (see the Errors and Exceptions lesson from Python creator Guido van Rossum's Python Tutorial for a more complete discussion of exceptions).

If your program detects an error condition, you can make it **raise** an exception. Here is an example that gets input from the user and checks that the number is non-negative.

```
#
# learn_exceptions.py
#
def get_age():
    age = int(raw_input('Please enter your age: '))
    if age < 0:
        raise ValueError, '%s is not a valid age' % age
    return age
```

The `raise` statement takes two arguments: the exception type, and specific information about the error. `ValueError` is the built-in exception which most closely matches the kind of error we want to raise. The complete listing of built-in exceptions is found in the Built-in Exceptions section of the Python Library Reference, again by Python's creator, Guido van Rossum.

If the function that called `get_age` handles the error, then the program can continue; otherwise, Python prints the traceback and exits:

```
>>> get_age()
Please enter your age: 42
42
>>> get_age()
Please enter your age: -2
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "learn_exceptions.py", line 4, in get_age
    raise ValueError, '%s is not a valid age' % age
ValueError: -2 is not a valid age
>>>
```

The error message includes the exception type and the additional information you provided.

Using exception handling, we can now modify `infinite_recursion.py` so that it stops

when it reaches the maximum recursion depth allowed:

```
#
# infinite_recursion.py
#
def recursion_depth(number):
    print("Recursion depth number %d." % (number))
    try:
        recursion_depth(number + 1)
    except:
        print("Maximum recursion depth exceeded.")

recursion_depth(0)
```

Run this version and observe the results.

Tail recursion

When the only thing returned from a function is a recursive call, it is refered to as **tail recursion**.

Here is a version of the `countdown` function from chapter 6 written using tail recursion:

```
def countdown(n):
    if n == 0:
        print("Blastoff!")
    else:
        print(n)
        countdown(n-1)
```

Any computation that can be made using iteration can also be made using recursion. Here is a version of `find_max` written using tail recursion:

```
def find_max(seq, max_so_far):
    if not seq:
        return max_so_far
    if max_so_far < seq[0]:
        return find_max(seq[1:], seq[0])
    else:
        return find_max(seq[1:], max_so_far)
```

Tail recursion is considered a bad practice in Python, since the Python compiler does not handle optimization for tail recursive calls. The recursive solution in cases like this use more system resources than the equivalent iterative solution.

Recursive mathematical functions

Several well known mathematical functions are defined recursively. Factorial, for example, is given the special operator, `!`, and is defined by:

```
0! = 1
n! = n(n-1)
```

We can easily code this into Python:

```
def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)
```

Another well know recursive relation in mathematics is the fibonacci sequence, which is defined by:

```
fibonacci(0) = 1
fibonacci(1) = 1
fibonacci(n) = fibonacci(n-1) + fibonacci(n-2)
```

This can also be written easily in Python:

```
def fibonacci (n):
    if n == 0 or n == 1:
        return 1
    else:
        return fibonacci(n-1) + fibonacci(n-2)
```

Calling `factorial(1000)` will exceed the maximum recursion depth. And try running `fibonacci(35)` and see how long it takes to complete (be patient, it will complete).

You will be asked to write an iterative version of `factorial` as an exercise, and we will see a better way to handle `fibonacci` in the next chapter.

List comprehensions

A **list comprehension** is a syntactic construct that enables lists to be created from other lists using a compact, mathematical syntax:

```
>>> numbers = [1, 2, 3, 4]
>>> [x**2 for x in numbers]
[1, 4, 9, 16]
>>> [x**2 for x in numbers if x**2 > 8]
[9, 16]
>>> [(x, x**2, x**3) for x in numbers]
```

```
[(1, 1, 1), (2, 4, 8), (3, 9, 27), (4, 16, 64)]
>>> files = ['bin', 'Data', 'Desktop', '.bashrc', '.ssh', '.vimrc']
>>> [name for name in files if name[0] != '.']
['bin', 'Data', 'Desktop']
>>> letters = ['a', 'b', 'c']
>>> [n*letter for n in numbers for letter in letters]
['a', 'b', 'c', 'aa', 'bb', 'cc', 'aaa', 'bbb', 'ccc', 'aaaa', 'bbbb', 'cccc']
>>>
```

The general syntax for a list comprehension expression is:

```
[expr for  item1 in  seq1 for item2 in seq2 ... for itemx in seqx if condition]
```

This list expression has the same effect as:

```
output_sequence = []
for item1 in seq1:
    for item2 in seq2:
        ...
            for itemx in seqx:
                if condition:
                    output_sequence.append(expr)
```

As you can see, the list comprehension is much more compact.

Mini case study: tree

The following program implements a subset of the behavior of the Unix tree program.

```
#!/usr/bin/env python

import os
import sys


def getroot():
    if len(sys.argv) == 1:
        path = ''
    else:
        path = sys.argv[1]

    if os.path.isabs(path):
        tree_root = path
    else:
        tree_root = os.path.join(os.getcwd(), path)

    return tree_root
```

```
def getdirlist(path):
    dirlist = os.listdir(path)
    dirlist = [name for name in dirlist if name[0] != '.']
    dirlist.sort()
    return dirlist


def traverse(path, prefix='|--', s='.\n', f=0, d=0):
    dirlist = getdirlist(path)

    for num, file in enumerate(dirlist):
        lastprefix = prefix[:-3] + '`--'
        dirsize = len(dirlist)

        if num < dirsize - 1:
            s += '%s %s\n' % (prefix, file)
        else:
            s += '%s %s\n' % (lastprefix, file)
        path2file = os.path.join(path, file)

        if os.path.isdir(path2file):
            d += 1
            if getdirlist(path2file):
                s, f, d = traverse(path2file, '|    ' + prefix, s, f, d)
        else:
            f += 1

    return s, f, d


if __name__ == '__main__':
    root = getroot()
    tree_str, files, dirs = traverse(root)

    if dirs == 1:
        dirstring = 'directory'
    else:
        dirstring = 'directories'
    if files == 1:
        filestring = 'file'
    else:
        filestring = 'files'

    print(tree_str)
    print('%d %s, %d %s' % (dirs, dirstring, files, filestring))
```

You will be asked to explore this program in several of the exercises below.

Glossary

base case A branch of the conditional statement in a recursive function that does not result in a recursive call.

data structure An organization of data for the purpose of making it easier to use.

exception An error that occurs at runtime.

handle an exception To prevent an exception from terminating a program using the `try` and `except` statements.

immutable data type A data type which cannot be modified. Assignments to elements or slices of immutable types cause a runtime error.

infinite recursion A function that calls itself recursively without ever reaching the base case. Eventually, an infinite recursion causes a runtime error.

list comprehension A syntactic construct which enables lists to be generated from other lists using a syntax analogous to the mathematical set-builder notation.

mutable data type A data type which can be modified. All mutable types are compound types. Lists and dictionaries (see next chapter) are mutable data types; strings and tuples are not.

raise To signal an exception using the `raise` statement.

recursion The process of calling the function that is currently executing.

recursive call The statement in a recursive function with is a call to itself.

recursive definition A definition which defines something in terms of itself. To be useful it must include *base cases* which are not recursive. In this way it differs from a *circular definition*. Recursive definitions often provide an elegant way to express complex data structures.

tail recursion A recursive call that occurs as the last statement (at the tail) of a function definition. Tail recursion is considered bad practice in Python programs since a logically equivalent function can be written using *iteration* which is more efficient (see the Wikipedia article on tail recursion for more information).

tuple A data type that contains a sequence of elements of any type, like a list, but is immutable. Tuples can be used wherever an immutable type is required, such as a key in a dictionary (see next chapter).

tuple assignment An assignment to all of the elements in a tuple using a single assignment statement. Tuple assignment occurs in parallel rather than in sequence, making it useful for swapping values.

Exercises

1.
```
def swap(x, y):       # incorrect version
    print("before swap statement: id(x): %s id(y): %s" % (id(x), id(y)))
    x, y = y, x
    print("after swap statement: id(x): %s id(y): %s" % (id(x), id(y)))

a, b = 0, 1
print("before swap function call: id(a): %s id(b): %s" % (id(a), id(b)))
swap(a, b)
print("after swap function call: id(a): %s id(b): %s" % (id(a), id(b)))
```

   Run this program and describe the results. Use the results to explain why this version of `swap` does not work as intended. What will be the values of `a` and `b` after the call to `swap`?

2. Create a module named `seqtools.py`. Add the functions `encapsulate` and `insert_in_middle` from the chapter. Add doctests which test that these two functions work as intended with all three sequence types.

3. Add each of the following functions to `seqtools.py`:

```
def make_empty(seq):
    """
      >>> make_empty([1, 2, 3, 4])
      []
      >>> make_empty(('a', 'b', 'c'))
      ()
      >>> make_empty("No, not me!")
      ''
    """

def insert_at_end(val, seq):
    """
      >>> insert_at_end(5, [1, 3, 4, 6])
      [1, 3, 4, 6, 5]
      >>> insert_at_end('x', 'abc')
      'abcx'
      >>> insert_at_end(5, (1, 3, 4, 6))
      (1, 3, 4, 6, 5)
    """

def insert_in_front(val, seq):
    """
      >>> insert_in_front(5, [1, 3, 4, 6])
      [5, 1, 3, 4, 6]
      >>> insert_in_front(5, (1, 3, 4, 6))
      (5, 1, 3, 4, 6)
      >>> insert_in_front('x', 'abc')
```

```
        'xabc'
    """

def index_of(val, seq, start=0):
    """
      >>> index_of(9, [1, 7, 11, 9, 10])
      3
      >>> index_of(5, (1, 2, 4, 5, 6, 10, 5, 5))
      3
      >>> index_of(5, (1, 2, 4, 5, 6, 10, 5, 5), 4)
      6
      >>> index_of('y', 'happy birthday')
      4
      >>> index_of('banana', ['apple', 'banana', 'cherry', 'date'])
      1
      >>> index_of(5, [2, 3, 4])
      -1
      >>> index_of('b', ['apple', 'banana', 'cherry', 'date'])
      -1
    """

def remove_at(index, seq):
    """
      >>> remove_at(3, [1, 7, 11, 9, 10])
      [1, 7, 11, 10]
      >>> remove_at(5, (1, 4, 6, 7, 0, 9, 3, 5))
      (1, 4, 6, 7, 0, 3, 5)
      >>> remove_at(2, "Yomrktown")
      'Yorktown'
    """

def remove_val(val, seq):
    """
      >>> remove_val(11, [1, 7, 11, 9, 10])
      [1, 7, 9, 10]
      >>> remove_val(15, (1, 15, 11, 4, 9))
      (1, 11, 4, 9)
      >>> remove_val('what', ('who', 'what', 'when', 'where', 'why', 'how'))
      ('who', 'when', 'where', 'why', 'how')
    """

def remove_all(val, seq):
    """
      >>> remove_all(11, [1, 7, 11, 9, 11, 10, 2, 11])
      [1, 7, 9, 10, 2]
      >>> remove_all('i', 'Mississippi')
      'Mssssppp'
```

```
        """

    def count(val, seq):
        """
          >>> count(5, (1, 5, 3, 7, 5, 8, 5))
          3
          >>> count('s', 'Mississippi')
          4
          >>> count((1, 2), [1, 5, (1, 2), 7, (1, 2), 8, 5])
          2
        """

    def reverse(seq):
        """
          >>> reverse([1, 2, 3, 4, 5])
          [5, 4, 3, 2, 1]
          >>> reverse(('shoe', 'my', 'buckle', 2, 1))
          (1, 2, 'buckle', 'my', 'shoe')
          >>> reverse('Python')
          'nohtyP'
        """

    def sort_sequence(seq):
        """
          >>> sort_sequence([3, 4, 6, 7, 8, 2])
          [2, 3, 4, 6, 7, 8]
          >>> sort_sequence((3, 4, 6, 7, 8, 2))
          (2, 3, 4, 6, 7, 8)
          >>> sort_sequence("nothappy")
          'ahnoppty'
        """

    if __name__ == "__main__":
        import doctest
        doctest.testmod()
```

As usual, work on each of these one at a time until they pass all of the doctests.

4. Write a function, `recursive_min`, that returns the smallest value in a nested number list:

```
    def recursive_min(nested_num_list):
        """
          >>> recursive_min([2, 9, [1, 13], 8, 6])
          1
          >>> recursive_min([2, [[100, 1], 90], [10, 13], 8, 6])
          1
          >>> recursive_min([2, [[13, -7], 90], [1, 100], 8, 6])
          -7
```

```
        >>> recursive_min([[[-13, 7], 90], 2, [1, 100], 8, 6])
        -13
    """
```

Your function should pass the doctests.

5. Write a function `recursive_count` that returns the number of occurences of `target` in `nested_number_list`:

```
def recursive_count(target, nested_num_list):
    """
      >>> recursive_count(2, [2, 9, [2, 1, 13, 2], 8, [2, 6]])
      4
      >>> recursive_count(7, [[9, [7, 1, 13, 2], 8], [7, 6]])
      2
      >>> recursive_count(15, [[9, [7, 1, 13, 2], 8], [2, 6]])
      0
      >>> recursive_count(5, [[5, [5, [1, 5], 5], 5], [5, 6]])
      6
    """
```

As usual, your function should pass the doctests.

6. Write a function `flatten` that returns a simple list of numbers containing all the values in a `nested_number_list`:

```
def flatten(nested_num_list):
    """
      >>> flatten([2, 9, [2, 1, 13, 2], 8, [2, 6]])
      [2, 9, 2, 1, 13, 2, 8, 2, 6]
      >>> flatten([[9, [7, 1, 13, 2], 8], [7, 6]])
      [9, 7, 1, 13, 2, 8, 7, 6]
      >>> flatten([[9, [7, 1, 13, 2], 8], [2, 6]])
      [9, 7, 1, 13, 2, 8, 2, 6]
      >>> flatten([[5, [5, [1, 5], 5], 5], [5, 6]])
      [5, 5, 1, 5, 5, 5, 5, 6]
    """
```

Run your function to confirm that the doctests pass.

7. Write a function named `readposint` that prompts the user for a positive integer and then checks the input to confirm that it meets the requirements. A sample session might look like this:

```
>>> num = readposint()
Please enter a positive integer: yes
yes is not a positive integer.  Try again.
Please enter a positive integer: 3.14
3.14 is not a positive integer.  Try again.
Please enter a positive integer: -6
```

```
-6 is not a positive integer.  Try again.
Please enter a positive integer: 42
>>> num
42
>>> num2 = readposint("Now enter another one: ")
Now enter another one: 31
>>> num2
31
>>>
```

Use Python's exception handling mechanisms in confirming that the user's input is valid.

8. Give the Python interpreter's response to each of the following:

(a)
```
>>> nums = [1, 2, 3, 4]
>>> [x**3 for x in nums]
```

(b)
```
>>> nums = [1, 2, 3, 4]
>>> [x**2 for x in nums if x**2 != 4]
```

(c)
```
>>> nums = [1, 2, 3, 4]
>>> [(x, y) for x in nums for y in nums]
```

(d)
```
>>> nums = [1, 2, 3, 4]
>>> [(x, y) for x in nums for y in nums if x != y]
```

You should anticipate the results *before* you try them in the interpreter.

9. Use either `pydoc` or the on-line documentation at http://pydoc.org to find out what `sys.getrecursionlimit()` and `sys.setrecursionlimit(n)` do. Create several *experiments* like what was done in `infinite_recursion.py` to test your understanding of how these module functions work.

10. Rewrite the `factorial` function using iteration instead of recursion. Call your new function with 1000 as an argument and make note of how fast it returns a value.

11. Write a program named `litter.py` that creates an empty file named `trash.txt` in each subdirectory of a directory tree given the root of the tree as an argument (or the current directory as a default). Now write a program named `cleanup.py` that removes all these files. *Hint:* Use the `tree` program from the mini case study as a basis for these two recursive programs.

Dictionaries

All of the compound data types we have studied in detail so far — strings, lists, and tuples—are sequence types, which use integers as indices to access the values they contain within them.

Dictionaries are a different kind of compound type. They are Python's built-in **mapping type**. They map **keys**, which can be any immutable type, to values, which can be any type, just like the values of a list or tuple.

As an example, we will create a dictionary to translate English words into Spanish. For this dictionary, the keys are strings.

One way to create a dictionary is to start with the empty dictionary and add **key-value pairs**. The empty dictionary is denoted `{}`:

```
>>> eng2sp = {}
>>> eng2sp['one'] = 'uno'
>>> eng2sp['two'] = 'dos'
```

The first assignment creates a dictionary named `eng2sp`; the other assignments add new key-value pairs to the dictionary. We can print the current value of the dictionary in the usual way:

```
>>> print(eng2sp)
{'two': 'dos', 'one': 'uno'}
```

The key-value pairs of the dictionary are seperated by commas. Each pair contains a key and a value separated by a colon.

The order of the pairs may not be what you expected. Python uses complex algorithms to determine where the key-value pairs are stored in a dictionary. For our purposes we can think of this ordering as unpredicatable.

Another way to create a dictionary is to provide a list of key-value pairs using the same syntax as the previous output:

```
>>> eng2sp = {'one': 'uno', 'two': 'dos', 'three': 'tres'}
```

It doesn't matter what order we write the pairs. The values in a dictionary are accessed with keys, not with indices, so there is no need to care about ordering.

Here is how we use a key to look up the corresponding value:

```
>>> print(eng2sp['two'])
'dos'
```

The key `'two'` yields the value `'dos'`.

Dictionary operations

The `del` statement removes a key-value pair from a dictionary. For example, the following dictionary contains the names of various fruits and the number of each fruit in stock:

```
>>> inventory = {'apples': 430, 'bananas': 312, 'oranges': 525, 'pears': 217}
>>> print(inventory)
{'oranges': 525, 'apples': 430, 'pears': 217, 'bananas': 312}
```

If someone buys all of the pears, we can remove the entry from the dictionary:

```
>>> del inventory['pears']
>>> print(inventory)
{'oranges': 525, 'apples': 430, 'bananas': 312}
```

Or if we're expecting more pears soon, we might just change the value associated with pears:

```
>>> inventory['pears'] = 0
>>> print(inventory)
{'oranges': 525, 'apples': 430, 'pears': 0, 'bananas': 312}
```

The `len` function also works on dictionaries; it returns the number of key-value pairs:

```
>>> len(inventory)
4
```

Dictionary methods

Dictionaries have a number of useful built-in methods.

The `keys` method takes a dictionary and returns a list of its keys.

```
>>> eng2sp.keys()
['three', 'two', 'one']
```

As we saw earlier with strings and lists, dictionary methods use dot notation, which specifies the name of the method to the right of the dot and the name of the object on which to apply the method immediately to the left of the dot. The parentheses indicate that this method takes no parameters.

A method call is called an **invocation**; in this case, we would say that we are invoking the `keys` method on the object `eng2sp`. As we will see in a few chapters when we talk about object oriented programming, the object on which a method is invoked is actually the first argument to the method.

The `values` method is similar; it returns a list of the values in the dictionary:

```
>>> eng2sp.values()
['tres', 'dos', 'uno']
```

The `items` method returns both, in the form of a list of tuples — one for each key-value pair:

```
>>> eng2sp.items()
[('three', 'tres'), ('two', 'dos'), ('one', 'uno')]
```

The `has_key` method takes a key as an argument and returns `True` if the key appears in the dictionary and `False` otherwise:

```
>>> eng2sp.has_key('one')
True
```

```
>>> eng2sp.has_key('deux')
False
```

This method can be very useful, since looking up a non-existant key in a dictionary causes a runtime error:

```
>>> eng2esp['dog']
Traceback (most recent call last):
  File "", line 1, in
KeyError: 'dog'
>>>
```

Aliasing and copying

Because dictionaries are mutable, you need to be aware of aliasing. Whenever two variables refer to the same object, changes to one affect the other.

If you want to modify a dictionary and keep a copy of the original, use the `copy` method. For example, `opposites` is a dictionary that contains pairs of opposites:

```
>>> opposites = {'up': 'down', 'right': 'wrong', 'true': 'false'}
>>> alias = opposites
>>> copy = opposites.copy()
```

`alias` and `opposites` refer to the same object; `copy` refers to a fresh copy of the same dictionary. If we modify `alias`, `opposites` is also changed:

```
>>> alias['right'] = 'left'
>>> opposites['right']
'left'
```

If we modify `copy`, `opposites` is unchanged:

```
>>> copy['right'] = 'privilege'
>>> opposites['right']
'left'
```

Sparse matrices

We previously used a list of lists to represent a matrix. That is a good choice for a matrix with mostly nonzero values, but consider a sparse matrix like this one:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \end{bmatrix}$$

The list representation contains a lot of zeroes:

```
matrix = [[0, 0, 0, 1, 0],
          [0, 0, 0, 0, 0],
          [0, 2, 0, 0, 0],
          [0, 0, 0, 0, 0],
          [0, 0, 0, 3, 0]]
```

An alternative is to use a dictionary. For the keys, we can use tuples that contain the row and column numbers. Here is the dictionary representation of the same matrix:

```
matrix = {(0, 3): 1, (2, 1): 2, (4, 3): 3}
```

We only need three key-value pairs, one for each nonzero element of the matrix. Each key is a tuple, and each value is an integer.

To access an element of the matrix, we could use the `[]` operator:

```
matrix[(0, 3)]
1
```

Notice that the syntax for the dictionary representation is not the same as the syntax for the nested list representation. Instead of two integer indices, we use one index, which is a tuple of integers.

There is one problem. If we specify an element that is zero, we get an error, because there is no entry in the dictionary with that key:

```
>>> matrix[(1, 3)]
KeyError: (1, 3)
```

The `get` method solves this problem:

```
>>> matrix.get((0, 3), 0)
1
```

The first argument is the key; the second argument is the value `get` should return if the key is not in the dictionary:

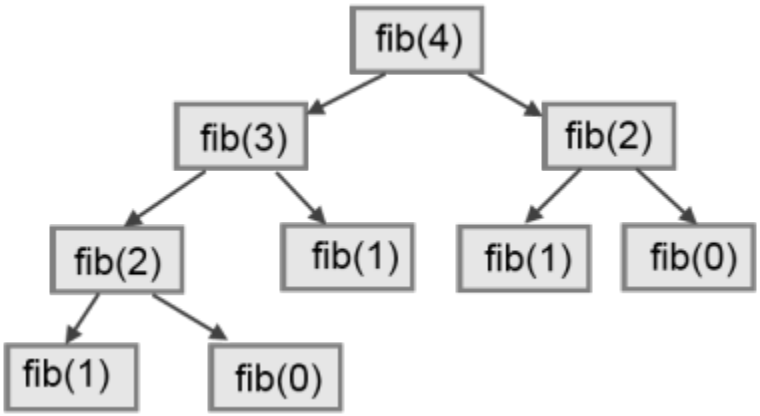
```
>>> matrix.get((1, 3), 0)
0
```

`get` definitely improves the semantics of accessing a sparse matrix. Shame about the syntax.

Hints

If you played around with the `fibonacci` function from the last chapter, you might have noticed that the bigger the argument you provide, the longer the function takes to run. Furthermore, the run time increases very quickly. On one of our machines, `fibonacci(20)` finishes instantly, `fibonacci(30)` takes about a second, and `fibonacci(40)` takes roughly forever.

To understand why, consider this **call graph** for `fibonacci` with `n = 4`:



A call graph shows a set function frames, with lines connecting each frame to the frames of the functions it calls. At the top of the graph, `fibonacci` with `n = 4` calls `fibonacci` with `n = 3` and `n = 2`. In turn, `fibonacci` with `n = 3` calls `fibonacci` with `n = 2` and `n = 1`. And so on.

Count how many times `fibonacci(0)` and `fibonacci(1)` are called. This is an inefficient solution to the problem, and it gets far worse as the argument gets bigger.

A good solution is to keep track of values that have already been computed by storing them in a dictionary. A previously computed value that is stored for later use is called a **hint**. Here is an implementation of `fibonacci` using hints:

```
previous = {0: 0, 1: 1}

def fibonacci(n):
    if previous.has_key(n):
        return previous[n]
    else:
        new_value = fibonacci(n-1) + fibonacci(n-2)
        previous[n] = new_value
        return new_value
```

The dictionary named `previous` keeps track of the Fibonacci numbers we already know. We start with only two pairs: 0 maps to 1; and 1 maps to 1.

Whenever `fibonacci` is called, it checks the dictionary to determine if it contains the result. If it's there, the function can return immediately without making any more recursive calls. If not, it has to compute the new value. The new value is added to the dictionary before the function returns.

Using this version of `fibonacci`, our machines can compute `fibonacci(100)` in an eye-blink.

```
>>> fibonacci(100)
354224848179261915075L
```

The `L` at the end of the number indicates that it is a `long` integer.

Long integers

Python provides a type called `long` that can handle any size integer (limited only by the amount of memory you have on your computer).

There are three ways to create a `long` value. The first one is to compute an arithmetic expression too large to fit inside an `int`. We already saw this in the `fibonacci(100)` example above. Another way is to write an integer with a capital `L` at the end of your number:

```
>>> type(1L)
```

The third is to call `long` with the value to be converted as an argument. `long`, just like `int` and `float`, can convert `int`s, `float`s, and even strings of digits to long integers:

```
>>> long(7)
7L
>>> long(3.9)
3L
>>> long('59')
59L
```

Counting letters

In Chapter 7, we wrote a function that counted the number of occurrences of a letter in a string. A more general version of this problem is to form a histogram of the letters in the string, that is, how many times each letter appears.

Such a histogram might be useful for compressing a text file. Because different letters appear with different frequencies, we can compress a file by using shorter codes for common letters and longer codes for letters that appear less frequently.

Dictionaries provide an elegant way to generate a histogram:

```
>>> letter_counts = {}
>>> for letter in "Mississippi":
...   letter_counts[letter] = letter_counts.get (letter, 0) + 1
...
>>> letter_counts
{'M': 1, 's': 4, 'p': 2, 'i': 4}
```

We start with an empty dictionary. For each letter in the string, we find the current count (possibly zero) and increment it. At the end, the dictionary contains pairs of letters and their frequencies.

It might be more appealing to display the histogram in alphabetical order. We can do that with the `items` and `sort` methods:

```
>>> letter_items = letter_counts.items()
>>> letter_items.sort()
>>> print(letter_items)
[('M', 1), ('i', 4), ('p', 2), ('s', 4)]
```

Case Study: Robots

The game

In this case study we will write a version of the classic console based game, robots.

Robots is a turn-based game in which the protagonist, you, are trying to stay alive while being chased by stupid, but relentless robots. Each robot moves one square toward you each time you move. If they catch you, you are dead, but if they collide they die, leaving a pile of dead robot junk in their wake. If other robots collide with the piles of junk, they die.

The basic strategy is to position yourself so that the robots collide with each other and with piles of junk as they move toward you. To make the game playable, you also are given the ability to teleport to another location on the screen – 3 times safely and randomly thereafter, so that you don't just get forced into a corner and loose every time.

Setting up the world, the player, and the main loop

Let's start with a program that places the player on the screen and has a function to move her around in response to keys pressed:

```
#
# robots.py
#
from gasp import *

SCREEN_WIDTH = 640
SCREEN_HEIGHT = 480
GRID_WIDTH = SCREEN_WIDTH/10 - 1
GRID_HEIGHT = SCREEN_HEIGHT/10 - 1


def place_player():
    x = random.randint(0, GRID_WIDTH)
    y = random.randint(0, GRID_HEIGHT)
    return {'shape': Circle((10*x+5, 10*y+5), 5, filled=True), 'x': x, 'y': y}


def move_player(player):
    update_when('key_pressed')
```

```
    if key_pressed('escape'):
        return True
    elif key_pressed('4'):
        if player['x'] > 0: player['x'] -= 1
    elif key_pressed('7'):
        if player['x'] > 0: player['x'] -= 1
        if player['y'] < GRID_HEIGHT: player['y'] += 1
    elif key_pressed('8'):
        if player['y'] < GRID_HEIGHT: player['y'] += 1
    elif key_pressed('9'):
        if player['x'] < GRID_WIDTH: player['x'] += 1
        if player['y'] < GRID_HEIGHT: player['y'] += 1
    elif key_pressed('6'):
        if player['x'] < GRID_WIDTH: player['x'] += 1
    elif key_pressed('3'):
        if player['x'] < GRID_WIDTH: player['x'] += 1
        if player['y'] > 0: player['y'] -= 1
    elif key_pressed('2'):
        if player['y'] > 0: player['y'] -= 1
    elif key_pressed('1'):
        if player['x'] > 0: player['x'] -= 1
        if player['y'] > 0: player['y'] -= 1
    else:
        return False

    move_to(player['shape'], (10*player['x']+5, 10*player['y']+5))

    return False


def play_game():
    begin_graphics(SCREEN_WIDTH, SCREEN_HEIGHT, title="Robots")
    player = place_player()
    finished = False
    while not finished:
        finished = move_player(player)
    end_graphics()


if __name__ == '__main__':
    play_game()
```

Programs like this one that involve interacting with the user through **events** such as key presses and mouse clicks are called event-driven programs.

The main **event loop** at this stage is simply:

```
while not finished:
    finished = move_player(player)
```

The event handling is done inside the `move_player` function. `update_when('key_pressed')` waits until a key has been pressed before moving to the next statement. The multi-way branching statement then handles the all keys relevent to game play.

Pressing the escape key causes `move_player` to return `True`, making `not finished` false, thus exiting the main loop and ending the game. The 4, 7, 8, 9, 6, 3, 2, and 1 keys all cause the player to move in the appropriate direction, if she isn't blocked by the edge of a window.

Adding a robot

Now let's add a single robot that heads toward the player each time the player moves.

Add the following `place_robot` function between `place_player` and `move_player`:

```
def place_robot():
    x = random.randint(0, GRID_WIDTH)
    y = random.randint(0, GRID_HEIGHT)
    return {'shape': Box((10*x, 10*y), 10, 10), 'x': x, 'y': y}
```

Add `move_robot` immediately after `move_player`:

```
def move_robot(robot, player):
    if robot['x'] < player['x']: robot['x'] += 1
    elif robot['x'] > player['x']: robot['x'] -= 1

    if robot['y'] < player['y']: robot['y'] += 1
    elif robot['y'] > player['y']: robot['y'] -= 1

    move_to(robot['shape'], (10*robot['x'], 10*robot['y']))
```

We need to pass both the robot and the player to this function so that it can compare their locations and move the robot toward the player.

Now add the line `robot = place_robot()` in the main body of the program immediately after the line `player = place_player()`, and add the `move_robot(robot, player)` call inside the main loop immediately after `finished = move_player(player)`.

Checking for Collisions

We now have a robot that moves relentlessly toward our player, but once it catches her it just follows her around wherever she goes. What we want to happen is for the game to end as soon as the player is caught. The following function will determine if that has happened:

```
def collided(robot, player):
    return player['x'] == robot['x'] and player['y'] == robot['y']
```

Place this new function immediately below the `move_player` function. Now let's modify `play_game` to check for collisions:

```
def play_game():
    begin_graphics(SCREEN_WIDTH, SCREEN_HEIGHT)
    player = place_player()
    robot = place_robot()
    defeated = False

    while not defeated:
        quit =  move_player(player)
        if quit:
            break
        move_robot(robot, player)
        defeated = collided(robot, player)

    if defeated:
        remove_from_screen(player['shape'])
        remove_from_screen(robot['shape'])
        Text("They got you!", (240, 240), size=32)
        sleep(3)

    end_graphics()
```

We rename the variable `finished` to `defeated`, which is now set to the result of `collided`. The main loop runs as long as `defeated` is false. Pressing the key still ends the program, since we check for `quit` and break out of the main loop if it is true. Finally, we check for `defeated` immediately after the main loop and display an appropriate message if it is true.

Adding more robots

There are several things we could do next:

- give the player the ability to *teleport* to another location to escape pursuit.
- provide safe placement of the player so that it never starts on top of a robot.
- add more robots.

Adding the ability to teleport to a random location is the easiest task, and it has been left to you to complete as an exercise.

How we provide safe placement of the player will depend on how we represent multiple robots, so it makes sense to tackle adding more robots first.

To add a second robot, we could just create another variable named something like `robot2` with another call to `place_robot`. The problem with this approach is that we will soon want lots of robots, and giving them all their own names will be cumbersome. A more elegant solution is to place all the robots in a list:

```
def place_robots(numbots):
    robots = []
    for i in range(numbots):
        robots.append(place_robot())
    return robots
```

Now instead of calling `place_robot` in `play_game`, call `place_robots`, which returns a single list containing all the robots:

```
robots = place_robots(2)
```

With more than one robot placed, we have to handle moving each one of them. We have already solved the problem of moving a single robot, however, so traversing the list and moving each one in turn does the trick:

```
def move_robots(robots, player):
    for robot in robots:
        move_robot(robot, player)
```

Add `move_robots` immediately after `move_robot`, and change `play_game` to call `move_robots` instead of `move_robot`.

We now need to check each robot to see if it has collided with the player:

```
def check_collisions(robots, player):
    for robot in robots:
        if collided(robot, player):
            return True
    return False
```

Add `check_collisions` immediately after `collided` and change the line in `play_game` that sets `defeated` to call `check_collisions` instead of `collided`.

Finally, we need to loop over `robots` to remove each one in turn if `defeated` becomes true. Adding this has been left as an exercise.

Winning the game

The biggest problem left in our game is that there is no way to win. The robots are both relentless and *indestructible*. With careful maneuvering and a bit of luck teleporting, we can reach the point where it appears there is only one robot chasing the player (all the robots will actually just be on top of each other). This moving pile of robots will continue chasing our hapless player until it catches it, either by a bad move on our part or a teleport that lands the player directly on the robots.

When two robots collide they are supposed to die, leaving behind a pile of junk. A robot (or the player) is also supposed to die when it collides with a pile of junk. The logic for doing this is quite tricky. After the player and each of the robots have moved, we need to:

1. Check whether the player has collided with a robot or a pile of junk. If so, set `defeated` to true and break out of the game loop.
2. Check each robot in the `robots` list to see if it has collided with a pile of junk. If it has, disgard the robot (remove it from the `robots` list).
3. Check each of the remaining robots to see if they have collided with another robot. If they have, disgard all the robots that have collided and place a pile of junk at the locations they occupied.
4. Check if any robots remain. If not, end the game and mark the player the winner.

Let's take on each of these tasks in turn.

Adding `junk`

Most of this work will take place inside our `check_collisions` function. Let's start by modifying `collided`, changing the names of the parameters to reflect its more general use:

```
def collided(thing1, thing2):
    return thing1['x'] == thing2['x'] and thing1['y'] == thing2['y']
```

We now introduce a new empty list named `junk` immediately after the call to `place_robots`:

```
junk = []
```

and modify `check_collisions` to incorporate the new list:

```
def check_collisions(robots, junk, player):
    # check whether player has collided with anything
    for thing in robots + junk:
        if collided(thing, player):
            return True
    return False
```

Be sure to modify the call to `check_collisions` (currently `defeated = check_collisions(robots, player)`) to include `junk` as a new argument.

Again, we need to fix the logic after `if defeated:` to remove the new `junk` from the screen before displaying the They got you! message:

```
for thing in robots + junk:
    remove_from_screen(thing['shape'])
```

Since at this point `junk` is always an empty list, we haven't changed the behavior of our program. To test whether our new logic is actually working, we could introduce a single junk pile and run

our player into it, at which point the game should remove all items from the screen and display the ending message.

It will be helpful to modify our program temporarily to change the random placement of robots and player to predetermined locations for testing. We plan to use solid boxes to represent junk piles. We observe that placing a robot is very similar to placing a junk pile, and modify `place_robot` to do both:

```
def place_robot(x, y, junk=False):
    return {'shape': Box((10*x, 10*y), 10, 10, filled=junk), 'x': x, 'y': y}
```

Notice that `x` and `y` are now parameters, along with a new parameter that we will use to set `filled` to true for piles of junk.

Our program is now broken, since the call in `place_robots` to `place_robot` does not pass arguments for `x` and `y`. Fixing this and setting up the program for testing is left to you as an exercise.

Removing robots that hit junk

To remove robots that collide with piles of junk, we add a *nested loop* to `check_collisions` between each robot and each pile of junk. Our first attempt at this does not work:

```
def check_collisions(robots, junk, player):
    # check whether player has collided with anything
    for thing in robots + junk:
        if collided(thing, player):
            return True

    # remove robots that have collided with a pile of junk
    for robot in robots:
        for pile in junk:
            if collided(robot, pile):
                robots.remove(robot)

    return False
```

Running this new code with the program as setup in exercise 11, we find a bug. It appears that the robots continue to pass through the pile of junk as before.

Actually, the bug is more subtle. Since we have two robots on top of each other, when the collision of the first one is detected and that robot is removed, we move the second robot into the first position in the list and *it is missed by the next iteration*. It is generally dangerous to modify a list while you are iterating over it. Doing so can introduce a host of difficult to find errors into your program.

The solution in this case is to loop over the `robots` list backwards, so that when we remove a robot from the list all the robots whose list indeces change as a result are robots we have already

evaluated.

As usual, Python provides an elegant way to do this. The built-in function, `reversed` provides for backward iteration over a sequence. Replacing:

```
for robot in robots:
```

with:

```
for robot in reversed(robots):
```

will make our program work the way we intended.

Turning robots into junk and enabling the player to win

We now want to check each robot to see if it has collided with any other robots. We will remove all robots that have collided, leaving a single pile of junk in their wake. If we reach a state where there are no more robots, the player wins.

Once again we have to be careful not to introduce bugs related to removing things from a list over which we are iterating.

Here is the plan:

1. Check each robot in `robots` (an outer loop, traversing forward).
2. Compare it with every robot that follows it (an inner loop, traversing backward).
3. If the two robots have collided, add a piece of junk at their location, mark the first robot as junk, and remove the second one.
4. Once all robots have been checked for collisions, traverse the robots list once again in reverse, removing all robots marked as junk.
5. Check to see if any robots remain. If not, declare the player the winner.

Adding the following to `check_collisions` will accomplish most of what we need to do:

```
# remove robots that collide and leave a pile of junk
for index, robot1 in enumerate(robots):
    for robot2 in reversed(robots[index+1:]):
        if collided(robot1, robot2):
            robot1['junk'] = True
            junk.append(place_robot(robot1['x'], robot1['y'], True))
            remove_from_screen(robot2['shape'])
            robots.remove(robot2)

for robot in reversed(robots):
    if robot['junk']:
        remove_from_screen(robot['shape'])
        robots.remove(robot)
```

We make use of the `enumerate` function we saw in Chapter 9 to get both the index and value of each robot as we traverse forward. Then a reverse traversal of the slice of the remaining robots, `reversed(robots[index+1:])`, sets up the collision check.

Whenever two robots collide, our plan calls for adding a piece of junk at that location, marking the first robot for later removal (we still need it to compare with the other robots), and immediately removing the second one. The body of the `if collided(robot1, robot2):` conditional is designed to do just that, but if you look carefully at the line:

```
robot1['junk'] = True
```

you should notice a problem. `robot1['junk']` will result in a systax error, since our robot dictionary does not yet contain a `'junk'` key. To fix this we modify `place_robot` to accomodate the new key:

```
def place_robot(x, y, junk=False):
    return {'shape': Box((10*x, 10*y), 10, 10, filled=junk),
            'x': x, 'y': y, 'junk': junk}
```

It is not at all unusual for data structures to change as program development proceeds. **Stepwise refinement** of both program data and logic is a normal part of the **structured programming** process.

After `robot1` is marked as junk, we add a pile of junk to the junk list at the same location with `junk.append(place_robot(robot1['x'], robot1['y'], True))`, and then remove `robot2` from the game by first removing its shape from the graphics window and then removing it from the robots list.

The next loop traverses backward over the robots list removing all the robots previously marked as junk. Since the player wins when all the robots die, and the robot list will be empty when it no longer contains live robots, we can simply check whether `robots` is empty to determine whether or not the player has won.

This can be done in `check_collisions` immediately after we finish checking robot collisions and removing dead robots by adding:

```
if not robots:
    return ...
```

Hmmm... What should we return? In its current state, `check_collisions` is a boolean function that returns true when the player has collided with something and lost the game, and false when the player has not lost and the game should continue. That is why the variable in the `play_game` function that catches the return value is called `defeated`.

Now we have three possible states:

1. `robots` is not empty and the player has not collided with anything – the game is still in play
2. the player has collided with something – the robots win

3. the player has not collided with anything and `robots` is empty – the player wins

In order to handle this with as few changes as possible to our present program, we will take advantage of the way that Python permits sequence types to live double lives as boolean values. We will return an empty string – which is false – when game play should continue, and either `"robots_win"` or `"player_wins"` to handle the other two cases. `check_collisions` should now look like this:

```
def check_collisions(robots, junk, player):
    # check whether player has collided with anything
    for thing in robots + junk:
        if collided(thing, player):
            return "robots_win"

    # remove robots that have collided with a pile of junk
    for robot in reversed(robots):
        for pile in junk:
            if collided(robot, pile):
                robots.remove(robot)

    # remove robots that collide and leave a pile of junk
    for index, robot1 in enumerate(robots):
        for robot2 in reversed(robots[index+1:]):
            if collided(robot1, robot2):
                robot1['junk'] = True
                junk.append(place_robot(robot1['x'], robot1['y'], True))
                remove_from_screen(robot2['shape'])
                robots.remove(robot2)

    for robot in reversed(robots):
        if robot['junk']:
            remove_from_screen(robot['shape'])
            robots.remove(robot)

    if not robots:
        return "player_wins"

    return ""
```

A few corresponding changes need to be made to `play_game` to use the new return values. These are left as an exercise.

Glossary

dictionary A collection of key-value pairs that maps from keys to values. The keys can be any immutable type, and the values can be any type.

mapping type A mapping type is a data type comprised of a collection of keys and associated values. Python's only built-in mapping type is the dictionary. Dictionaries implement the associative array abstract data type.

key A data item that is *mapped to* a value in a dictionary. Keys are used to look up values in a dictionary.

key-value pair One of the pairs of items in a dictionary. Values are looked up in a dictionary by key.

hint Temporary storage of a precomputed value to avoid redundant computation.

event A signal such as a keyboard press, mouse click, or message from another program.

event-driven program <fill in definition here>

event loop A programming construct that waits for events and processes them.

overflow A numerical result that is too large to be represented in a numerical format.

Exercises

1. Write a program that reads in a string on the command line and returns a table of the letters of the alphabet in alphabetical order which occur in the string together with the number of times each letter occurs. Case should be ignored. A sample run of the program would look this this:

```
$ python letter_counts.py "ThiS is String with Upper and lower case Letters."
a  2
c  1
d  1
e  5
g  1
h  2
i  4
l  2
n  2
o  1
p  2
r  4
s  5
t  5
u  1
w  2
$
```

2. Give the Python interpreter's response to each of the following from a continuous interpreter session:

(a)
```
>>> d = {'apples': 15, 'bananas': 35, 'grapes': 12}
>>> d['banana']
```

(b)
```
>>> d['oranges'] = 20
>>> len(d)
```

(c)
```
>>> d.has_key('grapes')
```

(d)
```
>>> d['pears']
```

(e)
```
>>> d.get('pears', 0)
```

(f)
```
>>> fruits = d.keys()
>>> fruits.sort()
>>> print(fruits)
```

(g)
```
>>> del d['apples']
>>> d.has_key('apples')
```

Be sure you understand why you get each result. Then apply what you have learned to fill in the body of the function below:

```
def add_fruit(inventory, fruit, quantity=0):
    """
    Adds quantity of fruit to inventory.

      >>> new_inventory = {}
      >>> add_fruit(new_inventory, 'strawberries', 10)
      >>> new_inventory.has_key('strawberries')
      True
      >>> new_inventory['strawberries']
      10
      >>> add_fruit(new_inventory, 'strawberries', 25)
      >>> new_inventory['strawberries']
    """
```

Your solution should pass the doctests.

3. Write a program called `alice_words.py` that creates a text file named `alice_words.txt` containing an alphabetical listing of all the words found in alice_in_wonderland.txt together with the number of times each word occurs. The first 10 lines of your output file should look something like this:

```
Word                Count
======================
a                   631
a-piece             1
abide               1
```

```
able            1
about           94
above           3
absence         1
absurd          2
```

How many times does the word, `alice`, occur in the book?

4. What is the longest word in Alice in Wonderland ? How many charactes does it have?
5. Copy the code from the *Setting up the world, the player, and the main loop* section into a file named `robots.py` and run it. You should be able to move the player around the screen using the numeric keypad and to quit the program by pressing the escape key.
6. Laptops usually have smaller keyboards than desktop computers that do not include a seperate numeric keypad. Modify the robots program so that it uses 'a', 'q', 'w', 'e', 'd', 'c', 'x', and 'z' instead of '4', '7', '8', '9', '6', '3', '2', and '1' so that it will work on a typical laptop keyboard.
7. Add all the code from the *Adding a robot* section in the places indicated. Make sure the program works and that you now have a robot following around your player.
8. Add all the code from the *Checking for Collisions* section in the places indicated. Verify that the program ends when the robot catches the player after displaying a They got you! message for 3 seconds.
9. Modify the `move_player` function to add the ability for the player to jump to a random location whenever the `0` key is pressed. (*hint:* `place_player` already has the logic needed to place the player in a random location. Just add another conditional branch to `move_player` that uses this logic when `key_pressed('0')` is true.) Test the program to verify that your player can now teleport to a random place on the screen to get out of trouble.
10. Make all the changes to your program indicated in *Adding more robots*. Be sure to loop over the `robots` list, removing each robot in turn, after `defeated` becomes true. Test your program to verify that there are now two robots chasing your player. Let a robot catch you to test whether you have correctly handled removing all the robots. Change the argument from 2 to 4 in `robots = place_robots(2)` and confirm that you have 4 robots.
11. Make the changes to your program indicated in *Adding ''junk''*. Fix `place_robots` by moving the random generation of values for `x` and `y` to the appropriate location and passing these values as arguments in the call to `place_robot`. Now we are ready to make temporary modifications to our program to remove the randomness so we can control it for testing. We can start by placing a pile of junk in the center of our game board. Change:

```
junk = []
```

to:

```
    junk = [place_robot(GRID_WIDTH/2, GRID_HEIGHT/2, junk=True)]
```

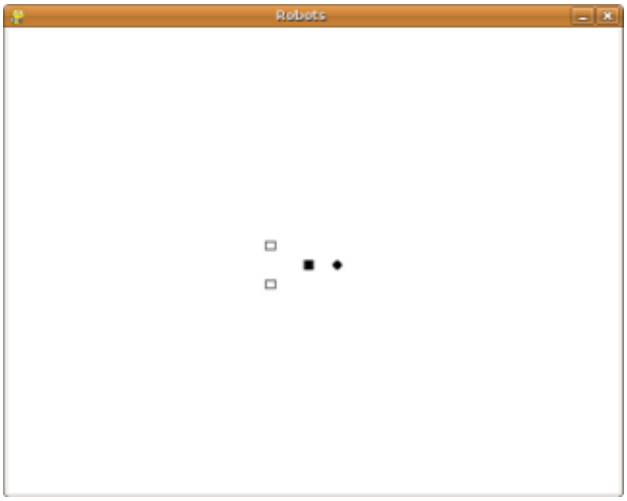
Run the program and confirm that there is a black box in the center of the board.Now change `place_player` so that it looks like this:

```
    def place_player():
        # x = random.randint(0, GRID_WIDTH)
        # y = random.randint(0, GRID_HEIGHT)
        x, y = GRID_WIDTH/2 + 3, GRID_HEIGHT/2
        return {'shape': Circle((10*x+5, 10*y+5), 5, filled=True), 'x': x, 'y': y}
```

Finally, temporarily **comment out** the random generation of `x` and `y` values in `place_robots` and the creation of `numbots` robots. Replace this logic with code to create two robots in fixed locations:

```
    def place_robots(numbots):
        robots = []
        # for i in range(numbots):
        #     x = random.randint(0, GRID_WIDTH)
        #     y = random.randint(0, GRID_HEIGHT)
        #     robots.append(place_robot(x, y))
        robots.append(place_robot(GRID_WIDTH/2 - 4, GRID_HEIGHT/2 + 2))
        robots.append(place_robot(GRID_WIDTH/2 - 4, GRID_HEIGHT/2 - 2))
        return robots
```

When you start your program now, it should look like this:



When you run this program and either stay still (by pressing the `5` repeatedly) or move away from the pile of junk, you can confirm that the robots move through it unharmed. When you move into the junk pile, on the other hand, you die.

12. Make the following modifications to `play_game` to integrate with the changes made in *Turning robots into junk and enabling the player to win*:

    (a) Rename `defeated` to `winner` and initialize it to the empty string instead of `False`.

Classes and objects

Object-oriented programming

Python is an **object-oriented programming language**, which means that it provides features that support object-oriented programming (**OOP**).

Object-oriented programming has its roots in the 1960s, but it wasn't until the mid 1980s that it became the main programming paradigm used in the creation of new software. It was developed as a way to handle the rapidly increasing size and complexity of software systems, and to make it easier to modify these large and complex systems over time.

Up to now we have been writing programs using a procedural programming paradigm. In procedural programming the focus is on writing functions or *procedures* which operate on data. In object-oriented programming the focus is on the creation of **objects** which contain both data and functionality together.

User-defined compound types

A class in essence defines a new **data type**. We have been using several of Python's built-in types throughout this book, we are now ready to create our own user-defined type: the `Point`.

Consider the concept of a mathematical point. In two dimensions, a point is two numbers (coordinates) that are treated collectively as a single object. In mathematical notation, points are often written in parentheses with a comma separating the coordinates. For example, `(0, 0)` represents the origin, and `(x, y)` represents the point `x` units to the right and `y` units up from the origin.

A natural way to represent a point in Python is with two numeric values. The question, then, is how to group these two values into a compound object. The quick and dirty solution is to use a list or tuple, and for some applications that might be the best choice.

An alternative is to define a new user-defined compound type, also called a **class**. This approach involves a bit more effort, but it has advantages that will be apparent soon.

A class definition looks like this:

```
class Point:
    pass
```

Class definitions can appear anywhere in a program, but they are usually near the beginning (after the `import` statements). The syntax rules for a class definition are the same as for other compound statements. There is a header which begins with the keyword, `class`, followed by the name of the class, and ending with a colon.

This definition creates a new class called `Point`. The **pass** statement has no effect; it is only necessary because a compound statement must have something in its body. A docstring could serve the same purpose:

```
class Point:
    "Point class for storing mathematical points."
```

By creating the `Point` class, we created a new type, also called `Point`. The members of this type are called **instances** of the type or **objects**. Creating a new instance is called **instantiation**, and is accomplished by **calling the class**. Classes, like functions, are callable, and we instantiate a `Point` object by calling the `Point` class:

```
>>> type(Point)
<type 'classobj'>
>>> p = Point()
>>> type(p)
<type 'instance'>
```

The variable `p` is assigned a reference to a new `Point` object.

It may be helpful to think of a class as a factory for making objects, so our `Point` class is a factory for making points. The class itself isn't an instance of a point, but it contains the machinary to make point instances.

Attributes

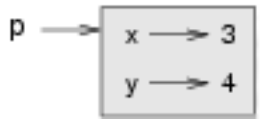
Like real world objects, object instances have both form and function. The form consists of data elements contained within the instance.

We can add new data elements to an instance using dot notation:

```
>>> p.x = 3
>>> p.y = 4
```

This syntax is similar to the syntax for selecting a variable from a module, such as `math.pi` or `string.uppercase`. Both modules and instances create their own namespaces, and the syntax for accessing names contained in each, called **attributes**, is the same. In this case the attribute we are selecting is a data item from an instance.

The following state diagram shows the result of these assignments:



The variable `p` refers to a Point object, which contains two attributes. Each attribute refers to a number.

We can read the value of an attribute using the same syntax:

```
>>> print(p.y)
4
```

```
>>> x = p.x
>>> print(x)
3
```

The expression `p.x` means, "Go to the object `p` refers to and get the value of `x`". In this case, we assign that value to a variable named `x`. There is no conflict between the variable `x` and the attribute `x`. The purpose of dot notation is to identify which variable you are referring to unambiguously.

You can use dot notation as part of any expression, so the following statements are legal:

```
print('(%d, %d)' % (p.x, p.y))
distance_squared = p.x * p.x + p.y * p.y
```

The first line outputs `(3, 4)`; the second line calculates the value 25.

The initialization method and `self`

Since our `Point` class is intended to represent two dimensional mathematical points, *all* point instances ought to have `x` and `y` attributes, but that is not yet so with our `Point` objects.

```
>>> p2 = Point()
>>> p2.x
Traceback (most recent call last):
  File "<stdin>", line 1, in ?
AttributeError: Point instance has no attribute 'x'
>>>
```

To solve this problem we add an **initialization method** to our class.

```
class Point:
    def __init__(self, x=0, y=0):
        self.x = x
        self.y = y
```

A **method** behaves like a function but it is part of an object. Like a data attribute it is accessed using dot notation. The initialization method is called automatically when the class is called.

Let's add another method, `distance_from_origin`, to see better how methods work:

```
class Point:
    def __init__(self, x=0, y=0):
        self.x = x
        self.y = y

    def distance_from_origin(self):
        return ((self.x ** 2) + (self.y ** 2)) ** 0.5
```

Let's create a few point instances, look at their attributes, and call our new method on them:

```
>>> p = Point(3, 4)
>>> p.x
3
>>> p.y
4
>>> p.distance_from_origin()
5.0
>>> q = Point(5, 12)
>>> q.x
5
>>> q.y
12
>>> q.distance_from_origin()
13.0
>>> r = Point()
>>> r.x
0
>>> r.y
0
>>> r.distance_from_origin()
0.0
```

When defining a method, the first parameter refers to the instance being created. It is customary to name this parameter **self**. In the example session above, the `self` parameter refers to the instances `p`, `q`, and `r` respectively.

Instances as parameters

You can pass an instance as a parameter to a function in the usual way. For example:

```
def print_point(p):
    print('(%s, %s)' % (str(p.x), str(p.y)))
```

`print_point` takes a point as an argument and displays it in the standard format. If you call `print_point(p)` with point `p` as defined previously, the output is `(3, 4)`.

Glossary

class A user-defined compound type. A class can also be thought of as a template for the objects that are instances of it.

instantiate To create an instance of a class.

instance An object that belongs to a class.

object A compound data type that is often used to model a thing or concept in the real world.

attribute One of the named data items that makes up an instance.

Exercises

1. Create and print a `Point` object, and then use `id` to print the object's unique identifier. Translate the hexadecimal form into decimal and confirm that they match.
2. Rewrite the `distance` function from chapter 5 so that it takes two `Points` as parameters instead of four numbers.

Classes and functions

Time

As another example of a user-defined type, we'll define a class called `Time` that records the time of day. The class definition looks like this:

```
class Time(object):
    pass
```

We can create a new `Time` object and assign attributes for hours, minutes, and seconds:

```
time = Time()
time.hours = 11
time.minutes = 59
time.seconds = 30
```

The state diagram for the `Time` object looks like this:

Pure functions

In the next few sections, we'll write two versions of a function called `add_time`, which calculates the sum of two `Times`. They will demonstrate two kinds of functions: pure functions and modifiers.

The following is a rough version of `add_time`:

```
def add_time(t1, t2):
    sum = Time()
    sum.hours = t1.hours + t2.hours
    sum.minutes = t1.minutes + t2.minutes
    sum.seconds = t1.seconds + t2.seconds
    return sum
```

The function creates a new `Time` object, initializes its attributes, and returns a reference to the new object. This is called a **pure function** because it does not modify any of the objects passed to it as parameters and it has no side effects, such as displaying a value or getting user input.

Here is an example of how to use this function. We'll create two `Time` objects: `current_time`, which contains the current time; and `bread_time`, which contains the amount of time it takes for a breadmaker to make bread. Then we'll use `add_time` to figure out when the bread will be done. If you haven't finished writing `print_time` yet, take a look ahead to Section before you try this:

```
>>> current_time = Time()
>>> current_time.hours = 9
>>> current_time.minutes = 14
>>> current_time.seconds =  30
>>> bread_time = Time()
>>> bread_time.hours = 3
>>> bread_time.minutes = 35
>>> bread_time.seconds = 0
>>> done_time = add_time(current_time, bread_time)
>>> print_time(done_time)
12:49:30
```

The output of this program is `12:49:30`, which is correct. On the other hand, there are cases where the result is not correct. Can you think of one?

The problem is that this function does not deal with cases where the number of seconds or minutes adds up to more than sixty. When that happens, we have to carry the extra seconds into the minutes column or the extra minutes into the hours column.

Here's a second corrected version of the function:

```
def add_time(t1, t2):
    sum = Time()
    sum.hours = t1.hours + t2.hours
    sum.minutes = t1.minutes + t2.minutes
    sum.seconds = t1.seconds + t2.seconds

    if sum.seconds >= 60:
        sum.seconds = sum.seconds - 60
        sum.minutes = sum.minutes + 1

    if sum.minutes >= 60:
```

```
        sum.minutes = sum.minutes - 60
        sum.hours = sum.hours + 1

    return sum
```

Although this function is correct, it is starting to get big. Later we will suggest an alternative approach that yields shorter code.

Modifiers

There are times when it is useful for a function to modify one or more of the objects it gets as parameters. Usually, the caller keeps a reference to the objects it passes, so any changes the function makes are visible to the caller. Functions that work this way are called **modifiers**.

`increment`, which adds a given number of seconds to a `Time` object, would be written most naturally as a modifier. A rough draft of the function looks like this:

```
def increment(time, seconds):
    time.seconds = time.seconds + seconds

    if time.seconds >= 60:
        time.seconds = time.seconds - 60
        time.minutes = time.minutes + 1

    if time.minutes >= 60:
        time.minutes = time.minutes - 60
        time.hours = time.hours + 1
```

The first line performs the basic operation; the remainder deals with the special cases we saw before.

Is this function correct? What happens if the parameter `seconds` is much greater than sixty? In that case, it is not enough to carry once; we have to keep doing it until `seconds` is less than sixty. One solution is to replace the `if` statements with `while` statements:

```
def increment(time, seconds):
    time.seconds = time.seconds + seconds

    while time.seconds >= 60:
        time.seconds = time.seconds - 60
        time.minutes = time.minutes + 1

    while time.minutes >= 60:
        time.minutes = time.minutes - 60
        time.hours = time.hours + 1
```

This function is now correct, but it is not the most efficient solution.

Prototype development versus planning

In this chapter, we demonstrated an approach to program development that we call **prototype development**. In each case, we wrote a rough draft (or prototype) that performed the basic calculation and then tested it on a few cases, correcting flaws as we found them.

Although this approach can be effective, it can lead to code that is unnecessarily complicated – since it deals with many special cases – and unreliable – since it is hard to know if you have found all the errors.

An alternative is **planned development**, in which high-level insight into the problem can make the programming much easier. In this case, the insight is that a `Time` object is really a three-digit number in base 60! The `second` component is the ones column, the `minute` component is the sixties column, and the `hour` component is the thirty-six hundreds column.

When we wrote `add_time` and `increment`, we were effectively doing addition in base 60, which is why we had to carry from one column to the next.

This observation suggests another approach to the whole problem – we can convert a `Time` object into a single number and take advantage of the fact that the computer knows how to do arithmetic with numbers. The following function converts a `Time` object into an integer:

```
def convert_to_seconds(t):
    minutes = t.hours * 60 + t.minutes
    seconds = minutes * 60 + t.seconds
    return seconds
```

Now, all we need is a way to convert from an integer to a `Time` object:

```
def make_time(seconds):
    time = Time()
    time.hours = seconds/3600
    seconds = seconds - time.hours * 3600
    time.minutes = seconds/60
    seconds = seconds - time.minutes * 60
    time.seconds = seconds
    return time
```

You might have to think a bit to convince yourself that this technique to convert from one base to another is correct. Assuming you are convinced, you can use these functions to rewrite `add_time`:

```
def add_time(t1, t2):
    seconds = convert_to_seconds(t1) + convert_to_seconds(t2)
    return make_time(seconds)
```

This version is much shorter than the original, and it is much easier to demonstrate that it is correct (assuming, as usual, that the functions it calls are correct).

Generalization

In some ways, converting from base 60 to base 10 and back is harder than just dealing with times. Base conversion is more abstract; our intuition for dealing with times is better.

But if we have the insight to treat times as base 60 numbers and make the investment of writing the conversion functions (`convert_to_seconds` and `make_time`), we get a program that is shorter, easier to read and debug, and more reliable.

It is also easier to add features later. For example, imagine subtracting two `Times` to find the duration between them. The naive approach would be to implement subtraction with borrowing. Using the conversion functions would be easier and more likely to be correct.

Ironically, sometimes making a problem harder (or more general) makes it easier (because there are fewer special cases and fewer opportunities for error).

Algorithms

When you write a general solution for a class of problems, as opposed to a specific solution to a single problem, you have written an **algorithm**. We mentioned this word before but did not define it carefully. It is not easy to define, so we will try a couple of approaches.

First, consider something that is not an algorithm. When you learned to multiply single-digit numbers, you probably memorized the multiplication table. In effect, you memorized 100 specific solutions. That kind of knowledge is not algorithmic.

But if you were lazy, you probably cheated by learning a few tricks. For example, to find the product of *n* and 9, you can write `n-1` as the first digit and `10-n` as the second digit. This trick is a general solution for multiplying any single-digit number by 9. That's an algorithm!

Similarly, the techniques you learned for addition with carrying, subtraction with borrowing, and long division are all algorithms. One of the characteristics of algorithms is that they do not require any intelligence to carry out. They are mechanical processes in which each step follows from the last according to a simple set of rules.

In our opinion, it is embarrassing that humans spend so much time in school learning to execute algorithms that, quite literally, require no intelligence.

On the other hand, the process of designing algorithms is interesting, intellectually challenging, and a central part of what we call programming.

Some of the things that people do naturally, without difficulty or conscious thought, are the hardest to express algorithmically. Understanding natural language is a good example. We all do it, but so far no one has been able to explain *how* we do it, at least not in the form of an algorithm.

Glossary

pure function A function that does not modify any of the objects it receives as parameters. Most pure functions are fruitful.

modifier A function that changes one or more of the objects it receives as parameters. Most modifiers are void.

functional programming style A style of program design in which the majority of functions are pure.

prototype development A way of developing programs starting with a prototype and gradually testing and improving it.

planned development A way of developing programs that involves high-level insight into the problem and more planning than incremental development or prototype development.

algorithm A set of instructions for solving a class of problems by a mechanical, unintelligent process.

Exercises

1. Write a function `print_time` that takes a `Time` object as an argument and prints it in the form `hours:minutes:seconds`.
2. Write a boolean function `after` that takes two `Time` objects, `t1` and `t2`, as arguments, and returns `True` if `t1` follows `t2` chronologically and `False` otherwise.
3. Rewrite the `increment` function so that it doesn't contain any loops.
4. Now rewrite `increment` as a pure function, and write function calls to both versions.

Classes and methods

Object-oriented features

Python is an **object-oriented programming language**, which means that it provides features that support **object-oriented programming**.

It is not easy to define object-oriented programming, but we have already seen some of its characteristics:

1. Programs are made up of object definitions and function definitions, and most of the computation is expressed in terms of operations on objects.
2. Each object definition corresponds to some object or concept in the real world, and the functions that operate on that object correspond to the ways real-world objects interact.

For example, the `Time` class defined in the last chapter corresponds to the way people record the time of day, and the functions we defined correspond to the kinds of things people do with times. Similarly, the `Point` and `Rectangle` classes correspond to the mathematical concepts of a point and a rectangle.

So far, we have not taken advantage of the features Python provides to support object-oriented programming. Strictly speaking, these features are not necessary. For the most part, they provide an alternative syntax for things we have already done, but in many cases, the alternative is more concise and more accurately conveys the structure of the program.

For example, in the `Time` program, there is no obvious connection between the class definition and the function definitions that follow. With some examination, it is apparent that every function takes at least one `Time` object as a parameter.

This observation is the motivation for **methods**. We have already seen some methods, such as `keys` and `values`, which were invoked on dictionaries. Each method is associated with a class and is intended to be invoked on instances of that class.

Methods are just like functions, with two differences:

1. Methods are defined inside a class definition in order to make the relationship between the class and the method explicit.
2. The syntax for invoking a method is different from the syntax for calling a function.

In the next few sections, we will take the functions from the previous two chapters and transform them into methods. This transformation is purely mechanical; you can do it simply by following a sequence of steps. If you are comfortable converting from one form to another, you will be able to choose the best form for whatever you are doing.

print_time

In the last chapter, we defined a class named `Time` and you wrote a function named `print_time`, which should have looked something like this:

```
class Time(object):
    pass

def print_time(time):
    print(str(time.hours) + ":" +
          str(time.minutes) + ":" +
          str(time.seconds))
```

To call this function, we passed a `Time` object as a parameter:

```
>>> current_time = Time()
>>> current_time.hours = 9
>>> current_time.minutes = 14
```

```
>>> current_time.seconds = 30
>>> print_time(current_time)
```

To make `print_time` a method, all we have to do is move the function definition inside the class definition. Notice the change in indentation.

```
class Time(object):
    def print_time(time):
        print(str(time.hours) + ":" +
              str(time.minutes) + ":" +
              str(time.seconds))
```

Now we can invoke `print_time` using dot notation.

```
>>> current_time.print_time()
```

As usual, the object on which the method is invoked appears before the dot and the name of the method appears after the dot.

The object on which the method is invoked is assigned to the first parameter, so in this case `current_time` is assigned to the parameter `time`.

By convention, the first parameter of a method is called `self`. The reason for this is a little convoluted, but it is based on a useful metaphor.

The syntax for a function call, `print_time(current_time)`, suggests that the function is the active agent. It says something like, Hey `print_time`! Here's an object for you to print.

In object-oriented programming, the objects are the active agents. An invocation like `current_time.print_time()` says Hey `current_time`! Please print yourself!

This change in perspective might be more polite, but it is not obvious that it is useful. In the examples we have seen so far, it may not be. But sometimes shifting responsibility from the functions onto the objects makes it possible to write more versatile functions, and makes it easier to maintain and reuse code.

Another example

Let's convert `increment` to a method. To save space, we will leave out previously defined methods, but you should keep them in your version:

```
class Time(object):
    #previous method definitions here...

    def increment(self, seconds):
        self.seconds = seconds + self.seconds

        while self.seconds >= 60:
```

```
            self.seconds = self.seconds - 60
            self.minutes = self.minutes + 1

        while self.minutes >= 60:
            self.minutes = self.minutes - 60
            self.hours = self.hours + 1
```

The transformation is purely mechanical - we move the method definition into the class definition and change the name of the first parameter.

Now we can invoke `increment` as a method.

```
current_time.increment(500)
```

Again, the object on which the method is invoked gets assigned to the first parameter, `self`. The second parameter, `seconds` gets the value `500`.

A more complicated example

The `after` function is slightly more complicated because it operates on two `Time` objects, not just one. We can only convert one of the parameters to `self`; the other stays the same:

```
class Time(object):
    #previous method definitions here...

    def after(self, time2):
        if self.hour > time2.hour:
            return True
        if self.hour < time2.hour:
            return False

        if self.minute > time2.minute:
            return True
        if self.minute < time2.minute:
            return False

        if self.second > time2.second:
            return True
        return False
```

We invoke this method on one object and pass the other as an argument:

```
if doneTime.after(current_time):
    print("The bread will be done after it starts.")
```

You can almost read the invocation like English: If the done-time is after the current-time, then...

Optional arguments

We have seen built-in functions that take a variable number of arguments. For example, `string.find` can take two, three, or four arguments.

It is possible to write user-defined functions with optional argument lists. For example, we can upgrade our own version of `find` to do the same thing as `string.find`.

This is the original version:

```
def find(str, ch):
    index = 0
    while index < len(str):
        if str[index] == ch:
            return index
        index = index + 1
    return -1
```

This is the new and improved version:

```
def find(str, ch, start=0):
    index = start
    while index < len(str):
        if str[index] == ch:
            return index
        index = index + 1
    return -1
```

The third parameter, `start`, is optional because a default value, `0`, is provided. If we invoke `find` with only two arguments, we use the default value and start from the beginning of the string:

```
>>> find("apple", "p")
1
```

If we provide a third parameter, it **overrides** the default:

```
>>> find("apple", "p", 2)
2
>>> find("apple", "p", 3)
-1
```

The initialization method

The **initialization method** is a special method that is invoked when an object is created. The name of this method is `__init__` (two underscore characters, followed by `init`, and then two more underscores). An initialization method for the `Time` class looks like this:

```
class Time(object):
    def __init__(self, hours=0, minutes=0, seconds=0):
        self.hours = hours
        self.minutes = minutes
        self.seconds = seconds
```

There is no conflict between the attribute `self.hours` and the parameter `hours`. Dot notation specifies which variable we are referring to.

When we invoke the `Time` constructor, the arguments we provide are passed along to `init`:

```
>>> current_time = Time(9, 14, 30)
>>> current_time.print_time()
>>> 9:14:30
```

Because the parameters are optional, we can omit them:

```
>>> current_time = Time()
>>> current_time.print_time()
>>> 0:0:0
```

Or provide only the first parameter:

```
>>> current_time = Time (9)
>>> current_time.print_time()
>>> 9:0:0
```

Or the first two parameters:

```
>>> current_time = Time (9, 14)
>>> current_time.print_time()
>>> 9:14:0
```

Finally, we can provide a subset of the parameters by naming them explicitly:

```
>>> current_time = Time(seconds = 30, hours = 9)
>>> current_time.print_time()
>>> 9:0:30
```

Points revisited

Let's rewrite the `Point` class from chapter 12 in a more object- oriented style:

```
class Point(object):
    def __init__(self, x=0, y=0):
        self.x = x
        self.y = y
```

```
    def __str__(self):
        return '(' + str(self.x) + ', ' + str(self.y) + ')'
```

The initialization method takes `x` and `y` values as optional parameters; the default for either parameter is 0.

The next method, `__str__`, returns a string representation of a `Point` object. If a class provides a method named `__str__`, it overrides the default behavior of the Python built-in `str` function.

```
>>> p = Point(3, 4)
>>> str(p)
'(3, 4)'
```

Printing a `Point` object implicitly invokes `__str__` on the object, so defining `__str__` also changes the behavior of `print`:

```
>>> p = Point(3, 4)
>>> print(p)
(3, 4)
```

When we write a new class, we almost always start by writing `__init__`, which makes it easier to instantiate objects, and `__str__`, which is almost always useful for debugging.

Operator overloading

Some languages make it possible to change the definition of the built- in operators when they are applied to user-defined types. This feature is called **operator overloading**. It is especially useful when defining new mathematical types.

For example, to override the addition operator +, we provide a method named `__add__`:

```
class Point(object):
    # previously defined methods here...

    def __add__(self, other):
        return Point(self.x + other.x, self.y + other.y)
```

As usual, the first parameter is the object on which the method is invoked. The second parameter is conveniently named `other` to distinguish it from `self`. To add two `Points`, we create and return a new `Point` that contains the sum of the `x` coordinates and the sum of the `y` coordinates.

Now, when we apply the + operator to `Point` objects, Python invokes `__add__`:

```
>>>  p1 = Point(3, 4)
>>>  p2 = Point(5, 7)
>>>  p3 = p1 + p2
>>>  print(p3)
(8, 11)
```

The expression `p1 + p2` is equivalent to `p1.__add__(p2)`, but obviously more elegant. As an exercise, add a method `__sub__(self, other)` that overloads the subtraction operator, and try it out. There are several ways to override the behavior of the multiplication operator: by defining a method named `__mul__`, or `__rmul__`, or both.

If the left operand of `*` is a `Point`, Python invokes `__mul__`, which assumes that the other operand is also a `Point`. It computes the **dot product** of the two points, defined according to the rules of linear algebra:

```
def __mul__(self, other):
    return self.x * other.x + self.y * other.y
```

If the left operand of `*` is a primitive type and the right operand is a `Point`, Python invokes `__rmul__`, which performs **scalar multiplication**:

```
def __rmul__(self, other):
    return Point(other * self.x,  other * self.y)
```

The result is a new `Point` whose coordinates are a multiple of the original coordinates. If `other` is a type that cannot be multiplied by a floating-point number, then `__rmul__` will yield an error.

This example demonstrates both kinds of multiplication:

```
>>> p1 = Point(3, 4)
>>> p2 = Point(5, 7)
>>> print(p1 * p2)
43
>>> print(2 * p2)
(10, 14)
```

What happens if we try to evaluate `p2 * 2`? Since the first parameter is a `Point`, Python invokes `__mul__` with `2` as the second argument. Inside `__mul__`, the program tries to access the `x` coordinate of `other`, which fails because an integer has no attributes:

```
>>> print(p2 * 2)
AttributeError: 'int' object has no attribute 'x'
```

Unfortunately, the error message is a bit opaque. This example demonstrates some of the difficulties of object-oriented programming. Sometimes it is hard enough just to figure out what code is running.

For a more complete example of operator overloading, see Appendix (reference overloading).

Polymorphism

Most of the methods we have written only work for a specific type. When you create a new object, you write methods that operate on that type.

But there are certain operations that you will want to apply to many types, such as the arithmetic operations in the previous sections. If many types support the same set of operations, you can write functions that work on any of those types.

For example, the `multadd` operation (which is common in linear algebra) takes three parameters; it multiplies the first two and then adds the third. We can write it in Python like this:

```
def multadd (x, y, z):
    return x * y + z
```

This method will work for any values of `x` and `y` that can be multiplied and for any value of `z` that can be added to the product.

We can invoke it with numeric values:

```
>>> multadd (3, 2, 1)
7
```

Or with `Points`:

```
>>> p1 = Point(3, 4)
>>> p2 = Point(5, 7)
>>> print(multadd (2, p1, p2))
(11, 15)
>>> print(multadd (p1, p2, 1))
44
```

In the first case, the `Point` is multiplied by a scalar and then added to another `Point`. In the second case, the dot product yields a numeric value, so the third parameter also has to be a numeric value.

A function like this that can take parameters with different types is called **polymorphic**.

As another example, consider the method `front_and_back`, which prints a list twice, forward and backward:

```
def front_and_back(front):
    import copy
    back = copy.copy(front)
    back.reverse()
    print(str(front) + str(back))
```

Because the `reverse` method is a modifier, we make a copy of the list before reversing it. That way, this method doesn't modify the list it gets as a parameter.

Here's an example that applies `front_and_back` to a list:

```
>>>   myList = [1, 2, 3, 4]
>>>   front_and_back(myList)
[1, 2, 3, 4][4, 3, 2, 1]
```

Of course, we intended to apply this function to lists, so it is not surprising that it works. What would be surprising is if we could apply it to a `Point`.

To determine whether a function can be applied to a new type, we apply the fundamental rule of polymorphism: *If all of the operations inside the function can be applied to the type, the function can be applied to the type.* The operations in the method include `copy`, `reverse`, and `print`.

`copy` works on any object, and we have already written a `__str__` method for `Points`, so all we need is a `reverse` method in the `Point` class:

```
def reverse(self):
    self.x , self.y = self.y, self.x
```

Then we can pass `Points` to `front_and_back`:

```
>>>   p = Point(3, 4)
>>>   front_and_back(p)
(3, 4)(4, 3)
```

The best kind of polymorphism is the unintentional kind, where you discover that a function you have already written can be applied to a type for which you never planned.

Glossary

object-oriented language A language that provides features, such as user-defined classes and inheritance, that facilitate object-oriented programming.

object-oriented programming A style of programming in which data and the operations that manipulate it are organized into classes and methods.

method A function that is defined inside a class definition and is invoked on instances of that class. :override:: To replace a default. Examples include replacing a default parameter with a particular argument and replacing a default method by providing a new method with the same name.

initialization method A special method that is invoked automatically when a new object is created and that initializes the object's attributes.

operator overloading Extending built-in operators (+, −, ⋆, >, <, etc.) so that they work with user-defined types.

dot product An operation defined in linear algebra that multiplies two `Points` and yields a numeric value.

scalar multiplication An operation defined in linear algebra that multiplies each of the coordinates of a `Point` by a numeric value.

polymorphic A function that can operate on more than one type. If all the operations in a function can be applied to a type, then the function can be applied to a type.

Exercises

1. Convert the function `convertToSeconds`:

```
def convertToSeconds(t):
    minutes = t.hours * 60 + t.minutes
    seconds = minutes * 60 + t.seconds
    return seconds
```

   to a method in the `Time` class.

2. Add a fourth parameter, `end`, to the `find` function that specifies where to stop looking. Warning: This exercise is a bit tricky. The default value of `end` should be `len(str)`, but that doesn't work. The default values are evaluated when the function is defined, not when it is called. When `find` is defined, `str` doesn't exist yet, so you can't find its length.

Sets of objects

Composition

By now, you have seen several examples of composition. One of the first examples was using a method invocation as part of an expression. Another example is the nested structure of statements; you can put an `if` statement within a `while` loop, within another `if` statement, and so on.

Having seen this pattern, and having learned about lists and objects, you should not be surprised to learn that you can create lists of objects. You can also create objects that contain lists (as attributes); you can create lists that contain lists; you can create objects that contain objects; and so on.

In this chapter and the next, we will look at some examples of these combinations, using `Card` objects as an example.

`Card` objects

If you are not familiar with common playing cards, now would be a good time to get a deck, or else this chapter might not make much sense. There are fifty-two cards in a deck, each of which belongs to one of four suits and one of thirteen ranks. The suits are Spades, Hearts, Diamonds, and Clubs (in descending order in bridge). The ranks are Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. Depending on the game that you are playing, the rank of Ace may be higher than King or lower than 2.

If we want to define a new object to represent a playing card, it is obvious what the attributes should be: `rank` and `suit`. It is not as obvious what type the attributes should be. One possibility is to use strings containing words like `"Spade"` for suits and `"Queen"` for ranks. One problem with this implementation is that it would not be easy to compare cards to see which had a higher rank or suit.

An alternative is to use integers to **encode** the ranks and suits. By encode, we do not mean what some people think, which is to encrypt or translate into a secret code. What a computer scientist means by encode is to define a mapping between a sequence of numbers and the items I want to represent. For example:

```
Spades   -->  3
Hearts   -->  2
Diamonds -->  1
Clubs    -->  0
```

An obvious feature of this mapping is that the suits map to integers in order, so we can compare suits by comparing integers. The mapping for ranks is fairly obvious; each of the numerical ranks maps to the corresponding integer, and for face cards:

```
Jack   -->  11
Queen  -->  12
King   -->  13
```

The reason we are using mathematical notation for these mappings is that they are not part of the Python program. They are part of the program design, but they never appear explicitly in the code. The class definition for the `Card` type looks like this:

```
class Card(object):
    def __init__(self, suit=0, rank=0):
        self.suit = suit
        self.rank = rank
```

As usual, we provide an initialization method that takes an optional parameter for each attribute.

To create an object that represents the 3 of Clubs, use this command:

```
three_of_clubs = Card(0, 3)
```

The first argument, `0`, represents the suit Clubs.

Class attributes and the `__str__` method

In order to print `Card` objects in a way that people can easily read, we want to map the integer codes onto words. A natural way to do that is with lists of strings. We assign these lists to **class attributes** at the top of the class definition:

```
class Card(object):
    suits = ["Clubs", "Diamonds", "Hearts", "Spades"]
    ranks = ["narf", "Ace", "2", "3", "4", "5", "6", "7",
             "8", "9", "10", "Jack", "Queen", "King"]

    def __init__(self, suit=0, rank=0):
        self.suit = suit
```

```
        self.rank = rank

    def __str__(self):
        return (self.ranks[self.rank] + " of " + self.suits[self.suit])
```

A class attribute is defined outside of any method, and it can be accessed from any of the methods in the class.

Inside `__str__`, we can use `suits` and `ranks` to map the numerical values of `suit` and `rank` to strings. For example, the expression `self.suits[self.suit]` means use the attribute `suit` from the object `self` as an index into the class attribute named `suits`, and select the appropriate string.

The reason for the `"narf"` in the first element in `ranks` is to act as a place keeper for the zero-eth element of the list, which will never be used. The only valid ranks are 1 to 13. This wasted item is not entirely necessary. We could have started at 0, as usual, but it is less confusing to encode 2 as 2, 3 as 3, and so on.

With the methods we have so far, we can create and print cards:

```
>>> card1 = Card(1, 11)
>>> print(card1)
Jack of Diamonds
```

Class attributes like `suits` are shared by all `Card` objects. The advantage of this is that we can use any `Card` object to access the class attributes:

```
>>> card2 = Card(1, 3)
>>> print(card2)
3 of Diamonds
>>> print(card2.suits[1])
Diamonds
```

The disadvantage is that if we modify a class attribute, it affects every instance of the class. For example, if we decide that Jack of Diamonds should really be called Jack of Swirly Whales, we could do this:

```
>>> card1.suits[1] = "Swirly Whales"
>>> print(card1)
Jack of Swirly Whales
```

The problem is that *all* of the Diamonds just became Swirly Whales:

```
>>> print(card2)
3 of Swirly Whales
```

It is usually not a good idea to modify class attributes.

Comparing cards

For primitive types, there are conditional operators (<, >, ==, etc.) that compare values and determine when one is greater than, less than, or equal to another. For user-defined types, we can override the behavior of the built-in operators by providing a method named __cmp__. By convention, __cmp__ takes two parameters, `self` and `other`, and returns 1 if the first object is greater, -1 if the second object is greater, and 0 if they are equal to each other.

Some types are completely ordered, which means that you can compare any two elements and tell which is bigger. For example, the integers and the floating-point numbers are completely ordered. Some sets are unordered, which means that there is no meaningful way to say that one element is bigger than another. For example, the fruits are unordered, which is why you cannot compare apples and oranges.

The set of playing cards is partially ordered, which means that sometimes you can compare cards and sometimes not. For example, you know that the 3 of Clubs is higher than the 2 of Clubs, and the 3 of Diamonds is higher than the 3 of Clubs. But which is better, the 3 of Clubs or the 2 of Diamonds? One has a higher rank, but the other has a higher suit.

In order to make cards comparable, you have to decide which is more important, rank or suit. To be honest, the choice is arbitrary. For the sake of choosing, we will say that suit is more important, because a new deck of cards comes sorted with all the Clubs together, followed by all the Diamonds, and so on.

With that decided, we can write __cmp__:

```
def __cmp__(self, other):
    # check the suits
    if self.suit > other.suit: return 1
    if self.suit < other.suit: return -1
    # suits are the same... check ranks
    if self.rank > other.rank: return 1
    if self.rank < other.rank: return -1
    # ranks are the same... it's a tie
    return 0
```

In this ordering, Aces appear lower than Deuces (2s).

Decks

Now that we have objects to represent `Card`s, the next logical step is to define a class to represent a `Deck`. Of course, a deck is made up of cards, so each `Deck` object will contain a list of cards as an attribute.

The following is a class definition for the `Deck` class. The initialization method creates the attribute `cards` and generates the standard set of fifty-two cards:

```
class Deck(object):
    def __init__(self):
        self.cards = []
        for suit in range(4):
            for rank in range(1, 14):
                self.cards.append(Card(suit, rank))
```

The easiest way to populate the deck is with a nested loop. The outer loop enumerates the suits from 0 to 3. The inner loop enumerates the ranks from 1 to 13. Since the outer loop iterates four times, and the inner loop iterates thirteen times, the total number of times the body is executed is fifty-two (thirteen times four). Each iteration creates a new instance of `Card` with the current suit and rank, and appends that card to the `cards` list.

The `append` method works on lists but not, of course, tuples.

Printing the deck

As usual, when we define a new type of object we want a method that prints the contents of an object. To print a `Deck`, we traverse the list and print each `Card`:

```
class Deck(object):
    ...
    def print_deck(self):
        for card in self.cards:
            print(card)
```

Here, and from now on, the ellipsis (`...`) indicates that we have omitted the other methods in the class.

As an alternative to `print_deck`, we could write a `__str__` method for the `Deck` class. The advantage of `__str__` is that it is more flexible. Rather than just printing the contents of the object, it generates a string representation that other parts of the program can manipulate before printing, or store for later use.

Here is a version of `__str__` that returns a string representation of a `Deck`. To add a bit of pizzazz, it arranges the cards in a cascade where each card is indented one space more than the previous card:

```
class Deck(object):
    ...
    def __str__(self):
        s = ""
        for i in range(len(self.cards)):
            s = s + " " * i + str(self.cards[i]) + "\n"
        return s
```

This example demonstrates several features. First, instead of traversing `self.cards` and as-

signing each card to a variable, we are using `i` as a loop variable and an index into the list of cards.

Second, we are using the string multiplication operator to indent each card by one more space than the last. The expression `" " * i` yields a number of spaces equal to the current value of `i`.

Third, instead of using the `print` command to print the cards, we use the `str` function. Passing an object as an argument to `str` is equivalent to invoking the `__str__` method on the object.

Finally, we are using the variable `s` as an **accumulator**. Initially, `s` is the empty string. Each time through the loop, a new string is generated and concatenated with the old value of `s` to get the new value. When the loop ends, `s` contains the complete string representation of the `Deck`, which looks like this:

```
>>> deck = Deck()
>>> print(deck)
Ace of Clubs
 2 of Clubs
  3 of Clubs
   4 of Clubs
     5 of Clubs
       6 of Clubs
        7 of Clubs
         8 of Clubs
          9 of Clubs
           10 of Clubs
            Jack of Clubs
             Queen of Clubs
              King of Clubs
               Ace of Diamonds
```

And so on. Even though the result appears on 52 lines, it is one long string that contains newlines.

Shuffling the deck

If a deck is perfectly shuffled, then any card is equally likely to appear anywhere in the deck, and any location in the deck is equally likely to contain any card.

To shuffle the deck, we will use the `randrange` function from the `random` module. With two integer arguments, `a` and `b`, `randrange` chooses a random integer in the range `a <= x < b`. Since the upper bound is strictly less than `b`, we can use the length of a list as the second parameter, and we are guaranteed to get a legal index. For example, this expression chooses the index of a random card in a deck:

```
random.randrange(0, len(self.cards))
```

An easy way to shuffle the deck is by traversing the cards and swapping each card with a randomly chosen one. It is possible that the card will be swapped with itself, but that is fine. In fact, if we

precluded that possibility, the order of the cards would be less than entirely random:

```
class Deck(object):
    ...
    def shuffle(self):
        import random
        num_cards = len(self.cards)
        for i in range(num_cards):
            j = random.randrange(i, num_cards)
            self.cards[i], self.cards[j] = self.cards[j], self.cards[i]
```

Rather than assume that there are fifty-two cards in the deck, we get the actual length of the list and store it in `num_cards`.

For each card in the deck, we choose a random card from among the cards that haven't been shuffled yet. Then we swap the current card (`i`) with the selected card (`j`). To swap the cards we use a tuple assignment:

```
self.cards[i], self.cards[j] = self.cards[j], self.cards[i]
```

Removing and dealing cards

Another method that would be useful for the `Deck` class is `remove`, which takes a card as a parameter, removes it, and returns `True` if the card was in the deck and `False` otherwise:

```
class Deck(object):
    ...
    def remove(self, card):
        if card in self.cards:
            self.cards.remove(card)
            return True
        else:
            return False
```

The `in` operator returns `True` if the first operand is in the second, which must be a list or a tuple. If the first operand is an object, Python uses the object's `__cmp__` method to determine equality with items in the list. Since the `__cmp__` in the `Card` class checks for deep equality, the `remove` method checks for deep equality.

To deal cards, we want to remove and return the top card. The list method `pop` provides a convenient way to do that:

```
class Deck(object):
    ...
    def pop(self):
        return self.cards.pop()
```

Actually, `pop` removes the *last* card in the list, so we are in effect dealing from the bottom of the deck.

One more operation that we are likely to want is the boolean function `is_empty`, which returns true if the deck contains no cards:

```
class Deck(object):
    ...
    def is_empty(self):
        return (len(self.cards) == 0)
```

Glossary

encode To represent one set of values using another set of values by constructing a mapping between them.

class attribute A variable that is defined inside a class definition but outside any method. Class attributes are accessible from any method in the class and are shared by all instances of the class.

accumulator A variable used in a loop to accumulate a series of values, such as by concatenating them onto a string or adding them to a running sum.

Exercises

1. Modify `__cmp__` so that Aces are ranked higher than Kings.

Inheritance

Inheritance

The language feature most often associated with object-oriented programming is **inheritance**. Inheritance is the ability to define a new class that is a modified version of an existing class.

The primary advantage of this feature is that you can add new methods to a class without modifying the existing class. It is called inheritance because the new class inherits all of the methods of the existing class. Extending this metaphor, the existing class is sometimes called the **parent** class. The new class may be called the **child** class or sometimes subclass.

Inheritance is a powerful feature. Some programs that would be complicated without inheritance can be written concisely and simply with it. Also, inheritance can facilitate code reuse, since you can customize the behavior of parent classes without having to modify them. In some cases, the inheritance structure reflects the natural structure of the problem, which makes the program easier to understand.

On the other hand, inheritance can make programs difficult to read. When a method is invoked, it is sometimes not clear where to find its definition. The relevant code may be scattered among several modules. Also, many of the things that can be done using inheritance can be done as elegantly (or more so) without it. If the natural structure of the problem does not lend itself to inheritance, this style of programming can do more harm than good.

In this chapter we will demonstrate the use of inheritance as part of a program that plays the card game Old Maid. One of our goals is to write code that could be reused to implement other card games.

A hand of cards

For almost any card game, we need to represent a hand of cards. A hand is similar to a deck, of course. Both are made up of a set of cards, and both require operations like adding and removing cards. Also, we might like the ability to shuffle both decks and hands.

A hand is also different from a deck. Depending on the game being played, we might want to perform some operations on hands that don't make sense for a deck. For example, in poker we might classify a hand (straight, flush, etc.) or compare it with another hand. In bridge, we might want to compute a score for a hand in order to make a bid.

This situation suggests the use of inheritance. If `Hand` is a subclass of `Deck`, it will have all the methods of `Deck`, and new methods can be added.

In the class definition, the name of the parent class appears in parentheses:

```
class Hand(Deck):
    pass
```

This statement indicates that the new `Hand` class inherits from the existing `Deck` class.

The `Hand` constructor initializes the attributes for the hand, which are `name` and `cards`. The string `name` identifies this hand, probably by the name of the player that holds it. The name is an optional parameter with the empty string as a default value. `cards` is the list of cards in the hand, initialized to the empty list:

```
class Hand(Deck):
    def __init__(self, name=""):
        self.cards = []
        self.name = name
```

For just about any card game, it is necessary to add and remove cards from the deck. Removing cards is already taken care of, since `Hand` inherits `remove` from `Deck`. But we have to write `add`:

```
class Hand(Deck):
    ...
```

```
    def add(self,card):
        self.cards.append(card)
```

Again, the ellipsis indicates that we have omitted other methods. The list `append` method adds the new card to the end of the list of cards.

Dealing cards

Now that we have a `Hand` class, we want to deal cards from the `Deck` into hands. It is not immediately obvious whether this method should go in the `Hand` class or in the `Deck` class, but since it operates on a single deck and (possibly) several hands, it is more natural to put it in `Deck`.

`deal` should be fairly general, since different games will have different requirements. We may want to deal out the entire deck at once or add one card to each hand.

`deal` takes two parameters, a list (or tuple) of hands and the total number of cards to deal. If there are not enough cards in the deck, the method deals out all of the cards and stops:

```
class Deck(object):
    ...
    def deal(self, hands, num_cards=999):
        num_hands = len(hands)
        for i in range(num_cards):
            if self.is_empty(): break   # break if out of cards
            card = self.pop()           # take the top card
            hand = hands[i % num_hands] # whose turn is next?
            hand.add(card)              # add the card to the hand
```

The second parameter, `num_cards`, is optional; the default is a large number, which effectively means that all of the cards in the deck will get dealt.

The loop variable `i` goes from 0 to `nCards-1`. Each time through the loop, a card is removed from the deck using the list method `pop`, which removes and returns the last item in the list.

The modulus operator (`%`) allows us to deal cards in a round robin (one card at a time to each hand). When `i` is equal to the number of hands in the list, the expression `i % nHands` wraps around to the beginning of the list (index 0).

Printing a Hand

To print the contents of a hand, we can take advantage of the `printDeck` and `__str__` methods inherited from `Deck`. For example:

```
>>> deck = Deck()
>>> deck.shuffle()
>>> hand = Hand("frank")
```

```
>>> deck.deal([hand], 5)
>>> print(hand)
Hand frank contains
2 of Spades
 3 of Spades
  4 of Spades
   Ace of Hearts
    9 of Clubs
```

It's not a great hand, but it has the makings of a straight flush.

Although it is convenient to inherit the existing methods, there is additional information in a `Hand` object we might want to include when we print one. To do that, we can provide a `__str__` method in the `Hand` class that overrides the one in the `Deck` class:

```
class Hand(Deck)
    ...
    def __str__(self):
        s = "Hand " + self.name
        if self.is_empty():
            s = s + " is empty\n"
        else:
            s = s + " contains\n"
        return s + Deck.__str__(self)
```

Initially, `s` is a string that identifies the hand. If the hand is empty, the program appends the words `is empty` and returns `s`.

Otherwise, the program appends the word `contains` and the string representation of the `Deck`, computed by invoking the `__str__` method in the `Deck` class on `self`.

It may seem odd to send `self`, which refers to the current `Hand`, to a `Deck` method, until you remember that a `Hand` is a kind of `Deck`. `Hand` objects can do everything `Deck` objects can, so it is legal to send a `Hand` to a `Deck` method.

In general, it is always legal to use an instance of a subclass in place of an instance of a parent class.

The `CardGame` class

The `CardGame` class takes care of some basic chores common to all games, such as creating the deck and shuffling it:

```
class CardGame(object):
    def __init__(self):
        self.deck = Deck()
        self.deck.shuffle()
```

This is the first case we have seen where the initialization method performs a significant computation, beyond initializing attributes.

To implement specific games, we can inherit from `CardGame` and add features for the new game. As an example, we'll write a simulation of Old Maid.

The object of Old Maid is to get rid of cards in your hand. You do this by matching cards by rank and color. For example, the 4 of Clubs matches the 4 of Spades since both suits are black. The Jack of Hearts matches the Jack of Diamonds since both are red.

To begin the game, the Queen of Clubs is removed from the deck so that the Queen of Spades has no match. The fifty-one remaining cards are dealt to the players in a round robin. After the deal, all players match and discard as many cards as possible.

When no more matches can be made, play begins. In turn, each player picks a card (without looking) from the closest neighbor to the left who still has cards. If the chosen card matches a card in the player's hand, the pair is removed. Otherwise, the card is added to the player's hand. Eventually all possible matches are made, leaving only the Queen of Spades in the loser's hand.

In our computer simulation of the game, the computer plays all hands. Unfortunately, some nuances of the real game are lost. In a real game, the player with the Old Maid goes to some effort to get their neighbor to pick that card, by displaying it a little more prominently, or perhaps failing to display it more prominently, or even failing to fail to display that card more prominently. The computer simply picks a neighbor's card at random.

`OldMaidHand` class

A hand for playing Old Maid requires some abilities beyond the general abilities of a `Hand`. We will define a new class, `OldMaidHand`, that inherits from `Hand` and provides an additional method called `remove_matches`:

```
class OldMaidHand(Hand):
    def remove_matches(self):
        count = 0
        original_cards = self.cards[:]
        for card in original_cards:
            match = Card(3 - card.suit, card.rank)
            if match in self.cards:
                self.cards.remove(card)
                self.cards.remove(match)
                print("Hand %s: %s matches %s" % (self.name, card, match))
                count = count + 1
        return count
```

We start by making a copy of the list of cards, so that we can traverse the copy while removing cards from the original. Since `self.cards` is modified in the loop, we don't want to use it to control the traversal. Python can get quite confused if it is traversing a list that is changing!

For each card in the hand, we figure out what the matching card is and go looking for it. The match card has the same rank and the other suit of the same color. The expression `3 - card.suit` turns a Club (suit 0) into a Spade (suit 3) and a Diamond (suit 1) into a Heart (suit 2). You should satisfy yourself that the opposite operations also work. If the match card is also in the hand, both cards are removed.

The following example demonstrates how to use `remove_matches`:

```
>>> game = CardGame()
>>> hand = OldMaidHand("frank")
>>> game.deck.deal([hand], 13)
>>> print(hand)
Hand frank contains
Ace of Spades
 2 of Diamonds
  7 of Spades
   8 of Clubs
    6 of Hearts
     8 of Spades
      7 of Clubs
       Queen of Clubs
        7 of Diamonds
         5 of Clubs
          Jack of Diamonds
           10 of Diamonds
            10 of Hearts
>>> hand.remove_matches()
Hand frank: 7 of Spades matches 7 of Clubs
Hand frank: 8 of Spades matches 8 of Clubs
Hand frank: 10 of Diamonds matches 10 of Hearts
>>> print(hand)
Hand frank contains
Ace of Spades
 2 of Diamonds
  6 of Hearts
   Queen of Clubs
    7 of Diamonds
     5 of Clubs
      Jack of Diamonds
```

Notice that there is no `__init__` method for the `OldMaidHand` class. We inherit it from `Hand`.

`OldMaidGame` class

Now we can turn our attention to the game itself. `OldMaidGame` is a subclass of `CardGame` with a new method called `play` that takes a list of players as a parameter.

Since __init__ is inherited from CardGame, a new OldMaidGame object contains a new shuffled deck:

```
class OldMaidGame(CardGame):
    def play(self, names):
        # remove Queen of Clubs
        self.deck.remove(Card(0,12))

        # make a hand for each player
        self.hands = []
        for name in names:
            self.hands.append(OldMaidHand(name))

        # deal the cards
        self.deck.deal(self.hands)
        print("---------- Cards have been dealt")
        self.printHands()

        # remove initial matches
        matches = self.removeAllMatches()
        print("---------- Matches discarded, play begins")
        self.printHands()

        # play until all 50 cards are matched
        turn = 0
        numHands = len(self.hands)
        while matches < 25:
            matches = matches + self.playOneTurn(turn)
            turn = (turn + 1) % numHands

        print("---------- Game is Over")
        self.printHands()
```

The writing of printHands() is left as an exercise.

Some of the steps of the game have been separated into methods. remove_all_matches traverses the list of hands and invokes remove_matches on each:

```
class OldMaidGame(CardGame):
    ...
    def remove_all_matches(self):
        count = 0
        for hand in self.hands:
            count = count + hand.remove_matches()
        return count
```

count is an accumulator that adds up the number of matches in each hand and returns the total.

When the total number of matches reaches twenty-five, fifty cards have been removed from the

hands, which means that only one card is left and the game is over.

The variable `turn` keeps track of which player's turn it is. It starts at 0 and increases by one each time; when it reaches `numHands`, the modulus operator wraps it back around to 0.

The method `playOneTurn` takes a parameter that indicates whose turn it is. The return value is the number of matches made during this turn:

```
class OldMaidGame(CardGame):
    ...
    def play_one_turn(self, i):
        if self.hands[i].is_empty():
            return 0
        neighbor = self.find_neighbor(i)
        pickedCard = self.hands[neighbor].popCard()
        self.hands[i].add(pickedCard)
        print("Hand %s picked %s" % (self.hands[i].name, pickedCard))
        count = self.hands[i].remove_matches()
        self.hands[i].shuffle()
        return count
```

If a player's hand is empty, that player is out of the game, so he or she does nothing and returns 0.

Otherwise, a turn consists of finding the first player on the left that has cards, taking one card from the neighbor, and checking for matches. Before returning, the cards in the hand are shuffled so that the next player's choice is random.

The method `find_neighbor` starts with the player to the immediate left and continues around the circle until it finds a player that still has cards:

```
class OldMaidGame(CardGame):
    ...
    def find_neighbor(self, i):
        numHands = len(self.hands)
        for next in range(1,numHands):
            neighbor = (i + next) % numHands
            if not self.hands[neighbor].is_empty():
                return neighbor
```

If `find_neighbor` ever went all the way around the circle without finding cards, it would return `None` and cause an error elsewhere in the program. Fortunately, we can prove that that will never happen (as long as the end of the game is detected correctly).

We have omitted the `print_hands` method. You can write that one yourself.

The following output is from a truncated form of the game where only the top fifteen cards (tens and higher) were dealt to three players. With this small deck, play stops after seven matches instead of twenty-five.

```
>>> import cards
>>> game = cards.OldMaidGame()
>>> game.play(["Allen","Jeff","Chris"])
---------- Cards have been dealt
Hand Allen contains
King of Hearts
 Jack of Clubs
  Queen of Spades
   King of Spades
    10 of Diamonds

Hand Jeff contains
Queen of Hearts
 Jack of Spades
  Jack of Hearts
   King of Diamonds
    Queen of Diamonds

Hand Chris contains
Jack of Diamonds
 King of Clubs
  10 of Spades
   10 of Hearts
    10 of Clubs

Hand Jeff: Queen of Hearts matches Queen of Diamonds
Hand Chris: 10 of Spades matches 10 of Clubs
---------- Matches discarded, play begins
Hand Allen contains
King of Hearts
 Jack of Clubs
  Queen of Spades
   King of Spades
    10 of Diamonds

Hand Jeff contains
Jack of Spades
 Jack of Hearts
  King of Diamonds

Hand Chris contains
Jack of Diamonds
 King of Clubs
  10 of Hearts

Hand Allen picked King of Diamonds
Hand Allen: King of Hearts matches King of Diamonds
```

```
Hand Jeff picked 10 of Hearts
Hand Chris picked Jack of Clubs
Hand Allen picked Jack of Hearts
Hand Jeff picked Jack of Diamonds
Hand Chris picked Queen of Spades
Hand Allen picked Jack of Diamonds
Hand Allen: Jack of Hearts matches Jack of Diamonds
Hand Jeff picked King of Clubs
Hand Chris picked King of Spades
Hand Allen picked 10 of Hearts
Hand Allen: 10 of Diamonds matches 10 of Hearts
Hand Jeff picked Queen of Spades
Hand Chris picked Jack of Spades
Hand Chris: Jack of Clubs matches Jack of Spades
Hand Jeff picked King of Spades
Hand Jeff: King of Clubs matches King of Spades
---------- Game is Over
Hand Allen is empty

Hand Jeff contains
Queen of Spades

Hand Chris is empty
```

So Jeff loses.

Glossary

inheritance The ability to define a new class that is a modified version of a previously defined class.

parent class The class from which a child class inherits.

child class A new class created by inheriting from an existing class; also called a subclass.

Exercises

1. Add a method, `print_hands`, to the `OldMaidGame` class which traverses `self.hands` and prints each hand.

Linked lists

Embedded references

We have seen examples of attributes that refer to other objects, which we called **embedded references**. A common data structure, the **linked list**, takes advantage of this feature.

Linked lists are made up of **nodes**, where each node contains a reference to the next node in the list. In addition, each node contains a unit of data called the **cargo**.

A linked list is considered a **recursive data structure** because it has a recursive definition.

A linked list is either:

1. the empty list, represented by `None`, or
2. a node that contains a cargo object and a reference to a linked list.

Recursive data structures lend themselves to recursive methods.

The `Node` class

As usual when writing a new class, we'll start with the initialization and `__str__` methods so that we can test the basic mechanism of creating and displaying the new type:

```
class Node(object):
    def __init__(self, cargo=None, next=None):
        self.cargo = cargo
        self.next  = next

    def __str__(self):
        return str(self.cargo)
```

As usual, the parameters for the initialization method are optional. By default, both the cargo and the link, `next`, are set to `None`.

The string representation of a node is just the string representation of the cargo. Since any value can be passed to the `str` function, we can store any value in a list.

We could also write getters and setters for the `cargo` and `next` instance variables:

```
class Node(object):
    ...
    ...
    def getCargo(self):
      return self.cargo
    def getNext(self):
      return self.next
```

```
    def setCargo(self,c):
      self.cargo = c
    def setNext(self,n):
      self.next = n
```

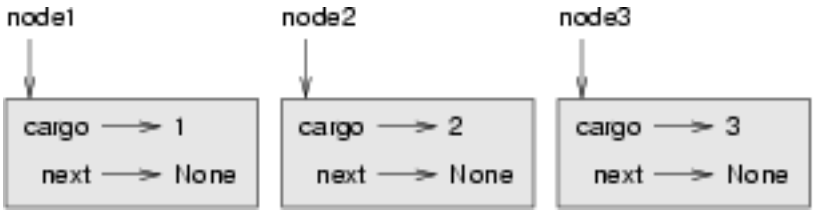
To test the implementation so far, we can create a `Node` and print it:

```
>>> node = Node("test")
>>> print(node)
test
>>> print(node.getNext())
None
```

To make it interesting, we need create more than one node, and *link* them together:

```
>>> node1 = Node(1)
>>> node2 = Node(2)
>>> node3 = Node(3)
```

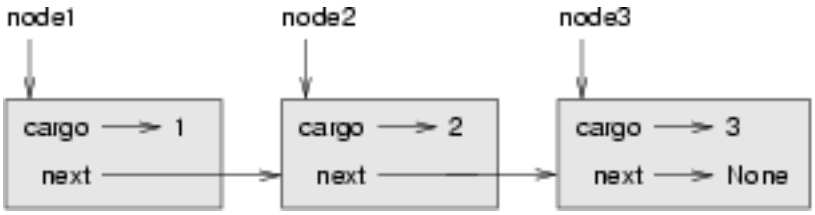
This code creates three nodes, but we don't have a linked-list yet because the nodes are not **linked**. The state diagram looks like this:



To link the nodes, we have to make the first node refer to the second and the second node refer to the third:

```
>>> node1.setNext(node2)
>>> node2.setNext(node3)
```

The reference of the third node is `None`, which indicates that it is the end of the list. Now the state diagram looks like this:



Now you know how to create nodes and link them together. What might be less clear at this point is *why*.

Lists as collections

Lists are useful because they provide a way to assemble multiple objects into a single entity, sometimes called a **collection**. In the example, the first node of the list serves as a reference to the entire list.

To pass the list as a parameter, we only have to pass a reference to the first node. For example, the function `print_list` takes a single node as an argument (`curr`, short for *current*). Assuming that is the head of the list, is *accumulates* the cargo from each node, printing out the contents of the list at the end:

```
def print_list(curr):
  output = ""
  while curr:
    output += str(curr) + " "
    curr = curr.getNext()
  print(output)
```

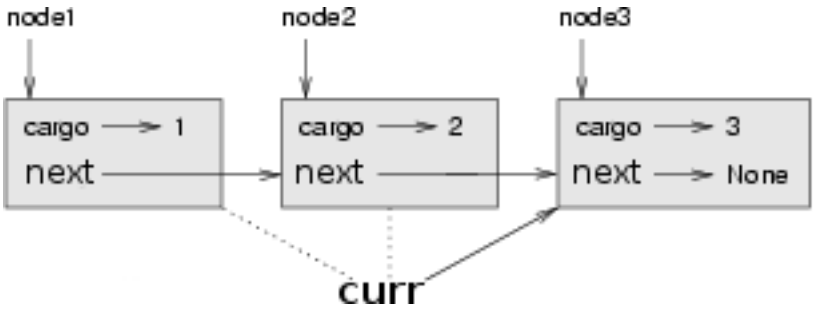
To invoke this method, we pass a reference to the first node:

```
>>> print_list(node1)
1 2 3
```

Inside `print_list` we have a reference to the first node of the list, but there is no variable that refers to the other nodes. We have to use the `next` value from each node to get to the next node.

To traverse a linked list, it is common to use a loop variable like `curr` to refer to each of the nodes in succession.

This diagram shows the full list and the values that `curr` takes on:



Lists and recursion

It is natural to express many list operations using recursive methods. For example, the following is a recursive algorithm for printing a list backwards:

1. Separate the list into two pieces: the first node (called the head); and the rest (called the tail).
2. Print the tail backward.

3. Print the head.

Of course, Step 2, the recursive call, assumes that we have a way of printing a list backward. But if we assume that the recursive call works – the leap of faith – then we can convince ourselves that this algorithm works.

All we need are a base case and a way of proving that for any list, we will eventually get to the base case. Given the recursive definition of a list, a natural base case is the empty list, represented by `None`.

```
def print_backward_helper(curr):
  if curr == None:
    return ""
  else:
    head = curr
    tail = curr.getNext()
    return print_backward_helper(tail) + str(head) + " "

def print_backward(firstnode):
  output = print_backward_helper(firstnode)
  print(output)
```

In the above example, we use a *helper* function, so we can just call `print_backward(node1)` the same as we called `print_list(node1)`. The helper function does the actual recursion and accumulating of the string. The first line of the helper function handles the base case by returning an empty string (and not recurring). The else case (if we are not at the end of the list) splits the list into `head` and `tail`. The final return line adds the head cargo and a space to the reversed rest of the list.

We invoke this method as we invoked `print_list`:

```
>>> print_backward(node1)
3 2 1
```

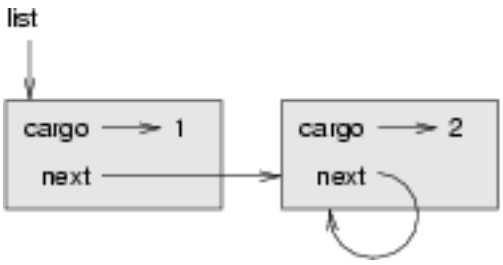
The result is the list contents printed in reverse.

You might wonder why `print_list` and `print_backward` are functions and not methods in the `Node` class. The reason is that we want to use `None` to represent the empty list and it is not legal to invoke a method on `None`. This limitation makes it awkward to write list-manipulating code in a clean object-oriented style.

Can we prove that `print_backward` will always terminate? In other words, will it always reach the base case? In fact, the answer is no. Some lists will make this method crash.

Infinite lists

There is nothing to prevent a node from referring back to an earlier node in the list, including itself. For example, this figure shows a list with two nodes, one of which refers to itself:



If we invoke `print_list` on this list, it will loop forever. If we invoke `print_backward`, it will recurse infinitely. This sort of behavior makes infinite lists difficult to work with.

Nevertheless, they are occasionally useful. For example, we might represent a number as a list of digits and use an infinite list to represent a repeating fraction.

Regardless, it is problematic that we cannot prove that `print_list` and `print_backward` terminate. The best we can do is the hypothetical statement, If the list contains no loops, then these methods will terminate. This sort of claim is called a **precondition**. It imposes a constraint on one of the parameters and describes the behavior of the method if the constraint is satisfied. You will see more examples soon.

The fundamental ambiguity theorem

One part of `print_backward` might have raised an eyebrow:

```
head = curr
tail = curr.getNext()
```

After the first assignment, `head` and `curr` have the same type and the same value. So why did we create a new variable?

The reason is that the two variables play different roles. We think of `head` as a reference to a single node, and we think of `curr` as a reference to the first node of a list. These roles are not part of the program; they are in the mind of the programmer.

In general we can't tell by looking at a program what role a variable plays. This ambiguity can be useful, but it can also make programs difficult to read. We often use variable names like `node` and `curr` to document how we intend to use a variable and sometimes create additional variables to disambiguate.

We could have written `print_backward_helper` without `head` and `tail`, which makes it more concise but possibly less clear:

```
def print_backward_helper(curr):
  if curr == None: return ""
  return print_backward_helper(curr.getNext()) + str(curr) + " "
```

The **fundamental ambiguity theorem** describes the ambiguity that is inherent in a reference to a node: *A variable that refers to a node might treat the node as a single object or as the first in a list of nodes.*

Modifying lists

There are two ways to modify a linked list. Obviously, we can change the cargo of one of the nodes, but the more interesting operations are the ones that add, remove, or reorder the nodes.

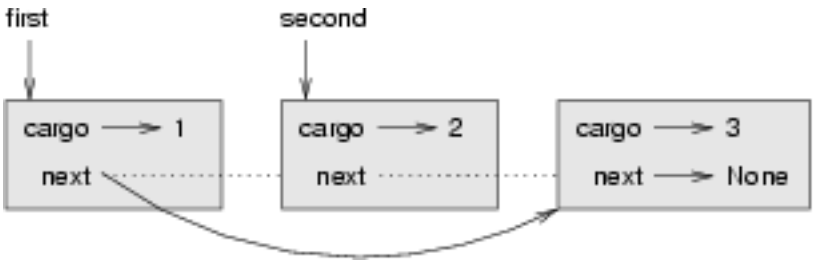
As an example, let's write a method that removes the second node in the list and returns a reference to the removed node:

```
def removeSecond(lst):
    if lst == None: return
    first = lst
    second = lst.getNext()
    # make the first node refer to the third
    first.setNext(second.getNext())
    # separate the second node from the rest of the list
    second.setNext(None)
    return second
```

Again, we are using temporary variables to make the code more readable. Here is how to use this method:

```
>>> print_list(node1)
1 2 3
>>> removed = removeSecond(node1)
>>> print_list(removed)
2
>>> print_list(node1)
1 3
```

This state diagram shows the effect of the operation:



What happens if you invoke this method and pass a list with only one element (a **singleton**)? What happens if you pass the empty list as an argument? Is there a precondition for this method? If so, fix the method to handle a violation of the precondition in a reasonable way.

Wrappers and helpers

It is often useful to divide a list operation into two methods, as we did above with the helper function. As an addition to that example, printing a list backward in the conventional list format `[3, 2, 1]` we can simply add code to the `print_backward` method:

```
def print_backward(firstnode):
  output = print_backward_helper(firstnode)
  lstbkwd = "[" + output[:len(output)-2] + "]"
  print(lstbkwd)
```

Again, it is a good idea to check methods like this to see if they work with special cases like an empty list or a singleton.

When we use this method, `print_backward` acts as a **wrapper**, and it uses `print_backward_helper` as a **helper**.

The `LinkedList` class

There are some subtle problems with the way we have been implementing lists. In a reversal of cause and effect, we'll propose an alternative implementation first and then explain what problems it solves.

First, we'll create a new class called `LinkedList`. Its attributes are an integer that contains the length of the list and a reference to the first node. `LinkedList` objects serve as handles for manipulating lists of `Node` objects:

```
class LinkedList(object):
    def __init__(self):
        self.length = 0
        self.head   = None
```

One nice thing about the `LinkedList` class is that it provides a natural place to put wrapper functions like `print_backward`, which we can make a method of the `LinkedList` class:

```
class LinkedList(object):
    ...
    def print_backward_helper(self,curr):
      if curr == None:
        return ""
      else:
        head = curr
        tail = curr.getNext()
        return self.print_backward_helper(tail) + str(head) + ", "

    def print_backward(self):
      output = self.print_backward_helper(self.head)
      lstbkwd = "[" + output[:len(output)-2] + "]"
      print(lstbkwd)
```

Another benefit of the `LinkedList` class is that it makes it easier to add or remove the first element of a list. For example, `addFirst` is a method for `LinkedLists`; it takes an item of cargo as an argument and puts it at the beginning of the list:

```
class LinkedList(object):
    ...
    def addFirst(self, cargo):
        node = Node(cargo)
        node.setNext(self.head)
        self.head = node
        self.length = self.length + 1
```

As usual, you should check code like this to see if it handles the special cases. For example, what happens if the list is initially empty?

Invariants

Some lists are well formed ; others are not. For example, if a list contains a loop, it will cause many of our methods to crash, so we might want to require that lists contain no loops. Another requirement is that the `length` value in the `LinkedList` object should be equal to the actual number of nodes in the list.

Requirements like these are called **invariants** because, ideally, they should be true of every object all the time. Specifying invariants for objects is a useful programming practice because it makes it easier to prove the correctness of code, check the integrity of data structures, and detect errors.

One thing that is sometimes confusing about invariants is that there are times when they are violated. For example, in the middle of `addFirst`, after we have added the node but before we have incremented `length`, the invariant is violated. This kind of violation is acceptable; in fact, it is often impossible to modify an object without violating an invariant for at least a little while. Normally, we require that every method that violates an invariant must restore the invariant.

If there is any significant stretch of code in which the invariant is violated, it is important for the comments to make that clear, so that no operations are performed that depend on the invariant.

Glossary

embedded reference A reference stored in an attribute of an object.

linked list A data structure that implements a collection using a sequence of linked nodes.

node An element of a list, usually implemented as an object that contains a reference to another object of the same type.

cargo An item of data contained in a node.

link An embedded reference used to link one object to another.

precondition An assertion that must be true in order for a method to work correctly.

fundamental ambiguity theorem A reference to a list node can be treated as a single object or as the first in a list of nodes.

singleton A linked list with a single node.

wrapper A method that acts as a middleman between a caller and a helper method, often making the method easier or less error-prone to invoke.

helper A method that is not invoked directly by a caller but is used by another method to perform part of an operation.

invariant An assertion that should be true of an object at all times (except perhaps while the object is being modified).

Exercises

1. By convention, lists are often printed in brackets with commas between the elements, as in `[1, 2, 3]`. Modify `print_list` so that it generates output in this format.

Stacks

Abstract data types

The data types you have seen so far are all concrete, in the sense that we have completely specified how they are implemented. For example, the `Card` class represents a card using two integers. As we discussed at the time, that is not the only way to represent a card; there are many alternative implementations.

An **abstract data type**, or ADT, specifies a set of operations (or methods) and the semantics of the operations (what they do), but it does not not specify the implementation of the operations. That's what makes it abstract.

Why is that useful?

1. It simplifies the task of specifying an algorithm if you can denote the operations you need without having to think at the same time about how the operations are performed.
2. Since there are usually many ways to implement an ADT, it might be useful to write an algorithm that can be used with any of the possible implementations.
3. Well-known ADTs, such as the Stack ADT in this chapter, are often implemented in standard libraries so they can be written once and used by many programmers.
4. The operations on ADTs provide a common high-level language for specifying and talking about algorithms.

When we talk about ADTs, we often distinguish the code that uses the ADT, called the **client** code, from the code that implements the ADT, called the **provider** code.

The Stack ADT

In this chapter, we will look at one common ADT, the **stack**. A stack is a collection, meaning that it is a data structure that contains multiple elements. Other collections we have seen include dictionaries and lists.

An ADT is defined by the operations that can be performed on it, which is called an **interface**. The interface for a stack consists of these operations:

`__init__` Initialize a new empty stack.

`push` Add a new item to the stack.

`pop` Remove and return an item from the stack. The item that is returned is always the last one that was added.

`is_empty` Check whether the stack is empty.

A stack is sometimes called a last in, first out or LIFO data structure, because the last item added is the first to be removed.

Implementing stacks with Python lists

The list operations that Python provides are similar to the operations that define a stack. The interface isn't exactly what it is supposed to be, but we can write code to translate from the Stack ADT to the built-in operations.

This code is called an **implementation** of the Stack ADT. In general, an implementation is a set of methods that satisfy the syntactic and semantic requirements of an interface.

Here is an implementation of the Stack ADT that uses a Python list:

```
class Stack(object):
    def __init__(self):
        self.items = []

    def push(self, item):
        self.items.append(item)

    def pop(self):
        return self.items.pop()

    def is_empty(self):
        return (self.items == [])
```

A `Stack` object contains an attribute named `items` that is a list of items in the stack. The initialization method sets `items` to the empty list.

To push a new item onto the stack, `push` appends it onto `items`. To pop an item off the stack, `pop` uses the homonymous (*same-named*) list method to remove and return the last item on the list.

Finally, to check if the stack is empty, `is_empty` compares `items` to the empty list.

An implementation like this, in which the methods consist of simple invocations of existing methods, is called a **veneer**. In real life, veneer is a thin coating of good quality wood used in furniture-making to hide lower quality wood underneath. Computer scientists use this metaphor to describe a small piece of code that hides the details of an implementation and provides a simpler, or more standard, interface.

Pushing and popping

A stack is a **generic data structure**, which means that we can add any type of item to it. The following example pushes two integers and a string onto the stack:

```
>>> s = Stack()
>>> s.push(54)
>>> s.push(45)
>>> s.push("+")
```

We can use `is_empty` and `pop` to remove and print all of the items on the stack:

```
output = ""
while not s.is_empty():
    output += "%s " % (s.pop())
print(output)
```

The output is `+ 45 54`. In other words, we just used a stack to print the items backward! Granted, it's not the standard format for printing a list, but by using a stack, it was remarkably easy to do.

You should compare this bit of code to the implementation of `print_backward` in the last chapter. There is a natural parallel between the recursive version of `print_backward` and the stack algorithm here. The difference is that `print_backward` uses the runtime stack to keep track of the nodes while it traverses the list, and then prints them on the way back from the recursion. The stack algorithm does the same thing, except that is use a `Stack` object instead of the runtime stack.

Using a stack to evaluate postfix

In most programming languages, mathematical expressions are written with the operator between the two operands, as in `1 + 2`. This format is called **infix**. An alternative used by some calculators is called **postfix**. In postfix, the operator follows the operands, as in `1 2 +`.

The reason postfix is sometimes useful is that there is a natural way to evaluate a postfix expression using a stack:

1. Starting at the beginning of the expression, get one term (operator or operand) at a time.
   - If the term is an operand, push it on the stack.
   - If the term is an operator, pop two operands off the stack, perform the operation on them, and push the result back on the stack.
2. When you get to the end of the expression, there should be exactly one operand left on the stack. That operand is the result.

Parsing

To implement the previous algorithm, we need to be able to traverse a string and break it into operands and operators. This process is an example of **parsing**, and the results—the individual chunks of the string – are called **tokens**. You might remember these words from Chapter 1.

Python provides a `split` method in both the `string` and `re` (regular expression) modules. The function `string.split` splits a string into a list using a single character as a **delimiter**. For example:

```
>>> import string
>>> string.split("Now is the time"," ")
['Now', 'is', 'the', 'time']
```

In this case, the delimiter is the space character, so the string is split at each space.

The function `re.split` is more powerful, allowing us to provide a regular expression instead of a delimiter. A regular expression is a way of specifying a set of strings. For example, `[A-z]` is the set of all letters and `[0-9]` is the set of all numbers. The `^` operator negates a set, so `[^0-9]` is the set of everything that is not a number, which is exactly the set we want to use to split up postfix expressions:

```
>>> import re
>>> re.split("([^0-9])", "123+456*/")
['123', '+', '456', '*', '', '/', '']
```

Notice that the order of the arguments is different from `string.split`; the delimiter comes before the string.

The resulting list includes the operands `123` and `456` and the operators `*` and `/`. It also includes two empty strings that are inserted after the operands.

Evaluating postfix

To evaluate a postfix expression, we will use the parser from the previous section and the algorithm from the section before that. To keep things simple, we'll start with an evaluator that only implements the operators + and *:

```
def eval_postfix(expr):
    import re
    token_list = re.split("([^0-9])", expr)
    stack = Stack()
    for token in token_list:
        if  token == '' or token == ' ':
            continue
        if  token == '+':
            sum = stack.pop() + stack.pop()
            stack.push(sum)
        elif token == '*':
            product = stack.pop() * stack.pop()
            stack.push(product)
        else:
            stack.push(int(token))
    return stack.pop()
```

The first condition takes care of spaces and empty strings. The next two conditions handle operators. We assume, for now, that anything else must be an operand. Of course, it would be better to check for erroneous input and report an error message, but we'll get to that later.

Let's test it by evaluating the postfix form of `(56+47)*2`:

```
>>> print(eval_postfix("56 47 + 2 \*"))
206
```

That's close enough.

Clients and providers

One of the fundamental goals of an ADT is to separate the interests of the provider, who writes the code that implements the ADT, and the client, who uses the ADT. The provider only has to worry about whether the implementation is correct – in accord with the specification of the ADT – and not how it will be used.

Conversely, the client *assumes* that the implementation of the ADT is correct and doesn't worry about the details. When you are using one of Python's built-in types, you have the luxury of thinking exclusively as a client.

Of course, when you implement an ADT, you also have to write client code to test it. In that case, you play both roles, which can be confusing. You should make some effort to keep track of which role you are playing at any moment.

Glossary

abstract data type (ADT) A data type (usually a collection of objects) that is defined by a set of operations but that can be implemented in a variety of ways.

interface The set of operations that define an ADT.

implementation Code that satisfies the syntactic and semantic requirements of an interface.

client A program (or the person who wrote it) that uses an ADT.

provider The code (or the person who wrote it) that implements an ADT.

veneer A class definition that implements an ADT with method definitions that are invocations of other methods, sometimes with simple transformations. The veneer does no significant work, but it improves or standardizes the interface seen by the client.

generic data structure A kind of data structure that can contain data of any type.

infix A way of writing mathematical expressions with the operators between the operands.

postfix A way of writing mathematical expressions with the operators after the operands.

parse To read a string of characters or tokens and analyze its grammatical structure.

token A set of characters that are treated as a unit for purposes of parsing, such as the words in a natural language.

delimiter A character that is used to separate tokens, such as punctuation in a natural language.

Exercises

1. Apply the postfix algorithm to the expression `1 2 + 3 *`.This example demonstrates one of the advantages of postfix—there is no need to use parentheses to control the order of operations. To get the same result in infix, we would have to write `(1 + 2) * 3`.
2. Write a postfix expression that is equivalent to `1 + 2 * 3`.

Queues

This chapter presents two ADTs: the Queue and the Priority Queue. In real life, a **queue** is a line of customers waiting for service of some kind. In most cases, the first customer in line is the next customer to be served. There are exceptions, though. At airports, customers whose flights are leaving soon are sometimes taken from the middle of the queue. At supermarkets, a polite customer might let someone with only a few items go first.

The rule that determines who goes next is called the **queueing policy**. The simplest queueing policy is called **FIFO**, for first- in-first-out. The most general queueing policy is **priority queueing**,

in which each customer is assigned a priority and the customer with the highest priority goes first, regardless of the order of arrival. We say this is the most general policy because the priority can be based on anything: what time a flight leaves; how many groceries the customer has; or how important the customer is. Of course, not all queueing policies are fair, but fairness is in the eye of the beholder.

The Queue ADT and the Priority Queue ADT have the same set of operations. The difference is in the semantics of the operations: a queue uses the FIFO policy; and a priority queue (as the name suggests) uses the priority queueing policy.

The Queue ADT

The Queue ADT is defined by the following operations:

__init__ Initialize a new empty queue.

insert Add a new item to the queue.

remove Remove and return an item from the queue. The item that is returned is the first one that was added.

is_empty Check whether the queue is empty.

Linked Queue

The first implementation of the Queue ADT we will look at is called a **linked queue** because it is made up of linked `Node` objects. Here is the class definition:

```
class Queue(object):
    def __init__(self):
        self.length = 0
        self.head = None

    def is_empty(self):
        return (self.length == 0)

    def insert(self, cargo):
        node = Node(cargo)
        node.next = None
        if self.head == None:
            # if list is empty the new node goes first
            self.head = node
        else:
            # find the last node in the list
            last = self.head
            while last.next: last = last.next
            # append the new node
```

```
            last.next = node
        self.length = self.length + 1

    def remove(self):
        cargo = self.head.cargo
        self.head = self.head.next
        self.length = self.length - 1
        return cargo
```

The methods `is_empty` and `remove` are identical to the `LinkedList` methods `is_empty` and `remove_first`. The `insert` method is new and a bit more complicated.

We want to insert new items at the end of the list. If the queue is empty, we just set `head` to refer to the new node.

Otherwise, we traverse the list to the last node and tack the new node on the end. We can identify the last node because its `next` attribute is `None`.

There are two invariants for a properly formed `Queue` object. The value of `length` should be the number of nodes in the queue, and the last node should have `next` equal to `None`. Convince yourself that this method preserves both invariants.

Performance characteristics

Normally when we invoke a method, we are not concerned with the details of its implementation. But there is one detail we might want to know—the performance characteristics of the method. How long does it take, and how does the run time change as the number of items in the collection increases?

First look at `remove`. There are no loops or function calls here, suggesting that the runtime of this method is the same every time. Such a method is called a **constant-time** operation. In reality, the method might be slightly faster when the list is empty since it skips the body of the conditional, but that difference is not significant.

The performance of `insert` is very different. In the general case, we have to traverse the list to find the last element.

This traversal takes time proportional to the length of the list. Since the runtime is a linear function of the length, this method is called **linear time**. Compared to constant time, that's very bad.

Improved Linked Queue

We would like an implementation of the Queue ADT that can perform all operations in constant time. One way to do that is to modify the Queue class so that it maintains a reference to both the first and the last node, as shown in the figure:

The `ImprovedQueue` implementation looks like this:

```
class ImprovedQueue(object):
    def __init__(self):
        self.length = 0
        self.head   = None
        self.last   = None

    def is_empty(self):
        return (self.length == 0)
```

So far, the only change is the attribute `last`. It is used in `insert` and `remove` methods:

```
class ImprovedQueue(object):
    ...
    def insert(self, cargo):
        node = Node(cargo)
        node.next = None
        if self.length == 0:
            # if list is empty, the new node is head and last
            self.head = self.last = node
        else:
            # find the last node
            last = self.last
            # append the new node
            last.next = node
            self.last = node
        self.length = self.length + 1
```

Since `last` keeps track of the last node, we don't have to search for it. As a result, this method is constant time.

There is a price to pay for that speed. We have to add a special case to `remove` to set `last` to `None` when the last node is removed:

```
class ImprovedQueue(object):
    ...
    def remove(self):
        cargo = self.head.cargo
        self.head = self.head.next
        self.length = self.length - 1
        if self.length == 0:
            self.last = None
        return cargo
```

This implementation is more complicated than the Linked Queue implementation, and it is more difficult to demonstrate that it is correct. The advantage is that we have achieved the goal – both `insert` and `remove` are constant-time operations.

Priority queue

The Priority Queue ADT has the same interface as the Queue ADT, but different semantics. Again, the interface is:

__init__ Initialize a new empty queue.

insert Add a new item to the queue.

remove Remove and return an item from the queue. The item that is returned is the one with the highest priority.

is_empty Check whether the queue is empty.

The semantic difference is that the item that is removed from the queue is not necessarily the first one that was added. Rather, it is the item in the queue that has the highest priority. What the priorities are and how they compare to each other are not specified by the Priority Queue implementation. It depends on which items are in the queue.

For example, if the items in the queue have names, we might choose them in alphabetical order. If they are bowling scores, we might go from highest to lowest, but if they are golf scores, we would go from lowest to highest. As long as we can compare the items in the queue, we can find and remove the one with the highest priority.

This implementation of Priority Queue has as an attribute a Python list that contains the items in the queue.

```
class PriorityQueue(object):
    def __init__(self):
        self.items = []

    def is_empty(self):
        return self.items == []

    def insert(self, item):
        self.items.append(item)
```

The initialization method, `is_empty`, and `insert` are all veneers on list operations. The only interesting method is `remove`:

```
class PriorityQueue(object):
    ...
    def remove(self):
        maxi = 0
        for i in range(1, len(self.items)):
            if self.items[i] > self.items[maxi]: maxi = i
        item = self.items[maxi]
        self.items[maxi:maxi+1] = []
        return item
```

At the beginning of each iteration, `maxi` holds the index of the biggest item (highest priority) we have seen *so far*. Each time through the loop, the program compares the `i`-eth item to the champion. If the new item is bigger, the value of `maxi` if set to `i`.

When the `for` statement completes, `maxi` is the index of the biggest item. This item is removed from the list and returned.

Let's test the implementation:

```
>>> q = PriorityQueue()
>>> q.insert(11)
>>> q.insert(12)
>>> q.insert(14)
>>> q.insert(13)
>>> while not q.is_empty(): print(q.remove())
14
13
12
11
```

If the queue contains simple numbers or strings, they are removed in numerical or alphabetical order, from highest to lowest. Python can find the biggest integer or string because it can compare them using the built-in comparison operators.

If the queue contains an object type, it has to provide a `__cmp__` method. When `remove` uses the > operator to compare items, it invokes the `__cmp__` for one of the items and passes the other as a parameter. As long as the `__cmp__` method works correctly, the Priority Queue will work.

The `Golfer` class

As an example of an object with an unusual definition of priority, let's implement a class called `Golfer` that keeps track of the names and scores of golfers. As usual, we start by defining `__init__` and `__str__`:

```
class Golfer(object):
    def __init__(self, name, score):
        self.name = name
        self.score= score

    def __str__(self):
        return "%-16s: %d" % (self.name, self.score)
```

`__str__` uses the format operator to put the names and scores in neat columns.

Next we define a version of `__cmp__` where the lowest score gets highest priority. As always, `__cmp__` returns 1 if `self` is greater than `other`, -1 if `self` is less than other, and 0 if they are equal.

```
class Golfer(object):
    ...
    def __cmp__(self, other):
        if self.score < other.score: return  1    # less is more
        if self.score > other.score: return -1
        return 0
```

Now we are ready to test the priority queue with the `Golfer` class:

```
>>> tiger = Golfer("Tiger Woods",     61)
>>> phil  = Golfer("Phil Mickelson", 72)
>>> hal   = Golfer("Hal Sutton",      69)
>>>
>>> pq = PriorityQueue()
>>> pq.insert(tiger)
>>> pq.insert(phil)
>>> pq.insert(hal)
>>> while not pq.is_empty(): print(pq.remove())
   Tiger Woods    : 61
   Hal Sutton     : 69
   Phil Mickelson : 72
```

Glossary

queue An ordered set of objects waiting for a service of some kind.

Queue An ADT that performs the operations one might perform on a queue.

queueing policy The rules that determine which member of a queue is removed next.

FIFO First In, First Out, a queueing policy in which the first member to arrive is the first to be removed.

priority queue A queueing policy in which each member has a priority determined by external factors. The member with the highest priority is the first to be removed.

Priority Queue An ADT that defines the operations one might perform on a priority queue.

linked queue An implementation of a queue using a linked list.

constant time An operation whose runtime does not depend on the size of the data structure.

linear time An operation whose runtime is a linear function of the size of the data structure.

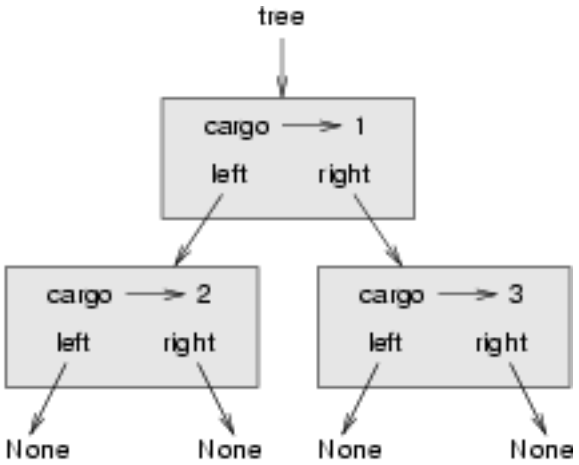
Exercises

1. Write an implementation of the Queue ADT using a Python list. Compare the performance of this implementation to the `ImprovedQueue` for a range of queue lengths. #. Write

an implementation of the Priority Queue ADT using a linked list. You should keep the list sorted so that removal is a constant time operation. Compare the performance of this implementation with the Python list implementation.

Trees

Like linked lists, trees are made up of nodes. A common kind of tree is a **binary tree**, in which each node contains a reference to two other nodes (possibly `None`). These references are referred to as the left and right subtrees. Like list nodes, tree nodes also contain cargo. A state diagram for a tree looks like this:



To avoid cluttering up the picture, we often omit the `None`s.

The top of the tree (the node `tree` refers to) is called the **root**. In keeping with the tree metaphor, the other nodes are called branches and the nodes at the tips with null references are called **leaves**. It may seem odd that we draw the picture with the root at the top and the leaves at the bottom, but that is not the strangest thing.

To make things worse, computer scientists mix in another metaphor – the family tree. The top node is sometimes called a **parent** and the nodes it refers to are its **children**. Nodes with the same parent are called **siblings**.

Finally, there is a geometric vocabulary for talking about trees. We already mentioned left and right, but there is also up (toward the parent/root) and down (toward the children/leaves). Also, all of the nodes that are the same distance from the root comprise a **level** of the tree.

We probably don't need three metaphors for talking about trees, but there they are.

Like linked lists, trees are recursive data structures because they are defined recursively. A tree is either:

1. the empty tree, represented by `None`, or
2. a node that contains an object reference (cargo) and two tree references.

Building trees

The process of assembling a tree is similar to the process of assembling a linked list. Each constructor invocation builds a single node.

```
class Tree(object):
    def __init__(self, cargo, left=None, right=None):
        self.cargo = cargo
        self.left  = left
        self.right = right

    def __str__(self):
        return str(self.cargo)
```

The `cargo` can be any type, but the `left` and `right` parameters should be tree nodes. `left` and `right` are optional; the default value is `None`.

To print a node, we just print the cargo.

One way to build a tree is from the bottom up. Allocate the child nodes first:

```
left = Tree(2)
right = Tree(3)
```

Then create the parent node and link it to the children:

```
tree = Tree(1, left, right);
```

We can write this code more concisely by nesting constructor invocations:

```
>>> tree = Tree(1, Tree(2), Tree(3))
```

Either way, the result is the tree at the beginning of the chapter.

Traversing trees

Any time you see a new data structure, your first question should be, How do I traverse it? The most natural way to traverse a tree is recursively. For example, if the tree contains integers as cargo, this function returns their sum:

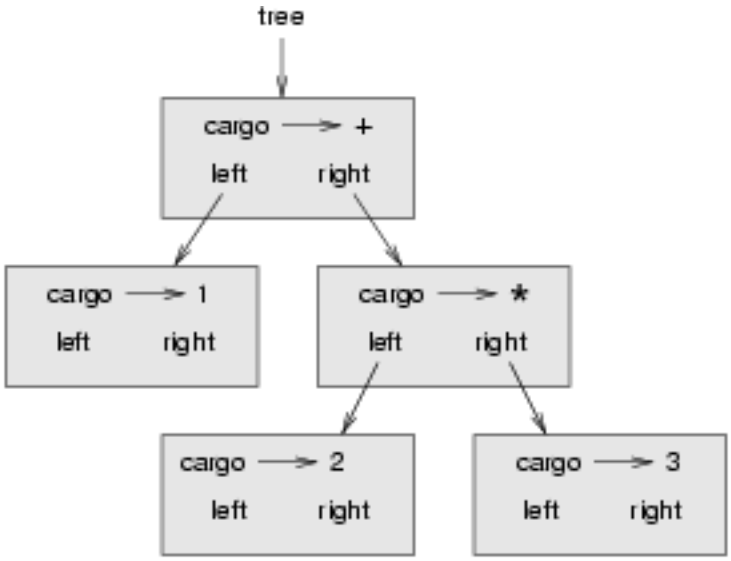
```
def total(tree):
    if tree == None: return 0
    return total(tree.left) + total(tree.right) + tree.cargo
```

The base case is the empty tree, which contains no cargo, so the sum is 0. The recursive step makes two recursive calls to find the sum of the child trees. When the recursive calls complete, we add the cargo of the parent and return the total.

Expression trees

A tree is a natural way to represent the structure of an expression. Unlike other notations, it can represent the computation unambiguously. For example, the infix expression `1 + 2 * 3` is ambiguous unless we know that the multiplication happens before the addition.

This expression tree represents the same computation:



The nodes of an expression tree can be operands like `1` and `2` or operators like `+` and `*`. Operands are leaf nodes; operator nodes contain references to their operands. (All of these operators are **binary**, meaning they have exactly two operands.)

We can build this tree like this:

```
>>> tree = Tree('+', Tree(1), Tree('*', Tree(2), Tree(3)))
```

Looking at the figure, there is no question what the order of operations is; the multiplication happens first in order to compute the second operand of the addition.

Expression trees have many uses. The example in this chapter uses trees to translate expressions to postfix, prefix, and infix. Similar trees are used inside compilers to parse, optimize, and translate programs.

Tree traversal

We can traverse an expression tree and print the contents like this:

```
def print_tree(tree):
    if tree == None: return
    sys.stdout.write("%s " % (tree.cargo))
    print_tree(tree.left)
    print_tree(tree.right)
```

In other words, to print a tree, first print the contents of the root, then print the entire left subtree, and then print the entire right subtree. This way of traversing a tree is called a **preorder**, because the contents of the root appear *before* the contents of the children. For the previous example, the output is:

```
>>> tree = Tree('+', Tree(1), Tree('*', Tree(2), Tree(3)))
>>> print_tree(tree)
+ 1 * 2 3
```

This format is different from both postfix and infix; it is another notation called **prefix**, in which the operators appear before their operands.

You might suspect that if you traverse the tree in a different order, you will get the expression in a different notation. For example, if you print the subtrees first and then the root node, you get:

```
def print_tree_postorder(tree):
    if tree == None: return
    print_tree_postorder(tree.left)
    print_tree_postorder(tree.right)
    sys.stdout.write("%s " % (tree.cargo))
```

The result, `1 2 3 * +`, is in postfix! This order of traversal is called **postorder**.

Finally, to traverse a tree **inorder**, you print the left tree, then the root, and then the right tree:

```
def print_tree_inorder(tree):
    if tree == None: return
    print_tree_inorder(tree.left)
    sys.stdout.write("%s " % (tree.cargo))
    print_tree_inorder(tree.right)
```

The result is `1 + 2 * 3`, which is the expression in infix.

To be fair, we should point out that we have omitted an important complication. Sometimes when we write an expression in infix, we have to use parentheses to preserve the order of operations. So an inorder traversal is not quite sufficient to generate an infix expression.

Nevertheless, with a few improvements, the expression tree and the three recursive traversals provide a general way to translate expressions from one format to another.

If we do an inorder traversal and keep track of what level in the tree we are on, we can generate a graphical representation of a tree:

```
def print_tree_indented(tree, level=0):
    if tree == None: return
    print_tree_indented(tree.right, level+1)
    print('   ' * level + str(tree.cargo))
    print_tree_indented(tree.left, level+1)
```

The parameter `level` keeps track of where we are in the tree. By default, it is initially 0. Each time we make a recursive call, we pass `level+1` because the child's level is always one greater

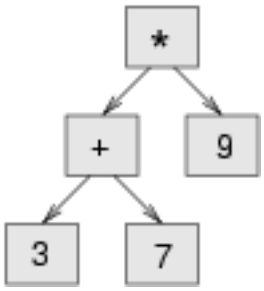
than the parent's. Each item is indented by two spaces per level. The result for the example tree is:

```
>>> print_tree_indented(tree)
    3
  *
    2
+
  1
```

If you look at the output sideways, you see a simplified version of the original figure.

Building an expression tree

In this section, we parse infix expressions and build the corresponding expression trees. For example, the expression `(3 + 7) * 9` yields the following tree:



Notice that we have simplified the diagram by leaving out the names of the attributes.

The parser we will write handles expressions that include numbers, parentheses, and the operators `+` and `*`. We assume that the input string has already been tokenized into a Python list (producing this list is left as an exercise). The token list for `(3 + 7) * 9` is:

```
['(', 3, '+', 7, ')', '*', 9, 'end']
```

The `end` token is useful for preventing the parser from reading past the end of the list.

The first function we'll write is `get_token`, which takes a token list and an expected token as parameters. It compares the expected token to the first token on the list: if they match, it removes the token from the list and returns `True`; otherwise, it returns `False`:

```
def get_token(token_list, expected):
    if token_list[0] == expected:
        del token_list[0]
        return True
    else:
        return False
```

Since `token_list` refers to a mutable object, the changes made here are visible to any other variable that refers to the same object.

The next function, `get_number`, handles operands. If the next token in `token_list` is a number, `get_number` removes it and returns a leaf node containing the number; otherwise, it returns `None`.

```
def get_number(token_list):
    x = token_list[0]
    if type(x) != type(0): return None
    del token_list[0]
    return Tree (x, None, None)
```

Before continuing, we should test `get_number` in isolation. We assign a list of numbers to `token_list`, extract the first, print the result, and print what remains of the token list:

```
>>> token_list = [9, 11, 'end']
>>> x = get_number(token_list)
>>> print_tree_postorder(x)
9
>>> print(token_list)
[11, 'end']
```

The next method we need is `get_product`, which builds an expression tree for products. A simple product has two numbers as operands, like `3 * 7`.

Here is a version of `get_product` that handles simple products.

```
def get_product(token_list):
    a = get_number(token_list)
    if get_token(token_list, '*'):
        b = get_number(token_list)
        return Tree ('*', a, b)
    else:
        return a
```

Assuming that `get_number` succeeds and returns a singleton tree, we assign the first operand to `a`. If the next character is `*`, we get the second number and build an expression tree with `a`, `b`, and the operator.

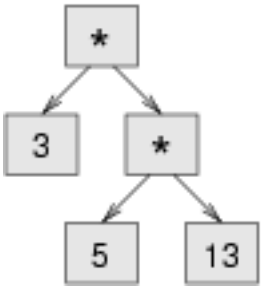
If the next character is anything else, then we just return the leaf node with `a`. Here are two examples:

```
>>> token_list = [9, '*', 11, 'end']
>>> tree = get_product(token_list)
>>> print_tree_postorder(tree)
9 11 *
```

```
>>> token_list = [9, '+', 11, 'end']
>>> tree = get_product(token_list)
>>> print_tree_postorder(tree)
9
```

The second example implies that we consider a single operand to be a kind of product. This definition of product is counterintuitive, but it turns out to be useful.

Now we have to deal with compound products, like like `3 * 5 * 13`. We treat this expression as a product of products, namely `3 * (5 * 13)`. The resulting tree is:



With a small change in `get_product`, we can handle an arbitrarily long product:

```
def get_product(token_list):
    a = get_number(token_list)
    if get_token(token_list, '*'):
        b = get_product(token_list)        # this line changed
        return Tree ('*', a, b)
    else:
        return a
```

In other words, a product can be either a singleton or a tree with * at the root, a number on the left, and a product on the right. This kind of recursive definition should be starting to feel familiar.

Let's test the new version with a compound product:

```
>>> token_list = [2, '*', 3, '*', 5 , '*', 7, 'end']
>>> tree = get_product(token_list)
>>> print_tree_postorder(tree)
2 3 5 7 * * *
```

Next we will add the ability to parse sums. Again, we use a slightly counterintuitive definition of sum. For us, a sum can be a tree with + at the root, a product on the left, and a sum on the right. Or, a sum can be just a product.

If you are willing to play along with this definition, it has a nice property: we can represent any expression (without parentheses) as a sum of products. This property is the basis of our parsing algorithm.

`get_sum` tries to build a tree with a product on the left and a sum on the right. But if it doesn't find a +, it just builds a product.

```
def get_sum(token_list):
    a = get_product(token_list)
    if get_token(token_list, '+'):
        b = get_sum(token_list)
        return Tree ('+', a, b)
```

```
    else:
        return a
```

Let's test it with `9 * 11 + 5 * 7`:

```
>>> token_list = [9, '*', 11, '+', 5, '*', 7, 'end']
>>> tree = get_sum(token_list)
>>> print_tree_postorder(tree)
9 11 * 5 7 * +
```

We are almost done, but we still have to handle parentheses. Anywhere in an expression where there can be a number, there can also be an entire sum enclosed in parentheses. We just need to modify `get_number` to handle **subexpressions**:

```
def get_number(token_list):
    if get_token(token_list, '('):
        x = get_sum(token_list)          # get the subexpression
        get_token(token_list, ')')       # remove the closing parenthesis
        return x
    else:
        x = token_list[0]
        if type(x) != type(0): return None
        token_list[0:1] = []
        return Tree (x, None, None)
```

Let's test this code with `9 * (11 + 5) * 7`:

```
>>> token_list = [9, '*', '(', 11, '+', 5, ')', '*', 7, 'end']
>>> tree = get_sum(token_list)
>>> print_tree_postorder(tree)
9 11 5 + 7 * *
```

The parser handled the parentheses correctly; the addition happens before the multiplication.

In the final version of the program, it would be a good idea to give `get_number` a name more descriptive of its new role.

Handling errors

Throughout the parser, we've been assuming that expressions are well-formed. For example, when we reach the end of a subexpression, we assume that the next character is a close parenthesis. If there is an error and the next character is something else, we should deal with it.

```
def get_number(token_list):
    if get_token(token_list, '('):
        x = get_sum(token_list)
        if not get_token(token_list, ')'):
            raise 'BadExpressionError', 'missing parenthesis'
```

```
        return x
    else:
        # the rest of the function omitted
```

The `raise` statement creates an exception; in this case we create a new kind of exception, called a `BadExpressionError`. If the function that called `get_number`, or one of the other functions in the traceback, handles the exception, then the program can continue. Otherwise, Python will print an error message and quit.

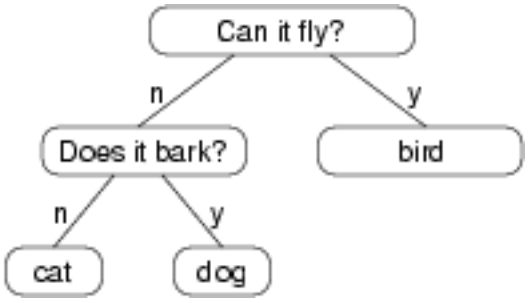
The animal tree

In this section, we develop a small program that uses a tree to represent a knowledge base.

The program interacts with the user to create a tree of questions and animal names. Here is a sample run:

```
Are you thinking of an animal? y
Is it a bird? n
What is the animals name? dog
What question would distinguish a dog from a bird? Can it fly
If the animal were dog the answer would be? n

Are you thinking of an animal? y
Can it fly? n
Is it a dog? n
What is the animals name? cat
What question would distinguish a cat from a dog? Does it bark
If the animal were cat the answer would be? n

Are you thinking of an animal? y
Can it fly? n
Does it bark? y
Is it a dog? y
I rule!

Are you thinking of an animal? n
```

Here is the tree this dialog builds:



At the beginning of each round, the program starts at the top of the tree and asks the first question. Depending on the answer, it moves to the left or right child and continues until it gets to a leaf node. At that point, it makes a guess. If the guess is not correct, it asks the user for the name of the new animal and a question that distinguishes the (bad) guess from the new animal. Then it adds a node to the tree with the new question and the new animal.

Here is the code:

```
def yes(ques):
    ans = raw_input(ques).lower()
    return ans[0] == 'y'

def animal():
    # start with a singleton
    root = Tree("bird")

    # loop until the user quits
    while True:
        print
        if not yes("Are you thinking of an animal? "): break

        # walk the tree
        tree = root
        while tree.left != None:
            prompt = tree.cargo + "? "
            if yes(prompt):
                tree = tree.right
            else:
                tree = tree.left

        # make a guess
        guess = tree.cargo
        prompt = "Is it a " + guess + "? "
        if yes(prompt):
            print("I rule!")
            continue

        # get new information
        prompt  = "What is the animal's name? "
        animal  = raw_input(prompt)
        prompt  = "What question would distinguish a %s from a %s? "
        question = raw_input(prompt % (animal, guess))

        # add new information to the tree
        tree.cargo = question
        prompt = "If the animal were %s the answer would be? "
        if yes(prompt % animal):
            tree.left = Tree(guess)
```

```
            tree.right = Tree(animal)
        else:
            tree.left = Tree(animal)
            tree.right = Tree(guess)
```

The function `yes` is a helper; it prints a prompt and then takes input from the user. If the response begins with *y* or *Y*, the function returns `True`.

The condition of the outer loop of `animal` is `True`, which means it will continue until the `break` statement executes, if the user is not thinking of an animal.

The inner `while` loop walks the tree from top to bottom, guided by the user's responses.

When a new node is added to the tree, the new question replaces the cargo, and the two children are the new animal and the original cargo.

One shortcoming of the program is that when it exits, it forgets everything you carefully taught it! Fixing this problem is left as an exercise.

Glossary

binary tree A tree in which each node refers to zero, one, or two dependent nodes.

root The topmost node in a tree, with no parent.

leaf A bottom-most node in a tree, with no children.

parent The node that refers to a given node.

child One of the nodes referred to by a node.

siblings Nodes that share a common parent.

level The set of nodes equidistant from the root.

binary operator An operator that takes two operands.

subexpression An expression in parentheses that acts as a single operand in a larger expression.

preorder A way to traverse a tree, visiting each node before its children.

prefix notation A way of writing a mathematical expression with each operator appearing before its operands.

postorder A way to traverse a tree, visiting the children of each node before the node itself.

inorder A way to traverse a tree, visiting the left subtree, then the root, and then the right subtree.

Exercises

1. Modify `print_tree_inorder` so that it puts parentheses around every operator and pair of operands. Is the output correct and unambiguous? Are the parentheses always necessary?
2. Write a function that takes an expression string and returns a token list.
3. Find other places in the expression tree functions where errors can occur and add appropriate `raise` statements. Test your code with improperly formed expressions.
4. Think of various ways you might save the animal knowledge tree in a file. Implement the one you think is easiest.

Debugging

Different kinds of errors can occur in a program, and it is useful to distinguish among them in order to track them down more quickly:

1. Syntax errors are produced by Python when it is translating the source code into byte code. They usually indicate that there is something wrong with the syntax of the program. Example: Omitting the colon at the end of a `def` statement yields the somewhat redundant message `SyntaxError: invalid syntax`.
2. Runtime errors are produced by the runtime system if something goes wrong while the program is running. Most runtime error messages include information about where the error occurred and what functions were executing. Example: An infinite recursion eventually causes a runtime error of maximum recursion depth exceeded.
3. Semantic errors are problems with a program that compiles and runs but doesn't do the right thing. Example: An expression may not be evaluated in the order you expect, yielding an unexpected result.

The first step in debugging is to figure out which kind of error you are dealing with. Although the following sections are organized by error type, some techniques are applicable in more than one situation.

Syntax errors

Syntax errors are usually easy to fix once you figure out what they are. Unfortunately, the error messages are often not helpful. The most common messages are `SyntaxError: invalid syntax` and `SyntaxError: invalid token`, neither of which is very informative.

On the other hand, the message does tell you where in the program the problem occurred. Actually, it tells you where Python noticed a problem, which is not necessarily where the error is. Sometimes the error is prior to the location of the error message, often on the preceding line.

If you are building the program incrementally, you should have a good idea about where the error is. It will be in the last line you added.

If you are copying code from a book, start by comparing your code to the book's code very carefully. Check every character. At the same time, remember that the book might be wrong, so if you see something that looks like a syntax error, it might be.

Here are some ways to avoid the most common syntax errors:

1. Make sure you are not using a Python keyword for a variable name.
2. Check that you have a colon at the end of the header of every compound statement, including `for`, `while`, `if`, and `def` statements.
3. Check that indentation is consistent. You may indent with either spaces or tabs but it's best not to mix them. Each level should be nested the same amount.
4. Make sure that any strings in the code have matching quotation marks.
5. If you have multiline strings with triple quotes (single or double), make sure you have terminated the string properly. An unterminated string may cause an `invalid token` error at the end of your program, or it may treat the following part of the program as a string until it comes to the next string. In the second case, it might not produce an error message at all!
6. An unclosed bracket – (, {, or [– makes Python continue with the next line as part of the current statement. Generally, an error occurs almost immediately in the next line.
7. Check for the classic = instead of == inside a conditional.

If nothing works, move on to the next section...

I can't get my program to run no matter what I do.

If the compiler says there is an error and you don't see it, that might be because you and the compiler are not looking at the same code. Check your programming environment to make sure that the program you are editing is the one Python is trying to run. If you are not sure, try putting an obvious and deliberate syntax error at the beginning of the program. Now run (or import) it again. If the compiler doesn't find the new error, there is probably something wrong with the way your environment is set up.

If this happens, one approach is to start again with a new program like Hello, World!, and make sure you can get a known program to run. Then gradually add the pieces of the new program to the working one.

Runtime errors

Once your program is syntactically correct, Python can import it and at least start running it. What could possibly go wrong?

My program does absolutely nothing.

This problem is most common when your file consists of functions and classes but does not actually invoke anything to start execution. This may be intentional if you only plan to import this module to supply classes and functions.

If it is not intentional, make sure that you are invoking a function to start execution, or execute one from the interactive prompt. Also see the Flow of Execution section below.

My program hangs.

If a program stops and seems to be doing nothing, we say it is hanging. Often that means that it is caught in an infinite loop or an infinite recursion.

1. If there is a particular loop that you suspect is the problem, add a `print` statement immediately before the loop that says entering the loop and another immediately after that says exiting the loop.
2. Run the program. If you get the first message and not the second, you've got an infinite loop. Go to the Infinite Loop section below.
3. Most of the time, an infinite recursion will cause the program to run for a while and then produce a RuntimeError: Maximum recursion depth exceeded error. If that happens, go to the Infinite Recursion section below.
4. If you are not getting this error but you suspect there is a problem with a recursive method or function, you can still use the techniques in the Infinite Recursion section.
5. If neither of those steps works, start testing other loops and other recursive functions and methods.
6. If that doesn't work, then it is possible that you don't understand the flow of execution in your program. Go to the Flow of Execution section below.

Infinite Loop

If you think you have an infinite loop and you think you know what loop is causing the problem, add a `print` statement at the end of the loop that prints the values of the variables in the condition and the value of the condition.

For example:

```
while x > 0 and y < 0:
    # do something to x
    # do something to y

    print  "x: ", x
```

```
    print  "y: ", y
    print  "condition: ", (x > 0 and y < 0)
```

Now when you run the program, you will see three lines of output for each time through the loop. The last time through the loop, the condition should be `false`. If the loop keeps going, you will be able to see the values of `x` and `y`, and you might figure out why they are not being updated correctly.

Infinite Recursion

Most of the time, an infinite recursion will cause the program to run for a while and then produce a `Maximum recursion depth exceeded` error.

If you suspect that a function or method is causing an infinite recursion, start by checking to make sure that there is a base case. In other words, there should be some condition that will cause the function or method to return without making a recursive invocation. If not, then you need to rethink the algorithm and identify a base case.

If there is a base case but the program doesn't seem to be reaching it, add a `print` statement at the beginning of the function or method that prints the parameters. Now when you run the program, you will see a few lines of output every time the function or method is invoked, and you will see the parameters. If the parameters are not moving toward the base case, you will get some ideas about why not.

Flow of Execution

If you are not sure how the flow of execution is moving through your program, add `print` statements to the beginning of each function with a message like entering function `foo`, where `foo` is the name of the function.

Now when you run the program, it will print a trace of each function as it is invoked.

When I run the program I get an exception.

If something goes wrong during runtime, Python prints a message that includes the name of the exception, the line of the program where the problem occurred, and a traceback.

The traceback identifies the function that is currently running, and then the function that invoked it, and then the function that invoked *that*, and so on. In other words, it traces the path of function invocations that got you to where you are. It also includes the line number in your file where each of these calls occurs.

The first step is to examine the place in the program where the error occurred and see if you can figure out what happened. These are some of the most common runtime errors:

NameError You are trying to use a variable that doesn't exist in the current environment. Remember that local variables are local. You cannot refer to them from outside the function where they are defined.

TypeError There are several possible causes:

1. You are trying to use a value improperly. Example: indexing a string, list, or tuple with something other than an integer.
2. There is a mismatch between the items in a format string and the items passed for conversion. This can happen if either the number of items does not match or an invalid conversion is called for.
3. You are passing the wrong number of arguments to a function or method. For methods, look at the method definition and check that the first parameter is `self`. Then look at the method invocation; make sure you are invoking the method on an object with the right type and providing the other arguments correctly.

KeyError You are trying to access an element of a dictionary using a key value that the dictionary does not contain.

AttributeError You are trying to access an attribute or method that does not exist.

IndexError The index you are using to access a list, string, or tuple is greater than its length minus one. Immediately before the site of the error, add a `print` statement to display the value of the index and the length of the array. Is the array the right size? Is the index the right value?

I added so many `print` statements I get inundated with output.

One of the problems with using `print` statements for debugging is that you can end up buried in output. There are two ways to proceed: simplify the output or simplify the program.

To simplify the output, you can remove or comment out `print` statements that aren't helping, or combine them, or format the output so it is easier to understand.

To simplify the program, there are several things you can do. First, scale down the problem the program is working on. For example, if you are sorting an array, sort a *small* array. If the program takes input from the user, give it the simplest input that causes the problem.

Second, clean up the program. Remove dead code and reorganize the program to make it as easy to read as possible. For example, if you suspect that the problem is in a deeply nested part of the program, try rewriting that part with simpler structure. If you suspect a large function, try splitting it into smaller functions and testing them separately.

Often the process of finding the minimal test case leads you to the bug. If you find that a program works in one situation but not in another, that gives you a clue about what is going on.

Similarly, rewriting a piece of code can help you find subtle bugs. If you make a change that you think doesn't affect the program, and it does, that can tip you off.

Semantic errors

In some ways, semantic errors are the hardest to debug, because the compiler and the runtime system provide no information about what is wrong. Only you know what the program is supposed to do, and only you know that it isn't doing it.

The first step is to make a connection between the program text and the behavior you are seeing. You need a hypothesis about what the program is actually doing. One of the things that makes that hard is that computers run so fast.

You will often wish that you could slow the program down to human speed, and with some debuggers you can. But the time it takes to insert a few well-placed `print` statements is often short compared to setting up the debugger, inserting and removing breakpoints, and walking the program to where the error is occurring.

My program doesn't work.

You should ask yourself these questions:

1. Is there something the program was supposed to do but which doesn't seem to be happening? Find the section of the code that performs that function and make sure it is executing when you think it should.
2. Is something happening that shouldn't? Find code in your program that performs that function and see if it is executing when it shouldn't.
3. Is a section of code producing an effect that is not what you expected? Make sure that you understand the code in question, especially if it involves invocations to functions or methods in other Python modules. Read the documentation for the functions you invoke. Try them out by writing simple test cases and checking the results.

In order to program, you need to have a mental model of how programs work. If you write a program that doesn't do what you expect, very often the problem is not in the program; it's in your mental model.

The best way to correct your mental model is to break the program into its components (usually the functions and methods) and test each component independently. Once you find the discrepancy between your model and reality, you can solve the problem.

Of course, you should be building and testing components as you develop the program. If you encounter a problem, there should be only a small amount of new code that is not known to be correct.

I've got a big hairy expression and it doesn't do what I expect.

Writing complex expressions is fine as long as they are readable, but they can be hard to debug. It is often a good idea to break a complex expression into a series of assignments to temporary

variables.

For example:

```
self.hands[i].addCard (self.hands[self.findNeighbor(i)].popCard())
```

This can be rewritten as:

```
neighbor = self.findNeighbor (i)
pickedCard = self.hands[neighbor].popCard()
self.hands[i].addCard (pickedCard)
```

The explicit version is easier to read because the variable names provide additional documentation, and it is easier to debug because you can check the types of the intermediate variables and display their values.

Another problem that can occur with big expressions is that the order of evaluation may not be what you expect. For example, if you are translating the expression `x/2pi` into Python, you might write:

```
y = x / 2 * math.pi;
```

That is not correct because multiplication and division have the same precedence and are evaluated from left to right. So this expression computes `(x/2)pi`.

A good way to debug expressions is to add parentheses to make the order of evaluation explicit:

```
y = x / (2 * math.pi);
```

Whenever you are not sure of the order of evaluation, use parentheses. Not only will the program be correct (in the sense of doing what you intended), it will also be more readable for other people who haven't memorized the rules of precedence.

I've got a function or method that doesn't return what I expect.

If you have a `return` statement with a complex expression, you don't have a chance to print the `return` value before returning. Again, you can use a temporary variable. For example, instead of:

```
return self.hands[i].removeMatches()
```

you could write:

```
count = self.hands[i].removeMatches()
return count
```

Now you have the opportunity to display the value of `count` before returning.

I'm really, really stuck and I need help.

First, try getting away from the computer for a few minutes. Computers emit waves that affect the brain, causing these effects:

1. Frustration and/or rage.
2. Superstitious beliefs (the computer hates me) and magical thinking (the program only works when I wear my hat backward).
3. Random-walk programming (the attempt to program by writing every possible program and choosing the one that does the right thing).

If you find yourself suffering from any of these symptoms, get up and go for a walk. When you are calm, think about the program. What is it doing? What are some possible causes of that behavior? When was the last time you had a working program, and what did you do next?

Sometimes it just takes time to find a bug. We often find bugs when we are away from the computer and let our minds wander. Some of the best places to find bugs are trains, showers, and in bed, just before you fall asleep.

No, I really need help.

It happens. Even the best programmers occasionally get stuck. Sometimes you work on a program so long that you can't see the error. A fresh pair of eyes is just the thing.

Before you bring someone else in, make sure you have exhausted the techniques described here. Your program should be as simple as possible, and you should be working on the smallest input that causes the error. You should have `print` statements in the appropriate places (and the output they produce should be comprehensible). You should understand the problem well enough to describe it concisely.

When you bring someone in to help, be sure to give them the information they need:

1. If there is an error message, what is it and what part of the program does it indicate?
2. What was the last thing you did before this error occurred? What were the last lines of code that you wrote, or what is the new test case that fails?
3. What have you tried so far, and what have you learned?

When you find the bug, take a second to think about what you could have done to find it faster. Next time you see something similar, you will be able to find the bug more quickly.

Remember, the goal is not just to make the program work. The goal is to learn how to make the program work.

Graphics API for Students of Python: GASP

Introduction

Describe gasp here...

Coordinates

(0, 0) is at the bottom left of the window. The window is 640 pixels by 480, by default. (You can make it a different size if you want to.) Coordinates are given in units of one pixel.

All functions that take coordinates take them as a tuple (x, y).

```
Circle((300, 200), 10)      # :) This is good
Circle(300, 200, 10)        # :( This is bad
```

Colors

To access the color module GASP has to offer. Call `color.*` where `*` is the color you wish to call. For example: ‘ color.BLACK ‘ This is the color black. Check out the gasp color refrence chart to see all of the availble color options.

The Essentials

```
from gasp import *

begin_graphics()

...                          # all of your code

end_graphics()
```

These are the essentials. ‘ from gasp import * ‘ imports the gasp module, `begin_graphics()` starts the graphics window, and `end_graphics()` quits the graphics window and ends the program. It’s dead simple, but also dead necessary.

Graphics Functions

begin_graphics()

```
begin_graphics(width=800, height=600, title="My Game", background=color.YELLOW)
```

This creates a graphics window with the dimensions 800x600, a title of My Game , and a background color of yellow. With no arguments you get a white 640x480 graphics window titled Gasp .

width The width of the window in pixels.

height The windows height in pixels.

title A string that will be the title of the window.

background It is the background of the graphics window. It can either be a color or an image

end_graphics()

```
endgraphics()
```

Ends a graphics window.

clear_screen()

```
clear_screen()
```

Clears everything off of the graphics window. It looks like a new graphcs window as if you just called begin_graphics().

remove_from_screen()

```
remove_from_screen(obj)
```

removes those objects from the screen

obj A screen object of a list of screen_objects you would like to remove from the screen

Screen Objects

The objects that you will be displayed in your graphics window. You can manipulate these objects using the screen object methods

Plot

```
Plot(pos, color=color.BLACK, size=1)
```

It puts a dot on the screen.

pos The coordinate on the screen that you wish to plot.

color The color you wish the dot to be.

size An integer that determinse the size the of the dot

Line

```
Line(start, end, color=color.BLACK)
```

Creates a line on the screen.

start The starting coordinate of the line.

end The coordinate at which the line will end.

color The color of the line

Box

```
Box(center, width, height, filled=False, color=color.BLACK, thickness=1)
```

This creates a Box on the screen

center A coorinate where the center of your box will be.

width The width in pixels of the box.

height The height of the box in pixels.

filled A boolean value that determines if your box will be filled

color The color of your box.

thickness The thickness in pixels of your box's lines.

Polygon

```
Polygon(points, filled=False, color=color.BLACK, thickness=1)
```

Creates a polygon on the screen

points A list of coorinates that is each point on the polygon. The must be more than two items in the list

filled A boolean value. If it is False the polygon will not be filled. Else, the polygon will not be filled

color The color of the polygon's lines

thickness An integer that determines the thickness of the lines.

Circle

```
Circle(center, radius, filled=False, color=color.BLACK, thickness=1)
```

Draws a circle, its `center` is a set of coordinates, and the `radius` is in pixels. It defaults to not being filled and the color black.

center The circle's center coordinate.

width An integer that is the radius of the circle

filled A boolean value that determines if your circle will be filled

color The color of your circle.

thickness The thickness in pixels of the circles lines.

Arc

```
Arc(center, radius, start_angle, end_angle, filled=False, color=color.BLACK, thickn
```

Creates an arc on the screen.

center A coordinate that is the center of the arc.

radius An integer that is the distance between the center and the outer edge of the arc.

start_angle The start angle in degrees of the arc

end_angle The end angle in degrees of your arc

filled A boolean value that if True it fills the arc

color The color the arc

thickness The thickness in pixels of the arc

Oval

```
Oval(center, width, height, filled=False, color=color.BLACK, thickness=1)
```

Puts an oval on the screen wherever you want.

center The center coordinate of the Oval

width The width in pixels of the oval

height The height of the oval in pixels

filled A boolean value determining if the oval will be filles or not.

color The oval's color

thickness The thickness of the ovals lines

Image

```
Image(file_path, center, width=None, height=None):
```

Loads an image onto the screen. If you only pass a width and not a height it automatically scales the height to fit the width you passed it. It behaves likewise when you pass just a height.

file_path The path to the image

center The center coordinates of the image

width The width of the image in pixels. If width equals None then it defaults to the image file's width

height The height of the image in pixels. If no height is passed it defaults to the image file's height

Screen Object Methods

The methods that manipulates screen objects

move_to()

```
move_to(obj, pos)
```

Move a screen object to a pos

obj A screen object you wish to move.

pos The coordinate on the screen that the object will move to

move_by()

```
move_by(obj, dx, dy)
```

Move a screen object relative to it's position

obj The screen object you wish to move

dx How much the object will move in the 'x' direction. Positive or negative.

dy How much the object will move in the 'y' direction. A pixel value.

rotate_to()

```
rotate_to(obj, angle)
```

Rotate an object to an angle

obj The screen object that will be rotated

angle The angle in degrees that the object will be rotated to

rotate_by()

```
rotate_by(obj, angle)
```

Rotate an object a certain degree.

obj The screen object you wish to rotate

angle The degree that the object will be rotate. Can be positive or negative.

Text

Text()

```
Text(text, pos, color=color.BLACK, size=12)
```

Puts text on the screen

text A string of the text that will be displayed

pos The center coordinate of the text

color The color of the text

size The font size

Mouse

mouse_position()

```
mouse_position()
```

Returns the current mouse coordinate

mouse_buttons()

```
mouse_buttons()
```

returns a dictionary of the buttons state. There is a 'left', 'middle', and 'right' key.

Keyboard

keys_pressed()

```
keys_pressed()
```

returns a list of all of the keys pressed at that moment.

Gasp Tools

screen_shot

```
screen_shot(filename)
```

Saves a screenshot of the current graphics screen to a png file.

filename The file path relative to the current directory that the image will be written to.

Configuring Ubuntu for Python Development

> *Note:* the following instructions assume that you are connected to the Internet and that you have both the `main` and `universe` package repositories enabled. All unix shell commands are assumed to be running from your home directory ($HOME). Finally, any command that begins with `sudo` assums that you have administrative rights on your machine. If you do not — please ask your system administrator about installing the software you need.

What follows are instructions for setting up an Ubuntu 9.10 (Karmic) home environment for use with this book. I use Ubuntu GNU/Linux for both development and testing of the book, so it is the only system about which I can personally answer setup and configuration questions.

In the spirit of software freedom and open collaboration, please contact me if you would like to maintain a similar appendix for your own favorite system. I'd be more than happy to link to it or put it on the Open Book Project site, provided you agree to answer user feedback concerning it.

Thanks!

Jeffrey Elkner
Governor's Career and Technical Academy in Arlington
Arlington, Virginia

Vim

Vim can be used very effectively for Python development, but Ubuntu only comes with the *vim-tiny* package installed by default, so it doesn't support color syntax highlighting or auto-indenting.

To use Vim, do the following:

1. From the unix command prompt, run:

```
$ sudo apt-get install vim-gnome
```

2. Create a file in your home directory named *.vimrc* that contains the following:

```
syntax enable
filetype indent on
set et
set sw=4
set smarttab
map <f2> :w\|!python %
```

When you edit a file with a *.py* extension, you should now have color systax highlighting and auto indenting. Pressing the key should run your program, and bring you back to the editor when the program completes.

To learn to use vim, run the following command at a unix command prompt:

```
$ vimtutor
```

GASP

Several of the case studies use GASP (Graphics API for Students for Python), which is the only additional library needed to use this book.

To install GASP on Ubuntu 9.04 (Jaunty) or later, run the following command at a unix command prompt:

```
$ sudo apt-get install python-gasp
```

or use the synaptic package manager.

Getting GASP from Launchpad

To install the latest version of GASP into your home directory, run the following commands at a unix command prompt:

```
$ sudo apt-get install bzr
$ bzr branch lp:gasp-code
```

$HOME environment

The following creates a useful environment in your home directory for adding your own Python libraries and executable scripts:

1. From the command prompt in your home directory, create *bin* and *lib/python* subdirectories by running the following commands:

```
$ mkdir bin lib
$ mkdir lib/python
```

2. Add the following lines to the bottom of your *.bashrc* in your home directory:

```
PYTHONPATH=$HOME/lib/python
EDITOR=vim

export PYTHONPATH EDITOR
```

   This will set your prefered editor to Vim, add your own *lib/python* subdirectory for your Python libraries to your Python path, and add your own *bin* directory as a place to put executable scripts. You need to logout and log back in before your local *bin* directory will be in your search path.

Making a python script executable and runnable from anywhere

On unix systems, Python scripts can be made *executable* using the following process:

1. Add this line as the first line in the script:

```
#!/usr/bin/env python
```

2. At the unix command prompt, type the following to make *myscript.py* executable:

```
$ chmod +x myscript.py
```

3. Move *myscript.py* into your *bin* directory, and it will be runnable from anywhere.

Customizing and Contributing to the Book

> *Note:* the following instructions assume that you are connected to the Internet and that you have both the `main` and `universe` package repositories enabled. All unix shell commands are assumed to be running from your home directory ($HOME). Finally, any command that begins with `sudo` assums that you have administrative rights on your machine. If you do not — please ask your system administrator about installing the software you need.

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That freedom lacks meaning, however, if you the tools needed to make a custom version or to contribute corrections and additions are not within your reach. This appendix attempts to put those tools in your hands.

Thanks!

Jeffrey Elkner
Governor's Career and Technical Academy in Arlington
Arlington, Virginia

Getting the Source

This book is marked up in ReStructuredText using a document generation system called Sphinx.

The source code is located on the Launchpad website at http://bazaar.launchpad.net/~thinkcspy/thinkcspy/english2e/files.

The easiest way to get the source code on an Ubuntu 9.10 computer is:

1. run `sudo apt-get install bzr` on your system to install bzr.
2. run `bzr branch lp:thinkcspy`.

The last command above will download the book source from Launchpad into a directory named `thinkcspy` which contains the Sphinx source and configuration information needed to build the book.

Making the HTML Version

To generate the html version of the book:

1. run `sudo apt-get install python-sphinx` to install the Sphinx documentation system.

2. `cd thinkcspy` - change into the `thinkcspy` directory containing the book source.
3. `make html`.

The last command will run sphinx and create a directory named `build` containing the html verion of the text.

> *Note*: Sphinx supports building other output types as well, such as PDF. This requires that LaTeX be present on your system. Since I only personally use the html version, I will not attempt to document that process here.

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